Radiation and Wormholes in AdS/CFT

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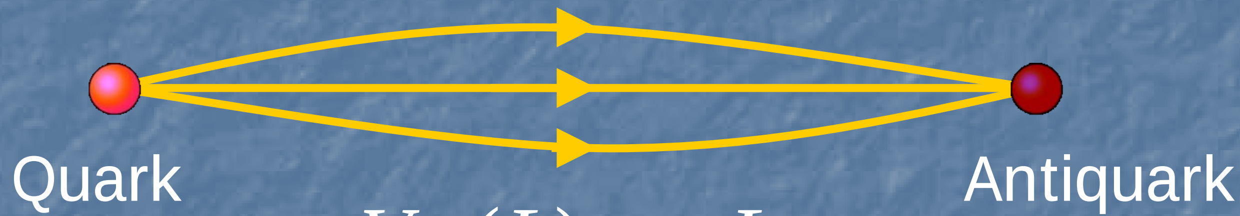
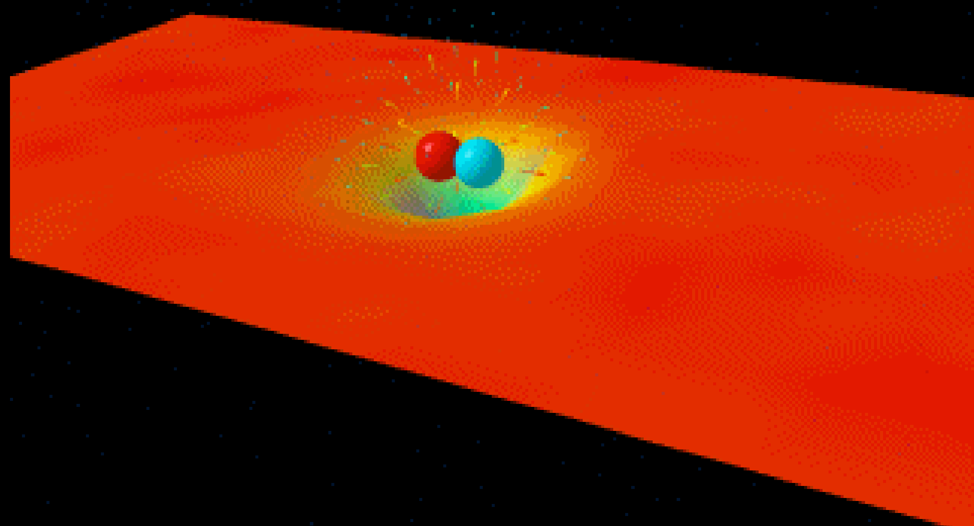
The association between **quarks** and **strings** goes back to

`**QCD string**' = gluonic **flux tube**

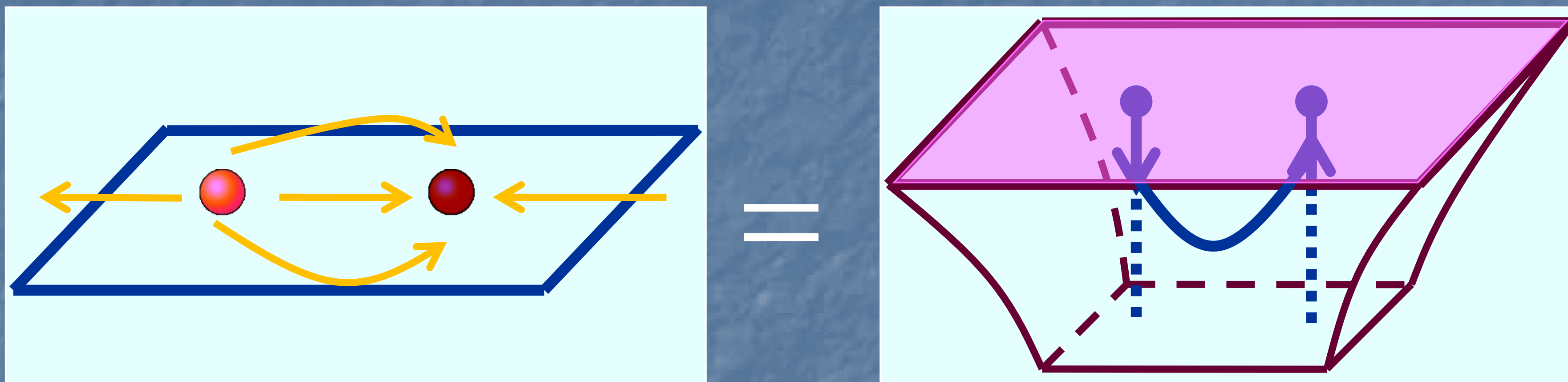


$$V_{q\bar{q}}(L) \simeq \sigma L$$

These flux tubes are visible in numerical calculations on discretized spacetime– lattice QCD

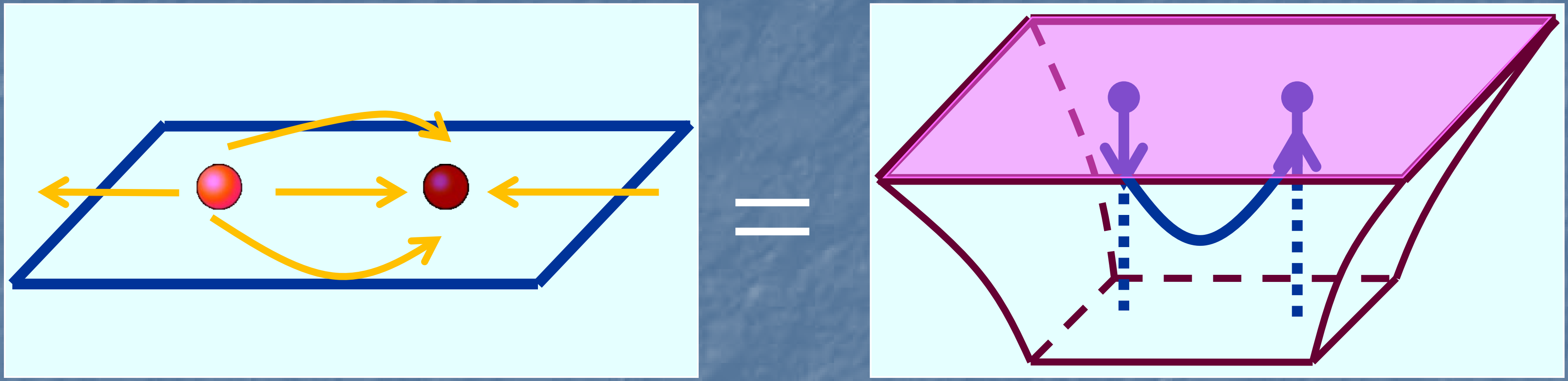


http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/



In the **gauge/gravity duality**, we also get a 'QCD string', with a few surprises:

- The string is not fat but **infinitesimally thin**
- It accomplishes this by **living in higher dimensions**
- It exists even in **nonconfining theories**



Over the years, **strings** have been found to efficiently encode various interesting (expected & unexpected) properties of **quarks in strongly coupled gauge theories**, including:

q-qbar potential, Wilson loops, scattering amplitudes, gluonic field profile, radiation rate, drag/damping, fluctuations, entanglement,...

Plan

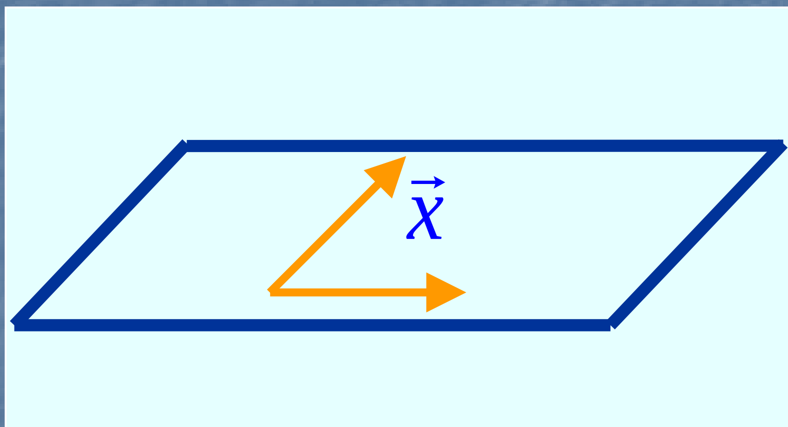
- Intro: Energy of Moving Quark
- Gluonic Radiation vs. `Near’ field
- ER=EPR and Wormholes on the Worldsheet

Based on:

C.A. Agón, AG, J.F. Pedraza	1402.5961
M. Chernicoff, AG, J.F. Pedraza	1308.3695
J.A. García, AG, E.J. Pulido	1210.4175
C.A. Agón, AG, B.O. Larios	1206.5005
M. Chernicoff, AG, J.F. Pedraza	1106.4059
M. Chernicoff, J.A. García, AG	0906.1592
	0903.2047

Intro

Basic Setup



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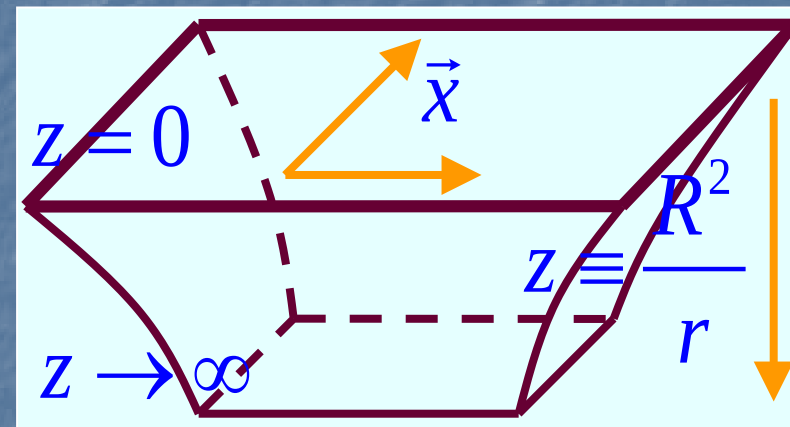


Conformal field theory (CFT) in d dimensions $=$ Gravity theory on $d+1$ asymptot. anti-de Sitter (AdS)

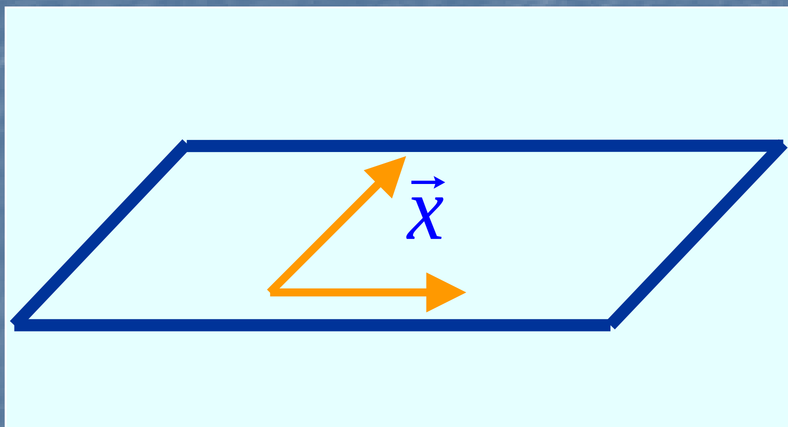
Vacuum $\longleftrightarrow$ Pure AdS

$$ds^2_{\mathrm{CFT}} = -dt^2 + d\vec{x}^2$$

$$ds^2_{\mathrm{grav}} = \left(\frac{R}{z}\right)^2 \left(-dt^2 + d\vec{x}^2 + dz^2\right)$$

Poincaré coordinates

Basic Setup



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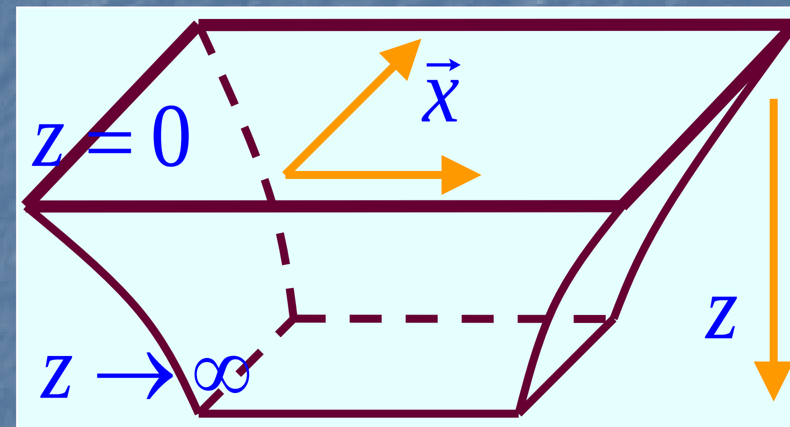
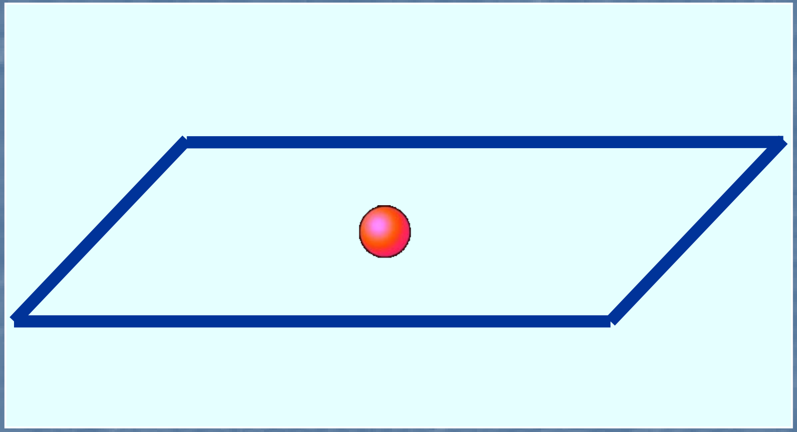


$SU(N_c)\ \mathcal{N}=4$ Yang-Mills (MSYM) in 3+1 dim $=$ IIB String Thy. on aAdS_5

$$\lambda \equiv g_{\mathrm{YM}}^2 N_c = \frac{R^4}{l_s^4}$$

Work with $N_c \gg 1, \quad \lambda \gg 1$ (easy regime on gravity side)

Quarks



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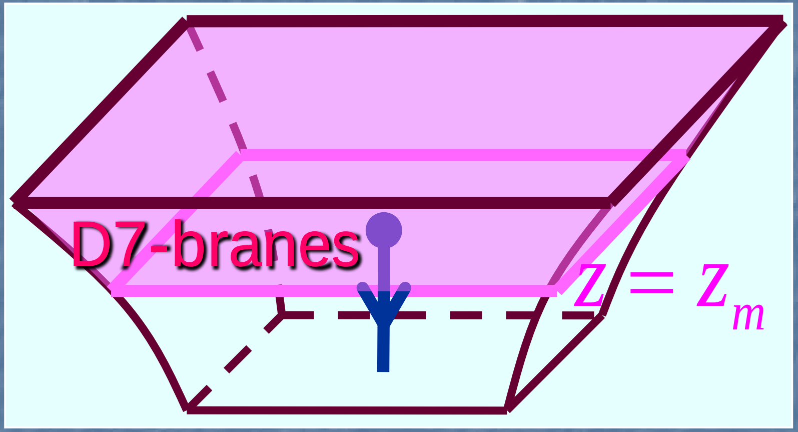
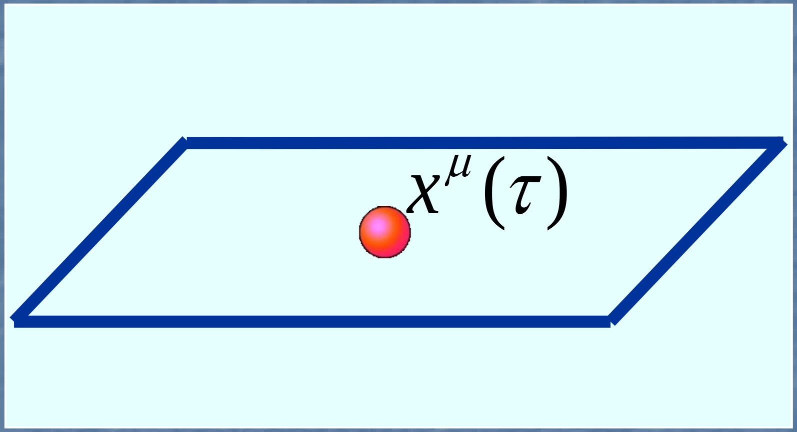


Quark with mass m $=$ String w/endpoint at $z_m \geq 0$

[Karch, Katz]

$$m = \frac{\sqrt{\lambda}}{2\pi z_m}$$

Quarks



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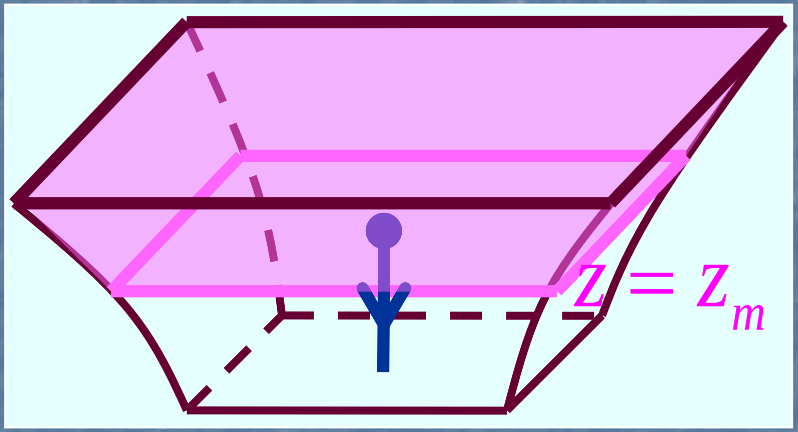


Quark with $m = \sqrt{\lambda} / 2\pi z_m$ $=$ String w/endpoint at $z_m \geq 0$

[Karch, Katz]

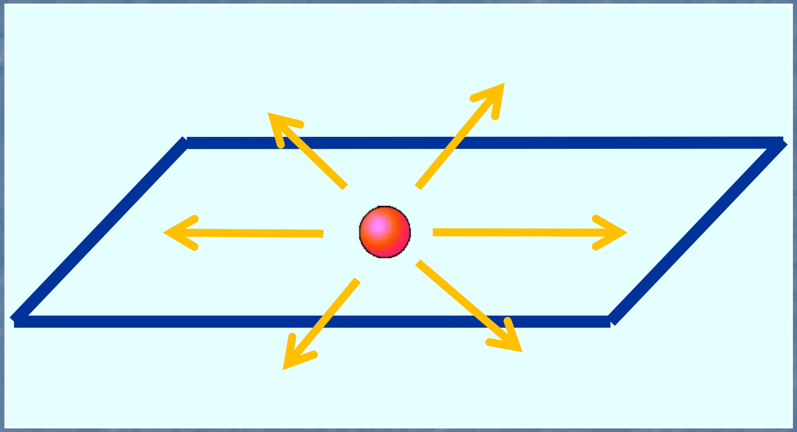
We are coupling `2nd-quantized' gluonic (+etc.) fields to **`1st-quantized' quark**

$$\int Dx(\tau) DA_{\mu}(x') \exp\left(iS[A(x'), x(\tau)]\right)$$

saddle point

carry out in full (=AdS)

Quarks



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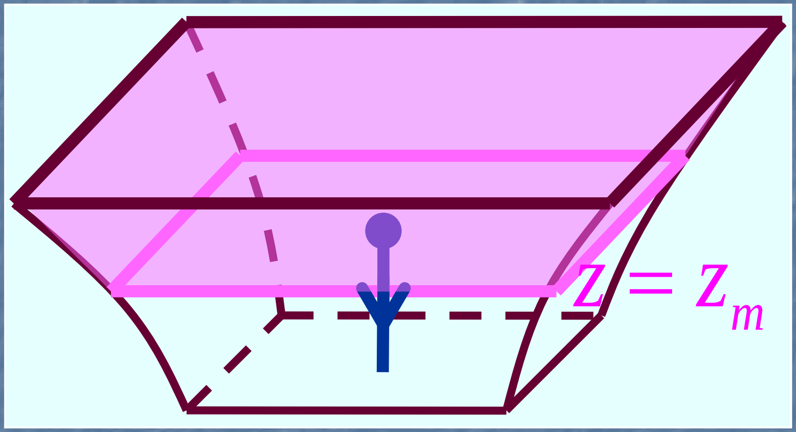


Quark with $m = \sqrt{\lambda}/2\pi z_m$ = String w/endpoint at $z_m > 0$

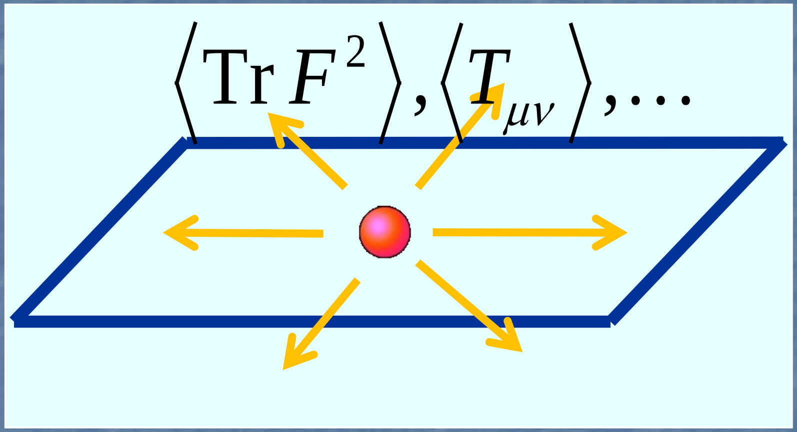
[Karch, Katz]

We are coupling `2nd-quantized' gluonic (+etc.) fields to **`1st-quantized' quark**

Endpoint $\leftrightarrow$ Quark , String $\leftrightarrow$ Gluonic (+etc.) field

both intrinsic/'near' and radiative/`far'

Static Quark



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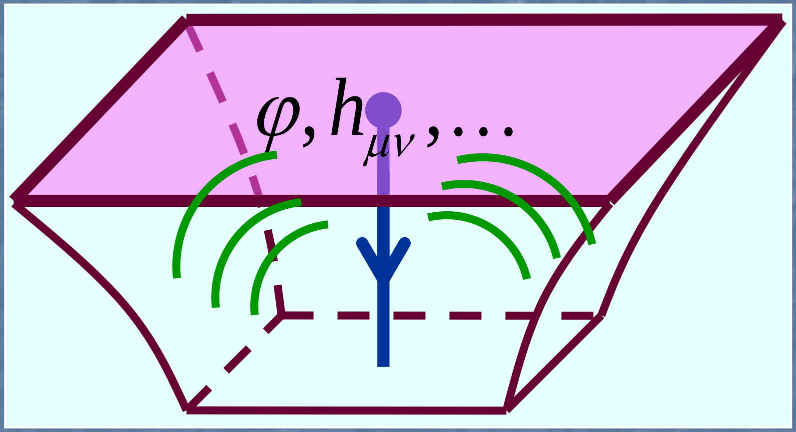


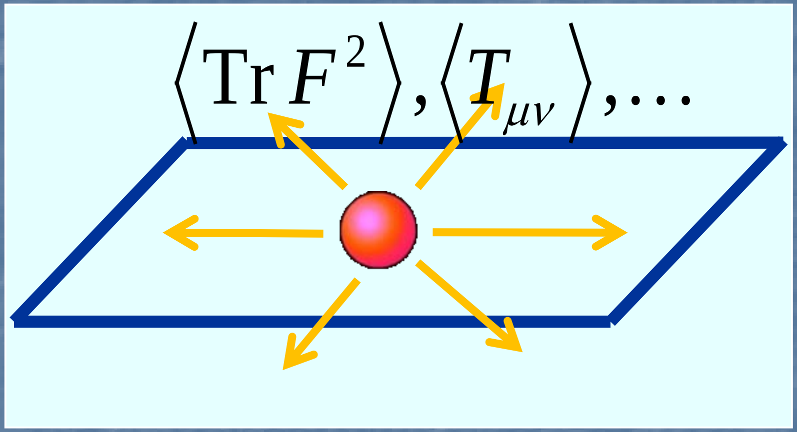
Infinitely heavy Quark = String w/endpoint at $z = 0$

E.g., $\left\langle \mathrm{Tr}\left[F^2(\vec{x},t) + \ldots \right] \right\rangle_q = \dfrac{\sqrt{\lambda}}{32\pi^2 |\vec{x}|^4}$

[Danielsson,Kruczenski,Keski-Vakkuri]

Coulombic profile (as expected by conformal invariance)

Static Quark



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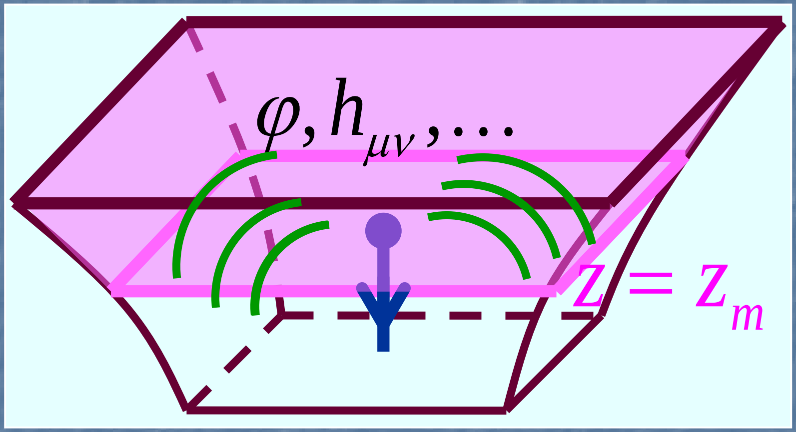
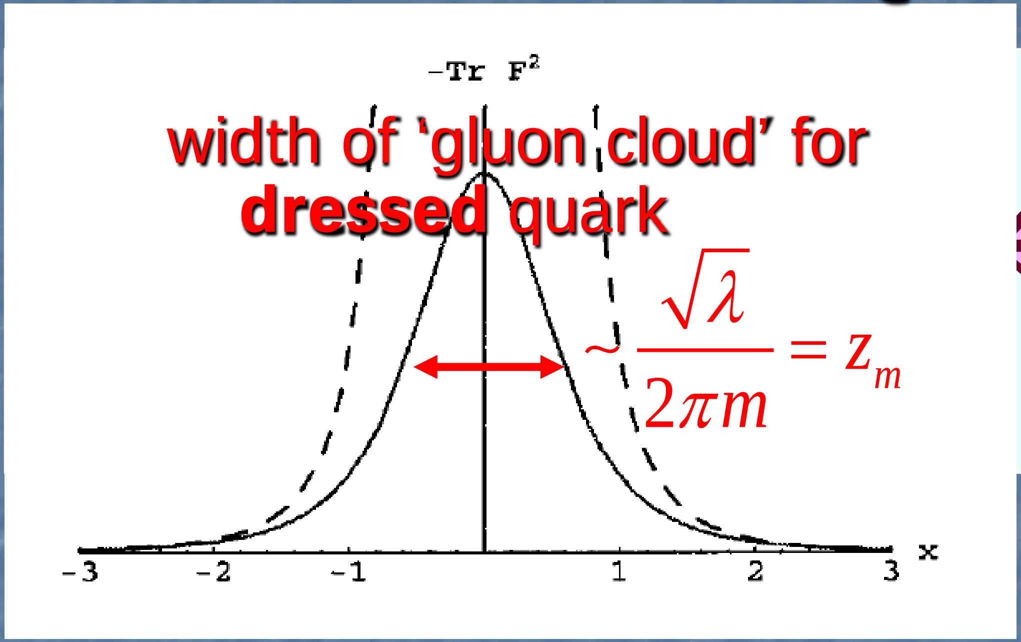


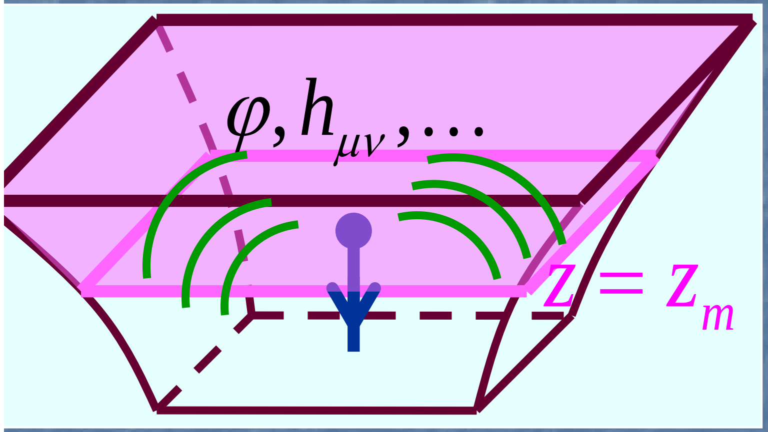
Quark with $m = \sqrt{\lambda} / 2\pi z_m$ $=$ String w/endpoint at $z_m > 0$

$$\left\langle \mathrm{Tr}\left[F^2(\vec{x},t) \right] + \ldots \right\rangle_q = \frac{\sqrt{\lambda}}{32\pi^2 |\vec{x}|^4} \left[1 - \frac{1 + \frac{5}{2}\left(\frac{2\pi m |\vec{x}|}{\sqrt{\lambda}} \right)^2}{\left(1 + \left(\frac{2\pi m |\vec{x}|}{\sqrt{\lambda}} \right)^2 \right)^{5/2}} \right]$$

[Hovdebo,Kruczenski,Mateos,Myers,Winters]

Fat Quark





$$\left\langle \mathrm{Tr}\left[F^2(\vec{x},t)\right]+\ldots\right\rangle_q = \frac{\sqrt{\lambda}}{32\pi^2|\vec{x}|^4}\left[1-\frac{1+\frac{5}{2}\left(\frac{2\pi m|\vec{x}|}{\sqrt{\lambda}}\right)^2}{\left(1+\left(\frac{2\pi m|\vec{x}|}{\sqrt{\lambda}}\right)^2\right)^{5/2}}\right]$$

[Hovdebo,Kruczenski,Mateos,Myers,Winters]

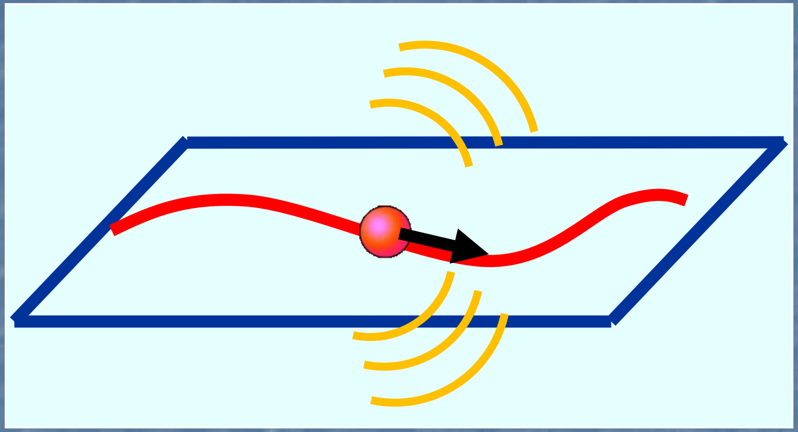
Fat Quark

Studying quarks with **finite** mass is interesting for various reasons:

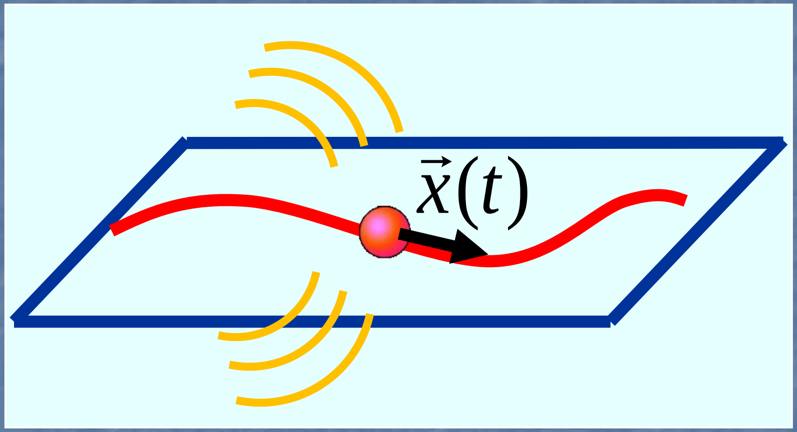
- More **realistic**
- **Nonpointlike** quark has **nontrivial dynamics**
- There exist **new physical effects** (e.g., damping, fluctuations) that are only visible at finite mass
- On the gravity side, the string endpoint in the **interior** of AdS serves as a useful probe of **UV/IR connection**

Accelerating Quark

When a charge accelerates in vacuum, we expect it to **radiate**, and experience **damping force** due to emitted radiation



Accelerating Quark



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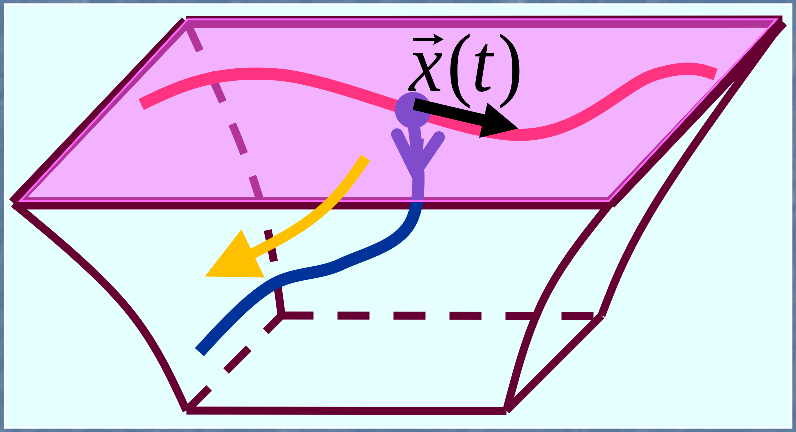


For **accelerating** quark, string trails behind endpoint i.e., quark has a ‘tail’, and it is this tail that **encodes radiation and is responsible for damping effect**

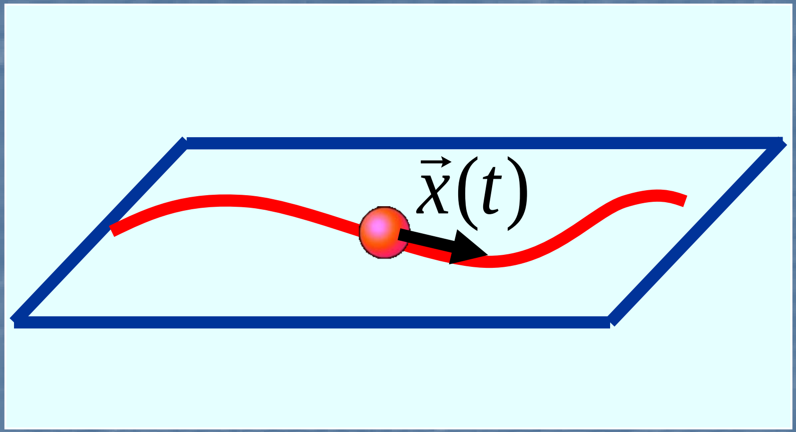
[Chernicoff,AG; Chernicoff,García,AG]

(This same mechanism is responsible for **drag force** in thermal plasma

[Herzog,Karch,Kovtun,Kozcaz,Yaffe; Gubser])

(Related work: [Casalderrey-Solana,Teaney])

Accelerating Quark



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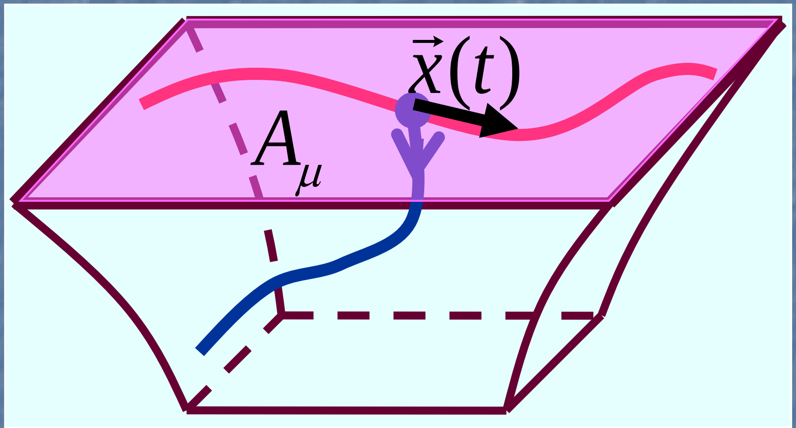
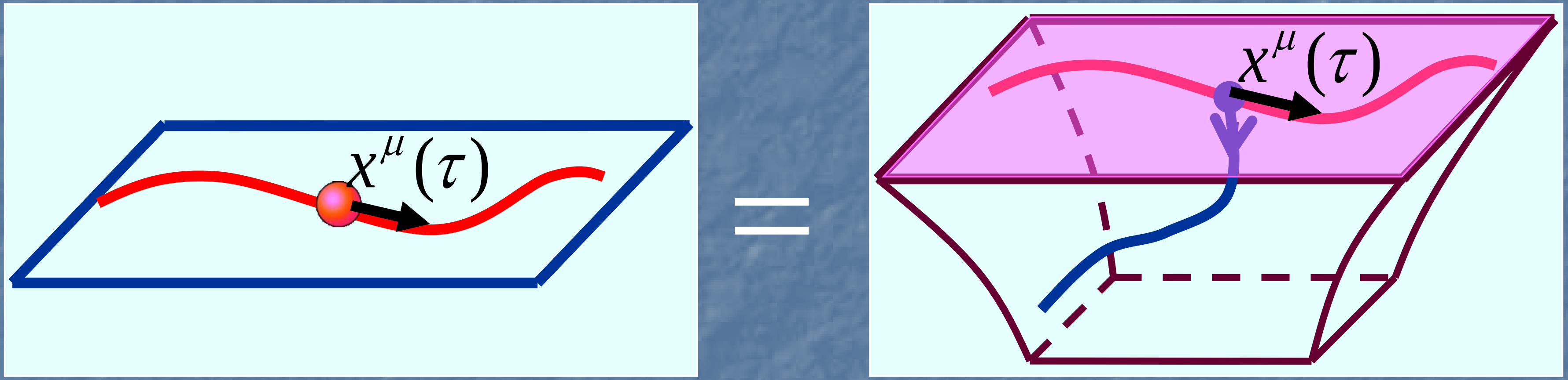


For **accelerating** quark, determine string embedding $\vec{X}(t,z)$ by extremizing Nambu-Goto+forcing action

$$S_{\mathrm{F1}} = -\frac{1}{2\pi l_s^2}\int d^2\sigma \sqrt{\left(\dot{X}\cdot X'\right)^2 - \dot{X}^2 X'^2} + \int d\tau A_\mu\left(x(\tau)\right)\frac{dx^\mu}{d\tau}$$

external 4-force: $F_\mu \equiv \gamma(\vec{f}\cdot\vec{v},\vec{f}) = \mathrm{F}_{\mu\nu}\dfrac{dx^\nu}{d\tau}$

Accelerating Quark



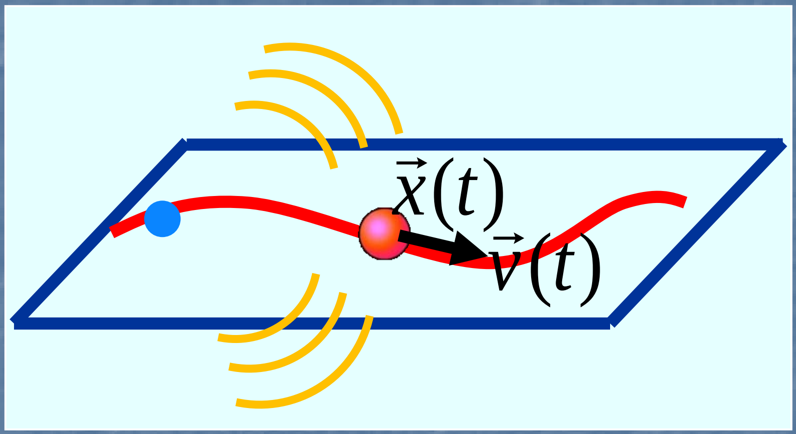
Solution for **arbitrary (!!)** timelike quark trajectory $x^{\mu}(\tau)$ in the case where quark is **infinitely massive**

$$X^{\mu}(\tau, z) = z\frac{dx^{\mu}(\tau)}{d\tau} + x^{\mu}(\tau)$$

[Mikhailov]

proper time

Accelerating Quark



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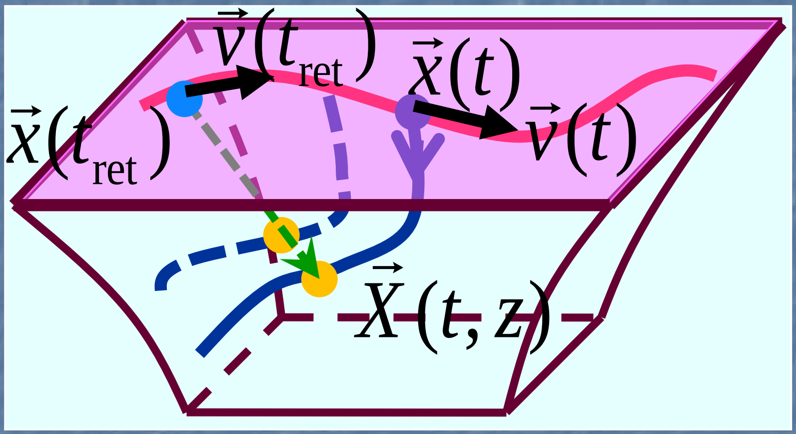


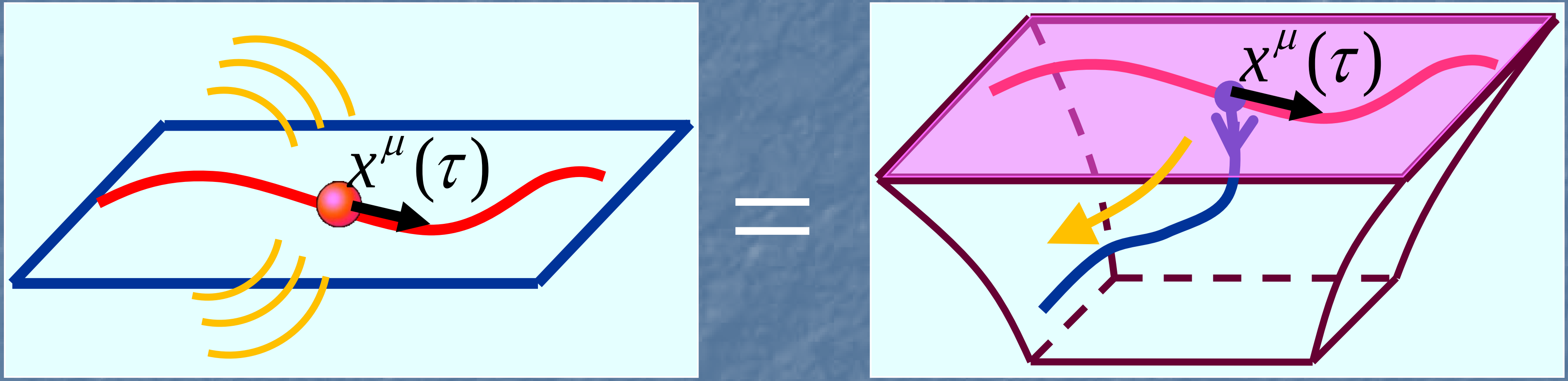
Solution for **arbitrary (!!)** timelike quark trajectory $\vec{x}(t)$

$$\vec{X}(t,z) = z\frac{\vec{v}(t_{\text{ret}})}{\sqrt{1-\vec{v}(t_{\text{ret}})^2}} + \vec{x}(t_{\text{ret}})$$

[Mikhailov]

$$t = z\frac{1}{\sqrt{1-\vec{v}(t_{\text{ret}})^2}} + t_{\text{ret}}$$

Accelerating Quark



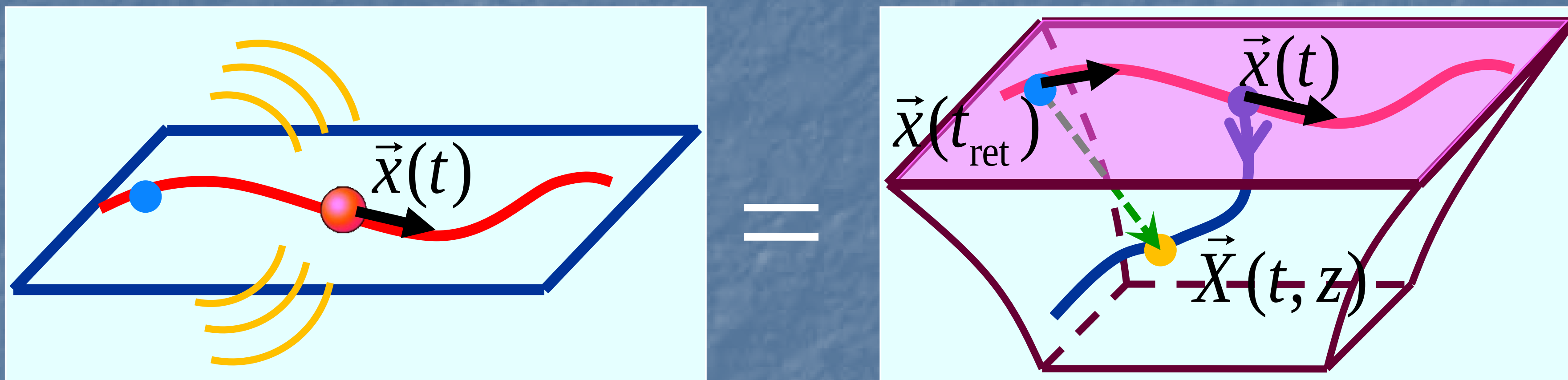
Solution for **arbitrary (!!)** timelike quark trajectory $x^{\mu}(\tau)$

$$X^{\mu}(\tau, z) = \pm z \frac{dx^{\mu}(\tau)}{d\tau} + x^{\mu}(\tau)$$

[Mikhailov]

This solution is ‘**retarded**’ (or ‘purely outgoing’)
‘**advanced**’ (or ‘purely ingoing’)

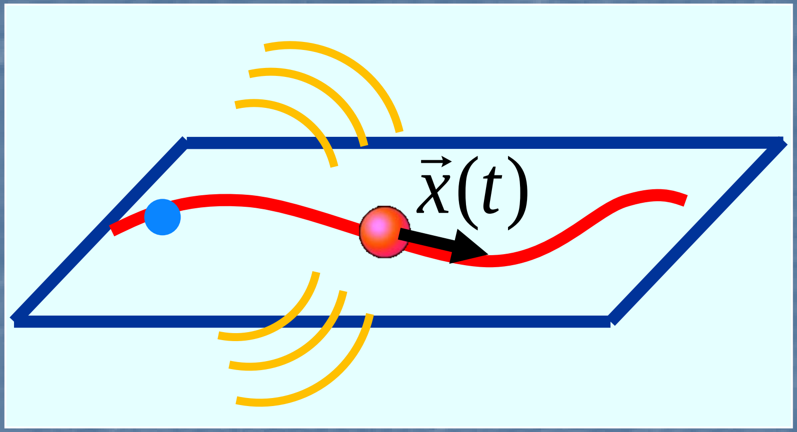
Energy Split on Worldsheet



Using Mikhailov's solution, can rewrite total string energy...

$$E(t)=\frac{\sqrt{\lambda}}{2\pi}\int_0^\infty \frac{dz\, z^{-2}\left[1+\left(\partial_z \vec{X}\right)^2\right]}{\sqrt{1-\left(\partial_t \vec{X}\right)^2+\left(\partial_z \vec{X}\right)^2-\left(\partial_t \vec{X}\right)^2\left(\partial_z \vec{X}\right)^2+\left(\partial_t \vec{X}\cdot\partial_z \vec{X}\right)^2}}$$

Energy Split on Worldsheet



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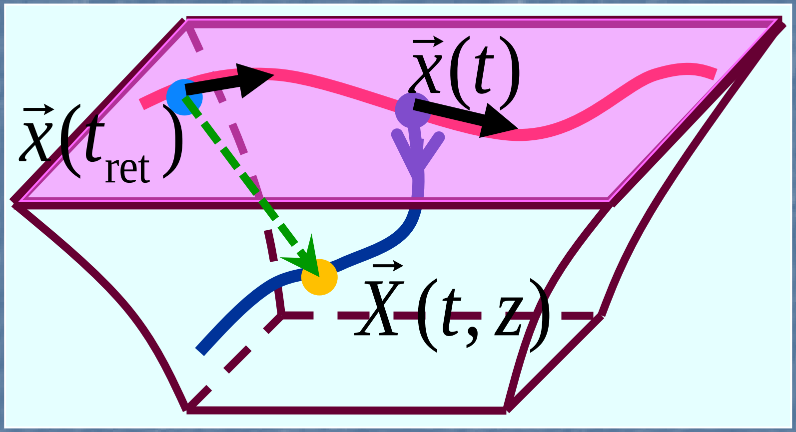


... through a change of variable $z \to t_{\mathrm{ret}}$, as

$$E(t) = \underbrace{\int_{-\infty}^{t} dt_{\mathrm{ret}} \frac{\sqrt{\lambda}}{2\pi} \frac{\vec{a}^2 - (\vec{v}\times\vec{a})^2}{(1-\vec{v}^2)^3}}_{} + \underbrace{\frac{\sqrt{\lambda}}{2\pi}\left(\frac{1}{\sqrt{1-\vec{v}^2}} \frac{1}{z} \right)\Bigg|_{z=\infty}^{z=0}}_{}$$

radiated E_{rad} (Lienard!) $E_q = \gamma m$ **intrinsic**

[Mikhailov] [Chernicoff,AG]

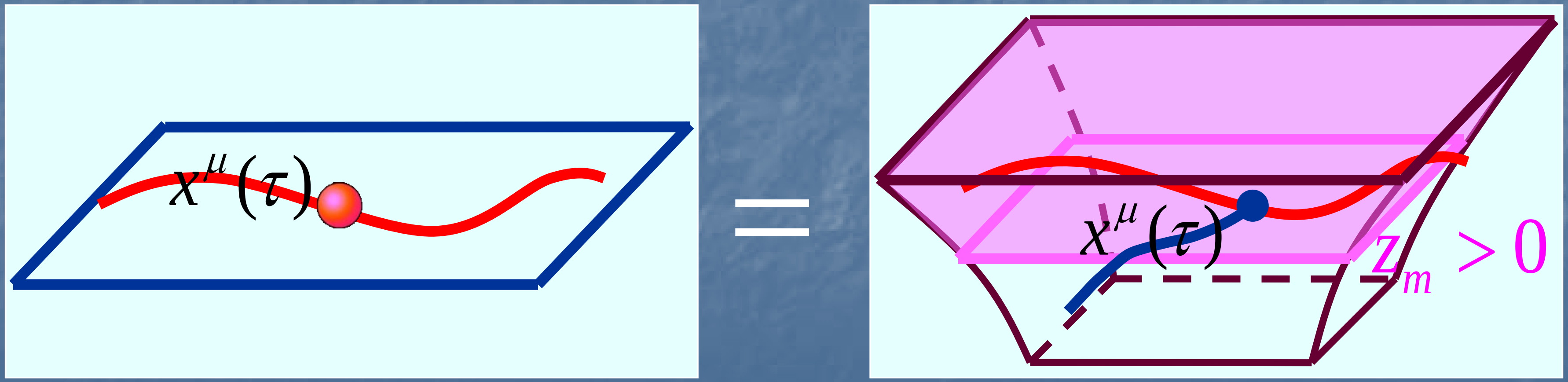
Energy Split on Worldsheet

In covariant form,

Intrinsic quark momentum $p_q^\mu = m v^\mu$

Radiation rate $\dfrac{dP_{\mathrm{rad}}^\mu}{d\tau} = \dfrac{\sqrt{\lambda}}{2\pi} a^2 v^\mu$

Modified for **finite quark mass** (**nonpointlike** quark), where b.c. for string must be enforced at $z = z_m$:



Energy Split on Worldsheet

In covariant form,

Intrinsic quark momentum $p_q^\mu = mv^\mu$

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Modified for **finite quark mass** (**nonpointlike** quark):

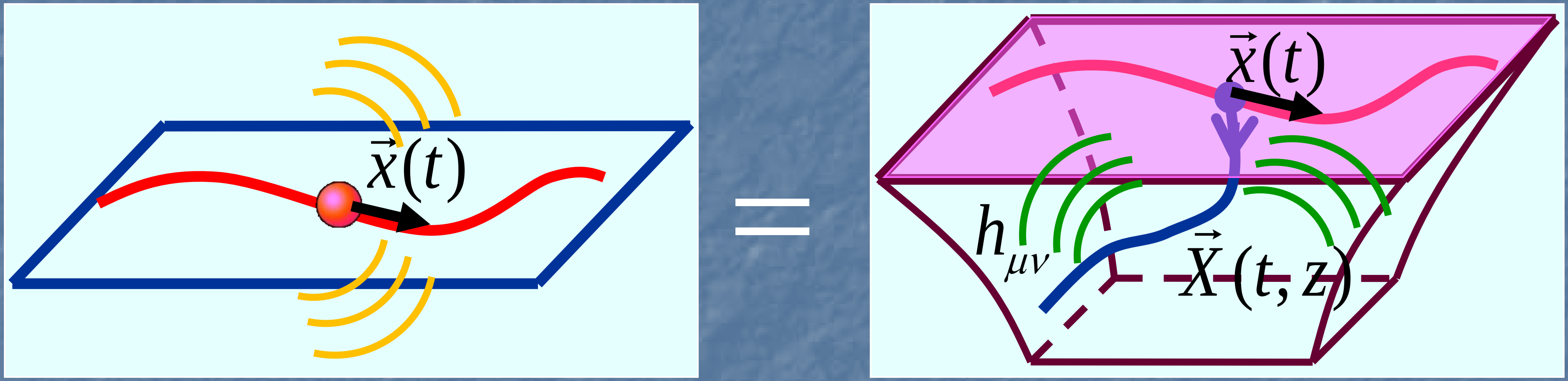
$$p_q^\mu = \frac{mv^\mu - \frac{\sqrt{\lambda}}{2\pi m} F^\mu}{\sqrt{1 - \frac{\lambda}{4\pi^2 m^4} F^2}}$$

$$\frac{dP_{\rm rad}^\mu}{d\tau} = \frac{\sqrt{\lambda}}{2\pi m^2} F^2 \left(\frac{v^\mu - \frac{\sqrt{\lambda}}{2\pi m^2} F^\mu}{1 - \frac{\lambda}{4\pi^2 m^4} F^2} \right)$$

($F^\mu =$ external force)

[Chernicoff, García, AG]

Bulk Calculation of Energy



- There is a second way to compute the energy of the system: use graviton field $h_{\mu\nu}$ sourced by string in GKPW recipe

$$\mathcal{E}(x) \equiv \left\langle T_{00}(x) \right\rangle_q = \frac{R^3}{12\kappa_5^2} \lim_{z\to 0} \left(\partial_z^4 h_{00}(x,z) \right)$$

This yields **spacetime distribution** of energy, but *a priori* does not distinguish **intrinsic vs. radiative**

Worldsheet vs. Bulk?

- It is natural to compare the results of the 2 approaches
- Hatta, Iancu, Mueller & Triantafyllopoulos used **bulk method** to compute $\mathcal{E}(x)$ for an **infinitely massive** quark undergoing arbitrary motion
- Surprise: in the far region, there are $1/r^2$ terms BEYOND those that reproduce the expected Lienard rate of **radiation**!
  - Hatta *et al.* noticed these in fact seem similar to **intrinsic energy** identified via **worldsheet method** by Chernicoff, García & AG
  - But the latter result is valid for **finite mass**, so to make a clean comparison, must upgrade Hatta *et al.* to finite mass...

Gluonic Profile at Finite Quark Mass

Energy Density at Finite Mass

$$\mathcal{E} = \mathcal{E}^{(1)} + \mathcal{E}^{(2)}$$

$$-(t-t_r)^2 + (\vec{x} - \vec{x}(t_r))^2 = z_m^2$$

$$\mathcal{E}^{(1)} = \frac{\sqrt{\lambda}}{8\pi^2} \left[\frac{\Xi A_1 - B_2}{|\Xi_m| \Xi^3} \right]_{\tilde{\tau} = \tilde{\tau}_r}$$

$$\mathcal{E}^{(2)} = \frac{\sqrt{\lambda}}{8\pi^2} \left[\frac{1}{|\Xi_m|} \frac{\partial}{\partial \tilde{\tau}} \left(\frac{2\Xi A_0 - B_1 + 2z_m (\Xi A_1 - B_2)}{2\Xi_m \Xi^2} \right) - \frac{1}{|\Xi_m|} \frac{\partial}{\partial \tilde{\tau}} \left(\frac{1}{\Xi_m} \frac{\partial}{\partial \tilde{\tau}} \left(\frac{B_0 + z_m B_1 + z_m^2 B_2}{2\Xi_m \Xi} \right) \right) \right]_{\tilde{\tau} = \tilde{\tau}_r}$$

where

$$A_0 \equiv 1 - 2\tilde{v}^0\tilde{v}_0 + \left[-(t-\tilde{t})\tilde{a}_0 + (x-\tilde{x})^i\,\tilde{a}_i\right]$$

$$A_1 \equiv -2\tilde{a}^0\tilde{v}_0 + \tilde{a}^2\left[-(t-\tilde{t})\tilde{v}_0 + (x-\tilde{x})^i\,\tilde{v}_i\right]$$

$$B_0 \equiv (x-\tilde{x})^i(x-\tilde{x})_i(\tfrac{4}{3} + \tilde{v}^j\tilde{v}_j) - \left[(x-\tilde{x})^i\tilde{v}_i\right]^2$$

$$B_1 \equiv 2(x-\tilde{x})^i(x-\tilde{x})_i\tilde{a}^j\tilde{v}_j - \tfrac{2}{3}(x-\tilde{x})^i\tilde{v}_i\left[4 + 3(x-\tilde{x})^j\,\tilde{a}_j\right]$$

$$B_2 \equiv \tilde{v}^i\tilde{v}_i\left[\tfrac{4}{3} + 2(x-\tilde{x})^j\,\tilde{a}_j\right] - 2(x-\tilde{x})^i\tilde{v}_i\tilde{a}^j\tilde{v}_j$$

$$+\tilde{a}^2\left\{(x-\tilde{x})^i(x-\tilde{x})_i\tilde{v}^j\tilde{v}_j - \left[(x-\tilde{x})^i\tilde{v}_i\right]^2\right\}$$

$$\Xi \equiv -(x-\tilde{x})^\mu\,\tilde{v}_\mu$$

$$\Xi_m \equiv -(x-\tilde{x})^\mu\,\tilde{v}_\mu - z_m\left[1 + (x-\tilde{x})^\mu\,\tilde{a}_\mu\right]$$

...which are in turn written in terms of **auxiliary** (tilde) variables related to the **physical** (nontilde) quark data through:

Proper time $d\tilde{\tau} = \dfrac{d\tau}{\sqrt{1 - z_m^4 \underline{F}^2(\tau)}}$ $\underline{F}^\mu \equiv \dfrac{2\pi F^\mu}{\sqrt{\lambda}}$

Position $\tilde{x}^\mu = x^\mu - z_m \dfrac{v^\mu - z_m^2 \underline{F}^\mu}{\sqrt{1 - z_m^4 \underline{F}^2}}$

Velocity $\tilde{v}^\mu = \dfrac{v^\mu - z_m^2 \underline{F}^\mu}{\sqrt{1 - z_m^4 \underline{F}^2}}$ Acceleration $\tilde{a}^\mu = z_m \dfrac{\underline{F}^\mu - z_m^2 \underline{F}^2 v^\mu}{\sqrt{1 - z_m^4 \underline{F}^2}}$

Jerk
$$\tilde{j}^\mu = z_m \dot{\underline{F}}^\mu - z_m^3 \underline{F}^2 a^\mu + \frac{z_m^5 (\underline{F} \cdot \dot{\underline{F}}) \underline{F}^\mu}{1 - z_m^4 \underline{F}^2} - \left[z_m^3 (\underline{F} \cdot \dot{\underline{F}}) + \frac{z_m^7 (\underline{F} \cdot \dot{\underline{F}}) \underline{F}^2}{1 - z_m^4 \underline{F}^2} \right] v^\mu$$

etc.

Energy Density at Finite Mass

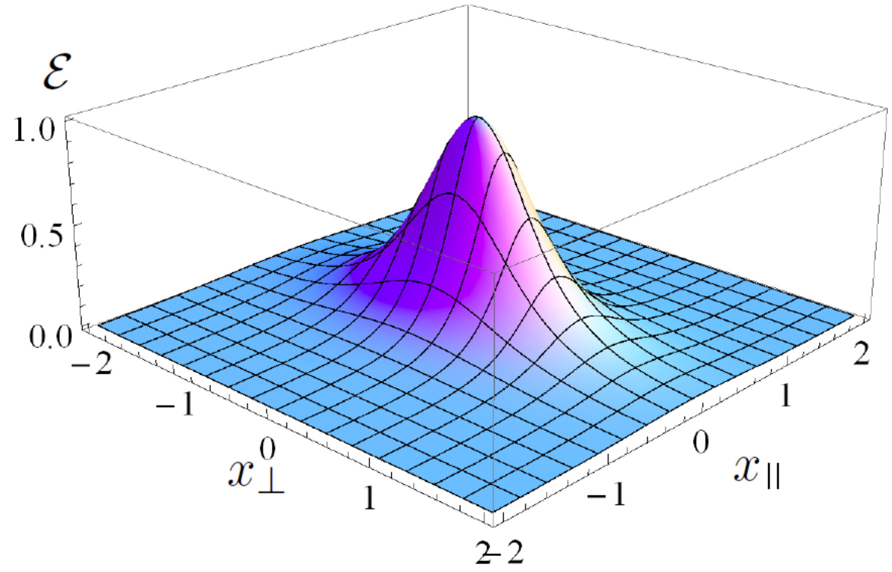
$$\mathcal{E} = \mathcal{E}^{(1)} + \mathcal{E}^{(2)}$$

$$-(t - t_r)^2 + (\vec{x} - \vec{x}(t_r))^2 = z_m^2$$

$$\mathcal{E}^{(1)} = \frac{\sqrt{\lambda}}{8\pi^2}\left[\frac{\Xi A_1 - B_2}{|\Xi_m|\Xi^3}\right]_{\tilde{\tau}=\tilde{\tau}_r}$$

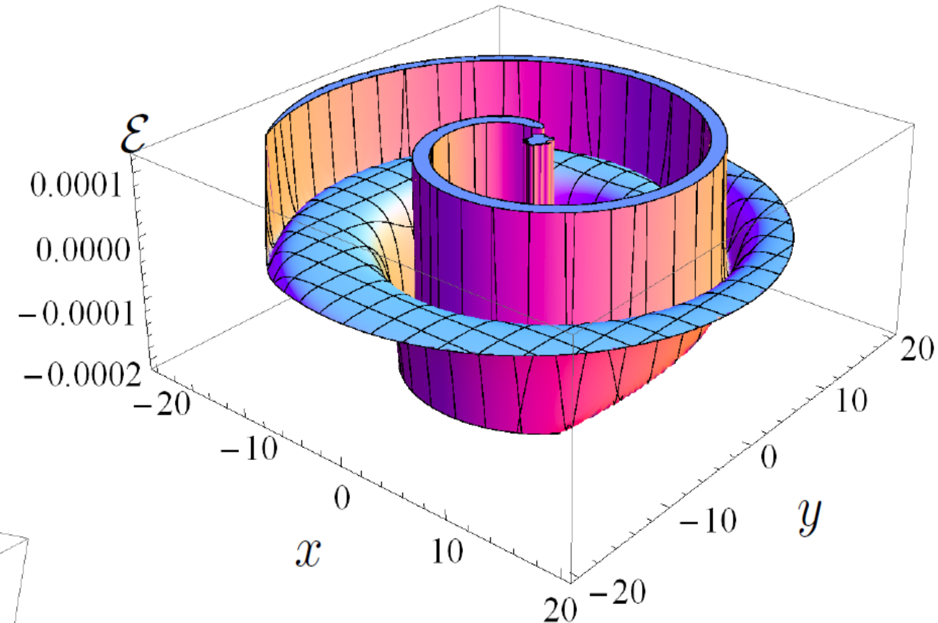
$$\mathcal{E}^{(2)} = \frac{\sqrt{\lambda}}{8\pi^2}\left[\frac{1}{|\Xi_m|}\frac{\partial}{\partial\tilde{\tau}}\left(\frac{2\Xi A_0 - B_1 + 2z_m(\Xi A_1 - B_2)}{2\Xi_m\Xi^2}\right)\right.$$

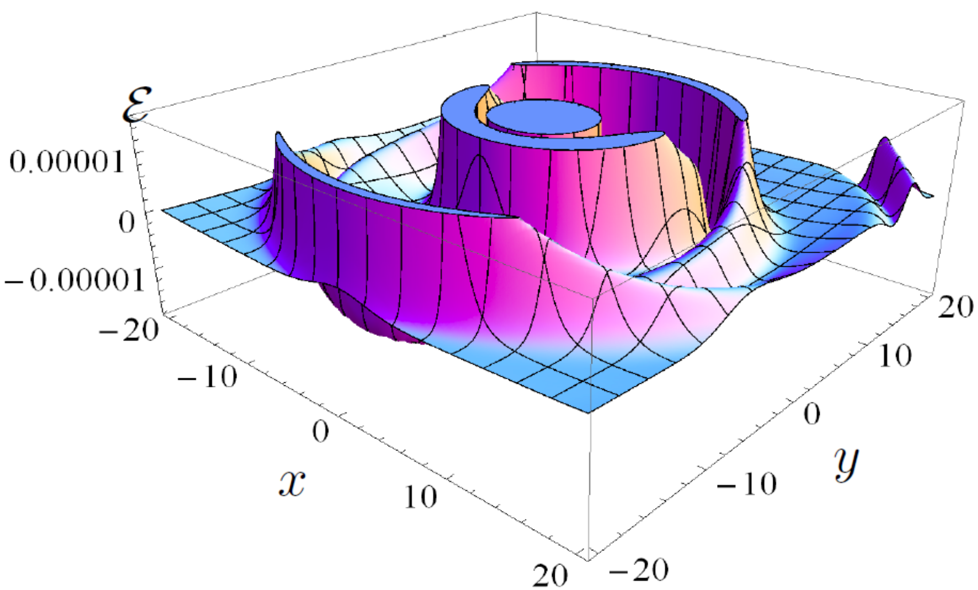
$$\left. - \frac{1}{|\Xi_m|}\frac{\partial}{\partial\tilde{\tau}}\left(\frac{1}{\Xi_m}\frac{\partial}{\partial\tilde{\tau}}\left(\frac{B_0 + z_m B_1 + z_m^2 B_2}{2\Xi_m\Xi}\right)\right)\right]_{\tilde{\tau}=\tilde{\tau}_r}$$



Constant velocity

Uniform circular motion





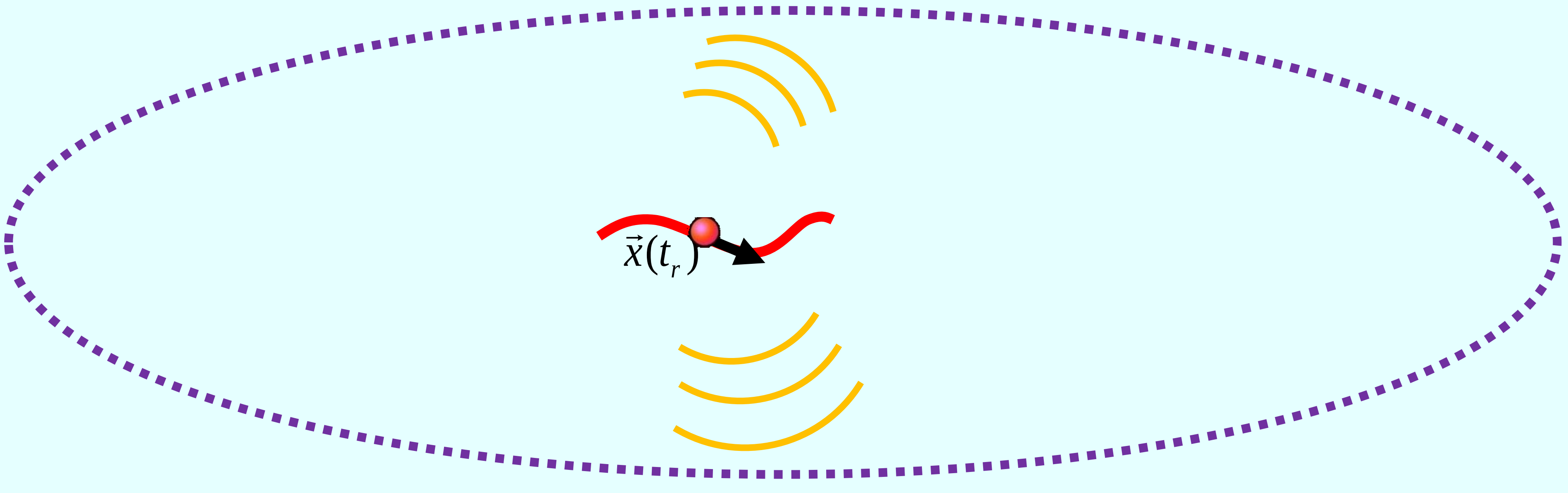
Harmonic oscillation

An Oddity: Intrinsic Far Field

Can work out total **power** at fixed **emission** time

$$P_{\mathrm{far}}(t_r) \equiv \frac{dE}{d\tau_r} = \frac{dt}{d\tau_r}\int d^3x\,\partial_t \mathcal{E}(t,\vec{x}) = \frac{dt}{d\tau_r}\int d^3x\,\partial_i \left\langle T^{i0}(t,\vec{x})\right\rangle$$

$$= \lim_{|\vec{x}|\to\infty}\frac{dt}{d\tau_r}\vec{x}^2\int d\Omega\, n^i \left\langle T^{i0}(t,\vec{x})\right\rangle = \lim_{|\vec{x}|\to\infty}\frac{dt}{d\tau_r}\vec{x}^2\int d\Omega\,\mathcal{E}_{\mathrm{far}}(t,\vec{x})$$



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Contribution from $\mathcal{E}^{(1)}$ is found to exactly match expected **radiation rate** from **worldsheet** method:

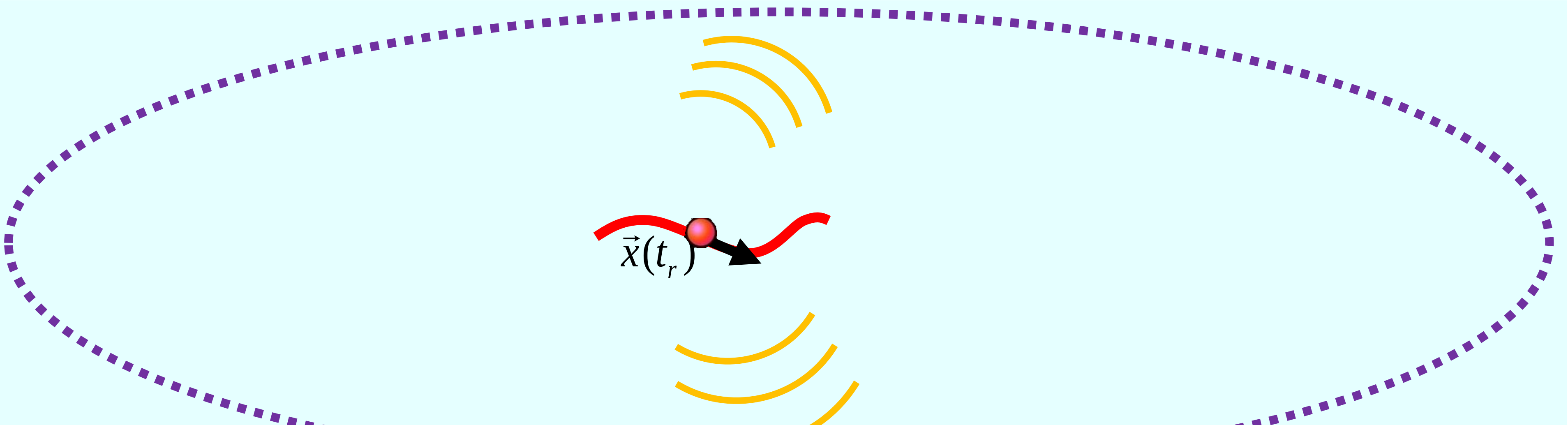
$$P^{(1)}_{\text{far}}(t_r) = \frac{\sqrt{\lambda}}{2\pi m^2} F^2 \left(\frac{v^0 - \frac{\sqrt{\lambda}}{2\pi m^2} F^0}{1 - \frac{\lambda}{4\pi^2 m^4} F^2} \right) = \frac{dE_{\text{rad}}}{d\tau}$$

An Oddity: Intrinsic Far Field

- But contribution from $\mathcal{E}^{(2)}$ is nonvanishing, so it must represent a **1/r² tail in INTRINSIC portion of the field**!!

$$P_{\mathrm{far}}^{(2)}(t_r) = \frac{\sqrt{\lambda}}{2\pi}\partial_{\tilde{\tau}_r}\left[-\frac{1}{9}\tilde{a}^0 + z_m \tilde{a}^2 \tilde{v}^0\left(\frac{17}{18}+\frac{1}{4|\tilde{\mathbf{v}}|^2}\right)+\ldots\right]$$

- Structure similar (but not identical) to intrinsic energy E_q deduced from worldsheet



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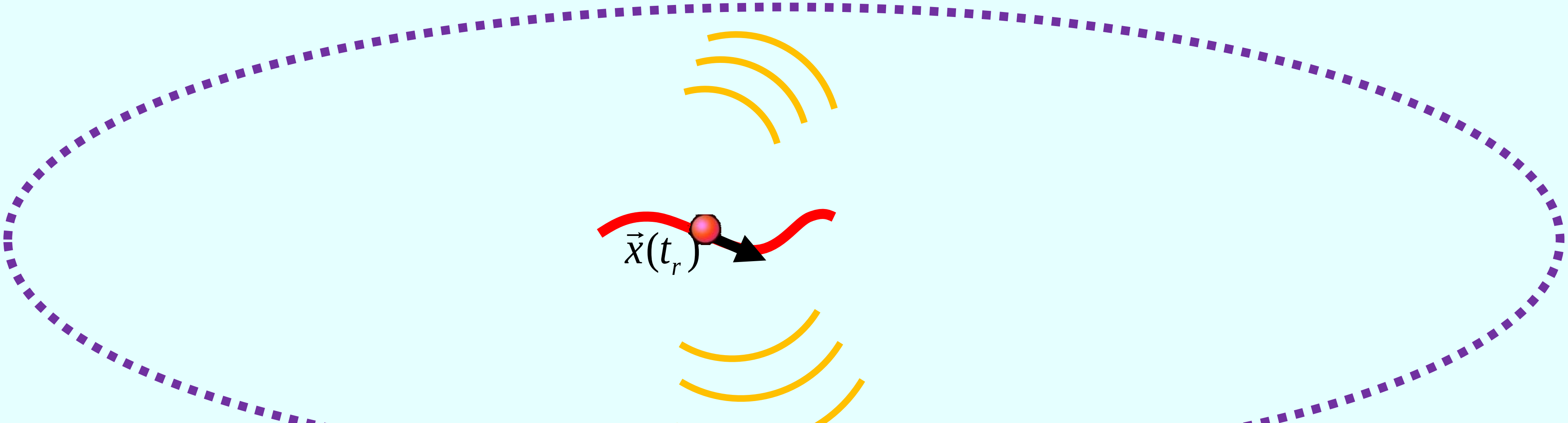
Notice this can be **negative**: by appropriate tug on quark, this portion of the energy can flow INWARD from infinity

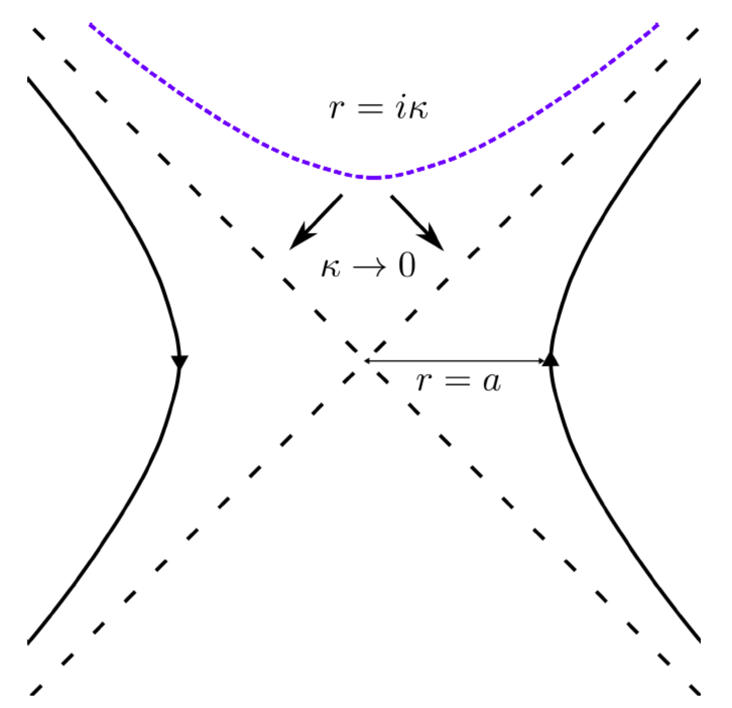


An Oddity: Intrinsic Far Field

Related to recent finding of Lewkowycz and Maldacena, who, for an **infinitely massive** quark undergoing **uniform acceleration**, noticed a discrepancy between:

- naive radiation rate as inferred from **exact** $\langle T_{\mu\nu} \rangle_q \propto h_w$
- **exact** Bremsstrahlung function B in $E_{\text{rad}} = 2\pi B \int dt\, a^2$

[Correa,Henn,Maldacena,Sever]



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[Correa,Henn,Maldacena,Sever]

Difference due to intrinsic/near field contribution

Discrepancy resolved upon **subtraction** based on expectation value of dim d-2 scalar operator:

$$T_{\mu\nu} \to T_{\mu\nu} - \frac{1}{(d-2)^2}\left(\nabla_\mu \nabla_\nu - \eta_{\mu\nu}\nabla^2\right) O_{d-2}$$

$$B = \frac{4\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)} \frac{d-1}{d-2} \frac{h_w}{4\pi^2}$$

[Fiol,Garolera,Lewkowycz; Lewkowycz,Maldacena]

An Oddity: Intrinsic Far Field

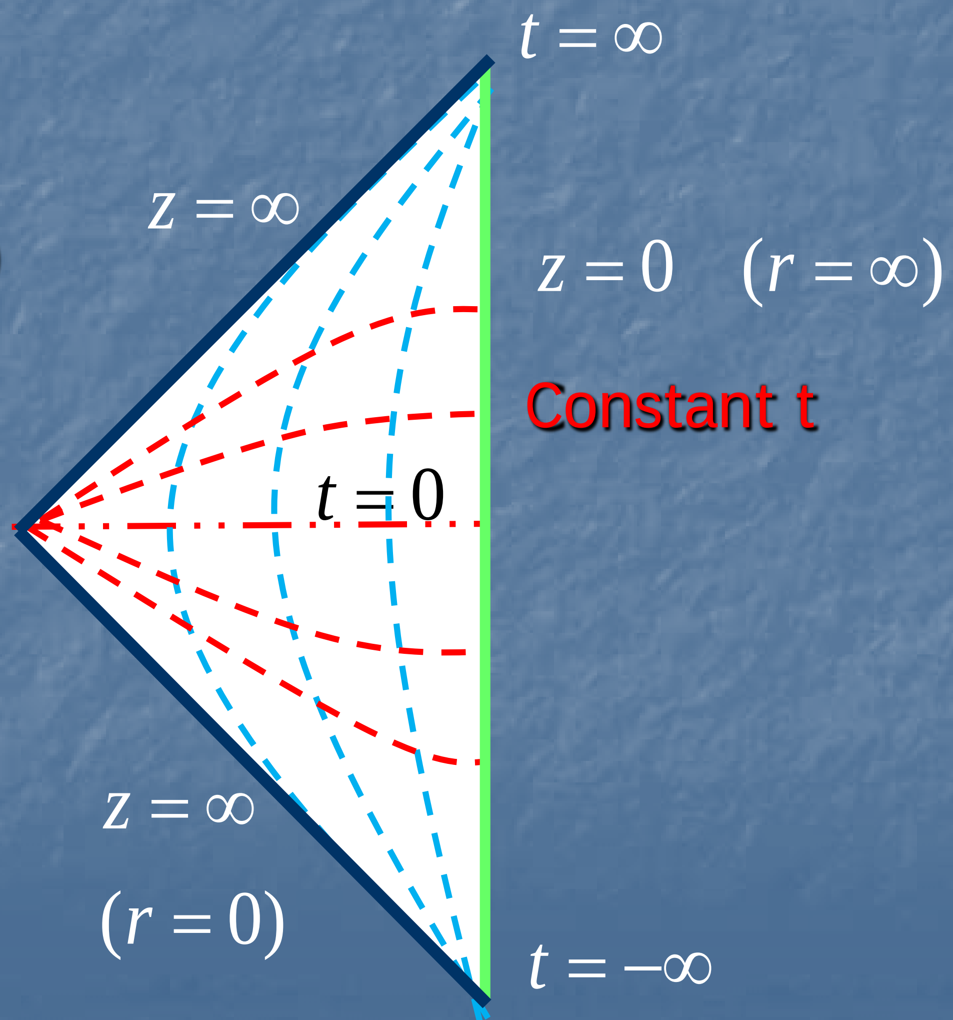
- Related to recent finding of Lewkowycz and Maldacena, for an **infinitely massive** quark undergoing **uniform acceleration**
- Our supergravity analysis shows that the discrepancy is not just a feature of uniform acceleration: it is due to **$1/r^2$ tail in intrinsic field**, for generic motion
- Their exact results show that this effect is not exclusive to the strong coupling regime of the CFT, and is due to **scalar** expectation value

ER=EPR and Wormholes on the Worldsheet

Poincaré AdS Causal Structure

Constant z observers share *acceleration horizon* (with T=0)

Penrose diagram for Poincaré AdS:



Poincaré AdS Causal Structure

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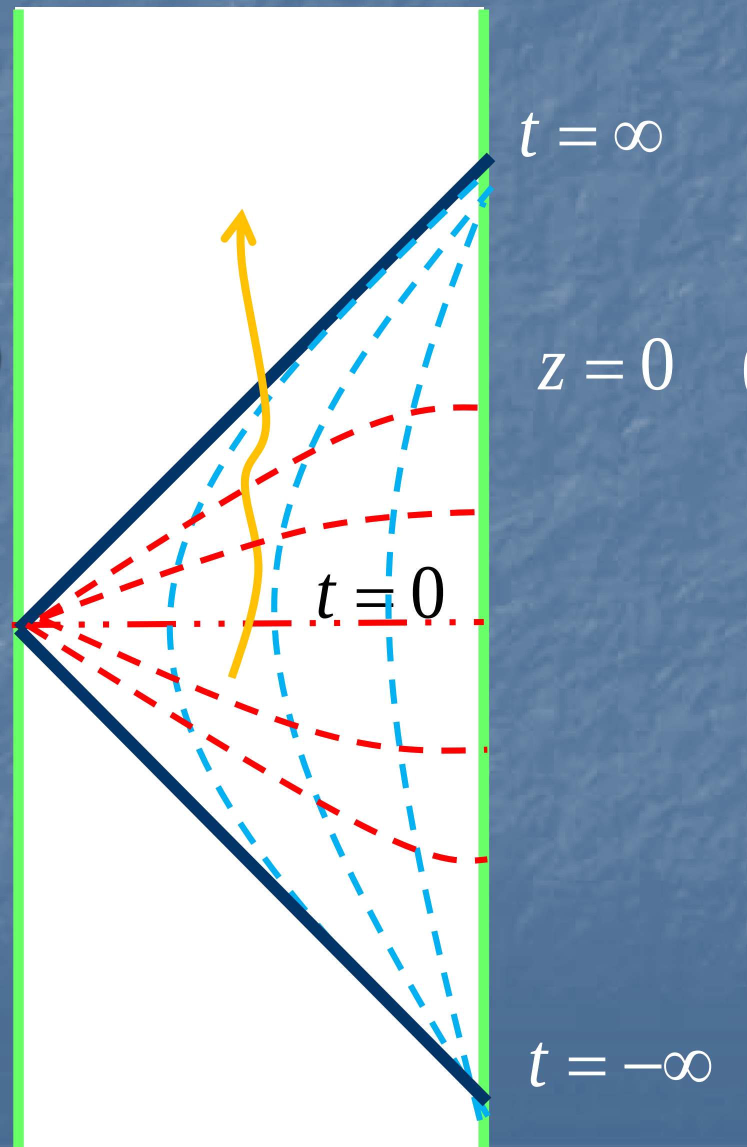
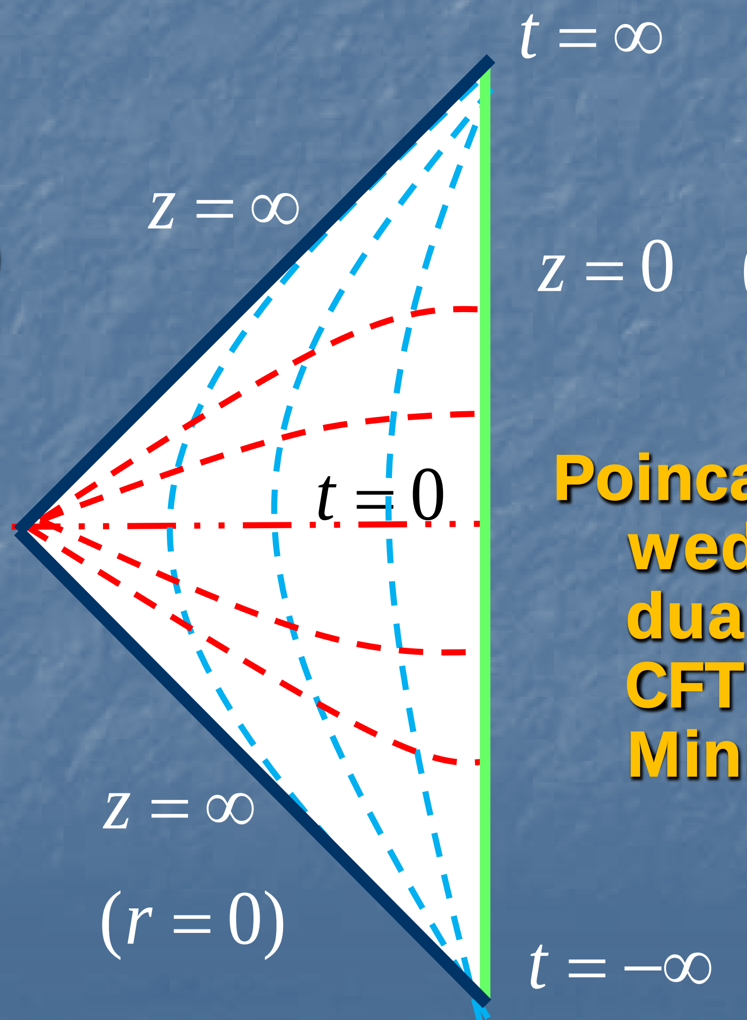


Timelike paths cross the horizon in finite proper time, but infinite Poincaré time

Poincaré AdS Causal Structure

Constant z observers share *acceleration horizon* (with T=0)

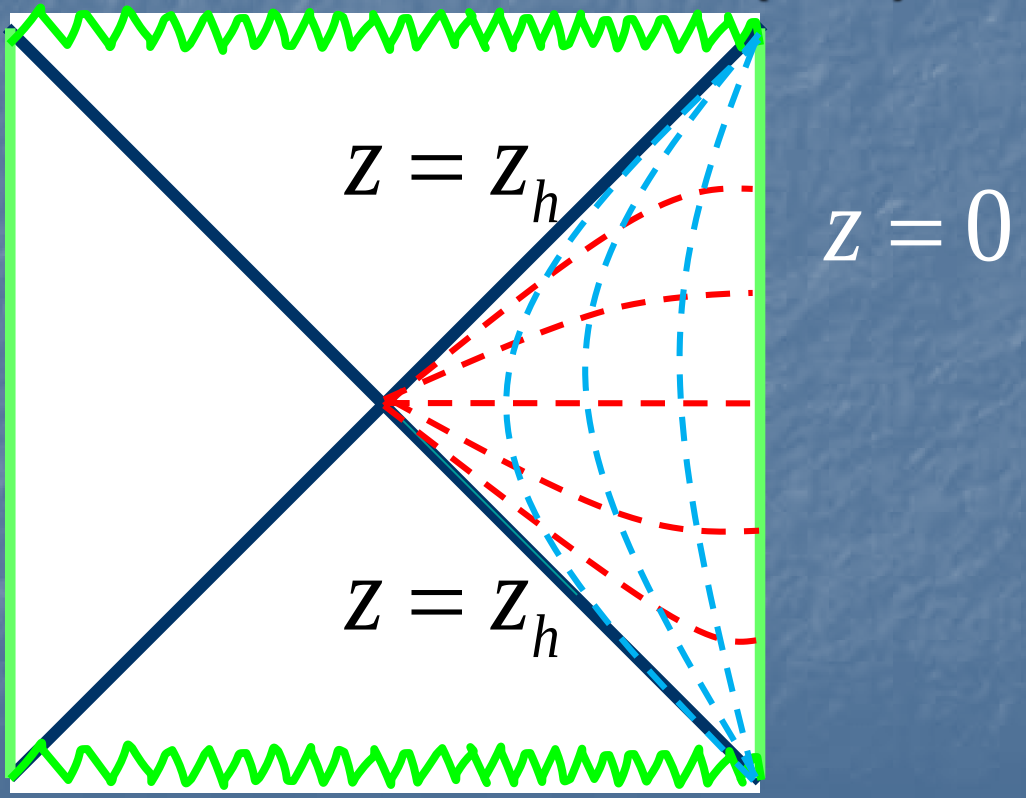
Poincaré time at the boundary corresponds to CFT time



Poincaré wedge is dual to CFT on Minkowski

Eternal AdS BH Causal Structure

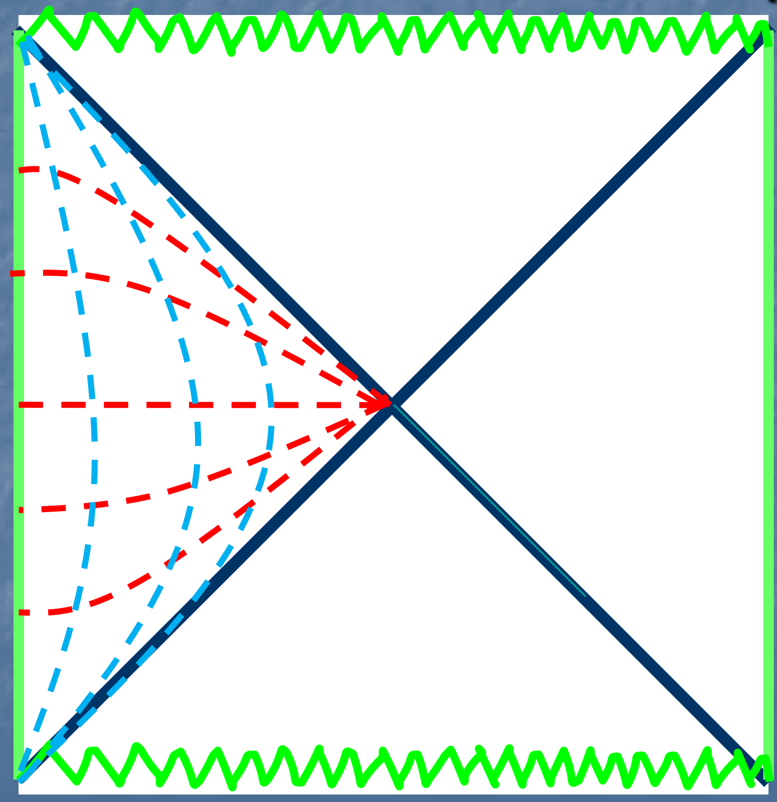
Penrose diagram for (maximal extension of) **eternal Schwarzschild black hole in AdS** (T>0):



Eternal AdS BH Causal Structure

Penrose diagram for (maximal extension of) **eternal Schwarzschild black hole in AdS** (T>0):

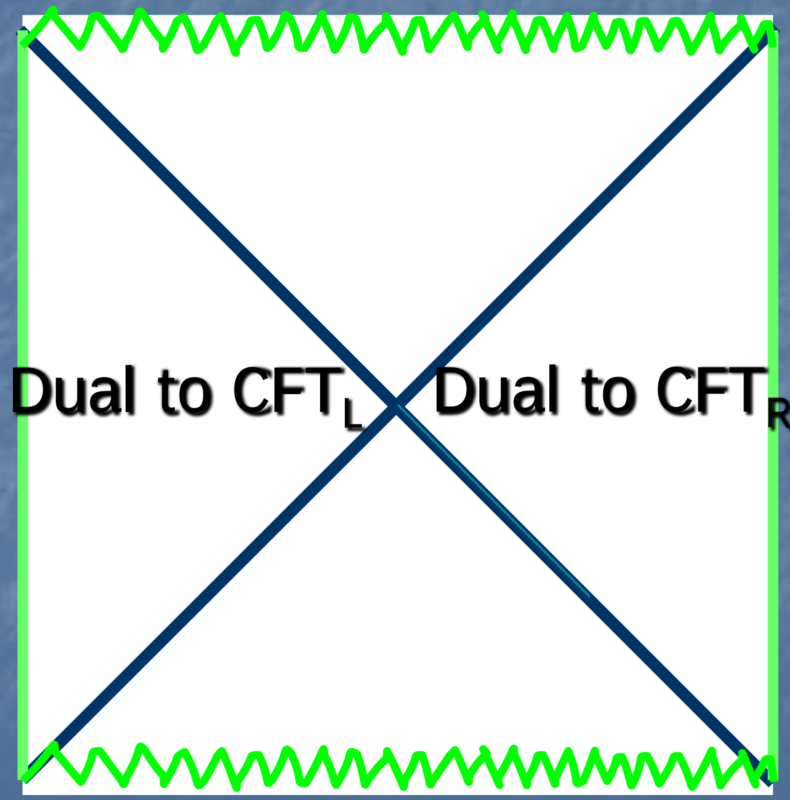
There is now a second exterior region (aAdS)



We now have 2 **noninteracting** copies of our CFT, in the particular **entangled state**

$$|\psi\rangle = \sum_n e^{-E_n/2T} |E_n\rangle_L \otimes |E_n\rangle_R$$

[Israel; Maldacena; Herzog,Son]



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[Israel; Maldacena; Herzog,Son]

If we trace this pure state over CFT_L, we get the expected thermal density matrix for CFT_R:

$$\rho_R = \mathrm{Tr}_L \left(\left|\psi\right\rangle\left\langle\psi\right|\right) = \sum_n e^{-E_n/T} \left|E_n\right\rangle_R \left\langle E_n\right|_R$$

We can regard this as just a trick, with CFT_L an auxiliary, fictitious system (“thermofield double”) to describe thermal physics of CFT_R in real time (Schwinger-Keldysh formalism)

But if we interpret this directly in terms of quantum states on the gravity side, we see that **entanglement is geometrized into a (nontraversable) wormhole**:

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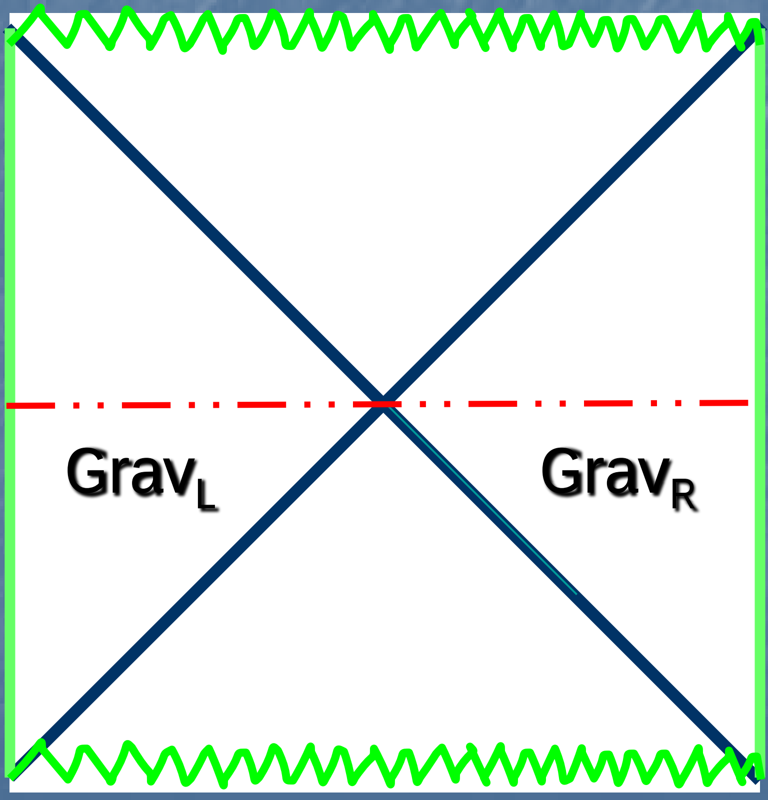
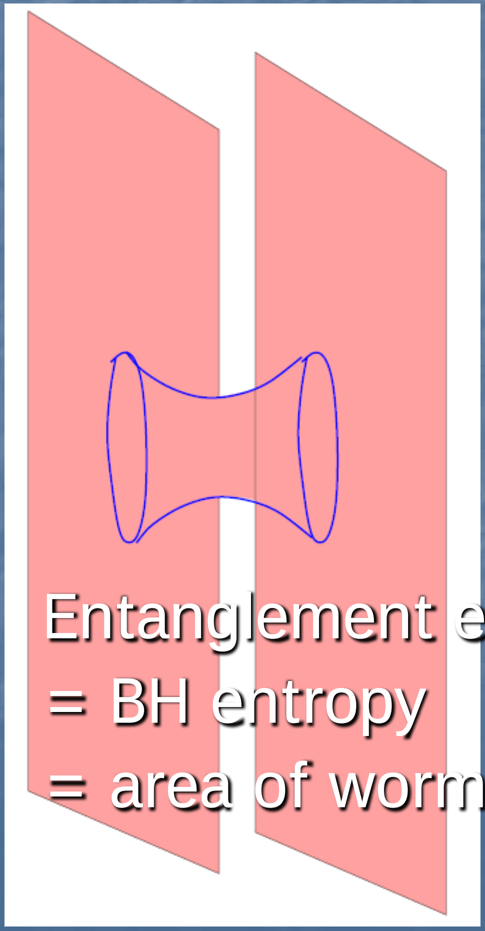
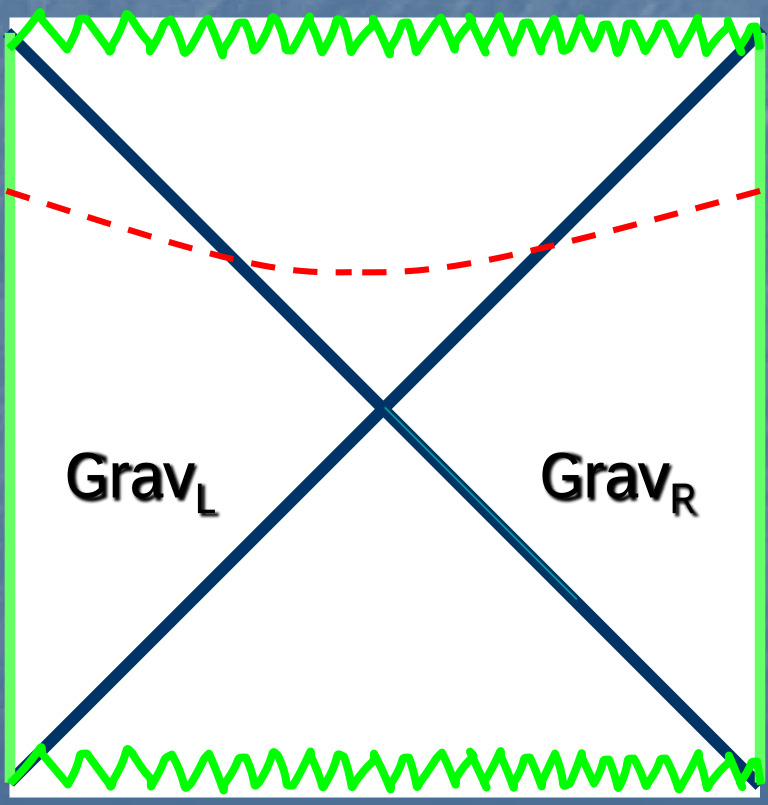


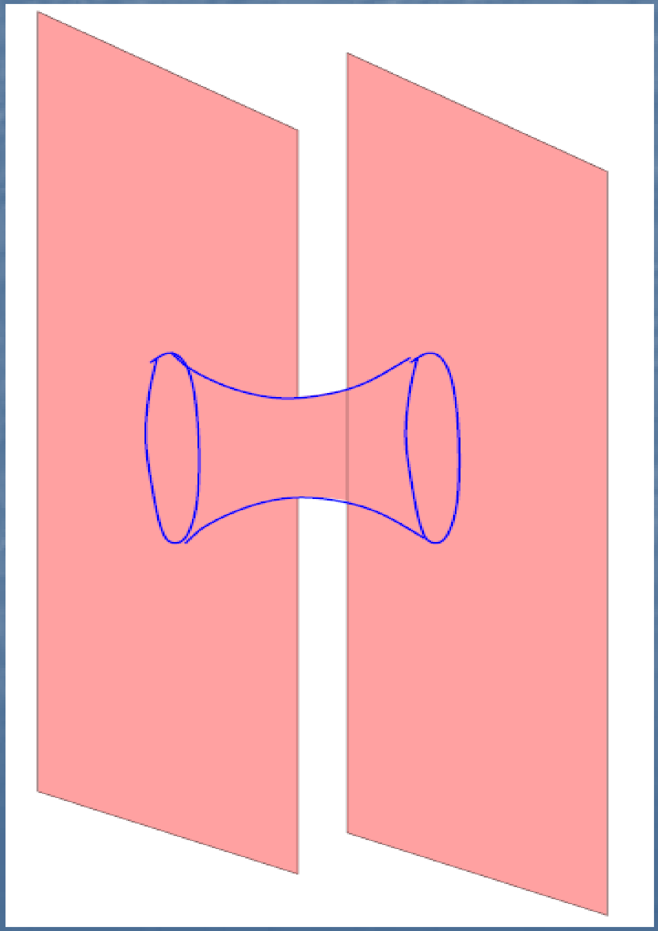


Entanglement entropy
= BH entropy
= area of wormhole

But if we interpret this directly in terms of quantum states on the gravity side, we see that **entanglement is geometrized into a (nontraversable) wormhole**:

$$e^{-iHt}\left|\psi\right\rangle = \sum_n e^{-i2E_n t} e^{-E_n/2T} \left|E_n\right\rangle_L \otimes \left|E_n\right\rangle_R$$





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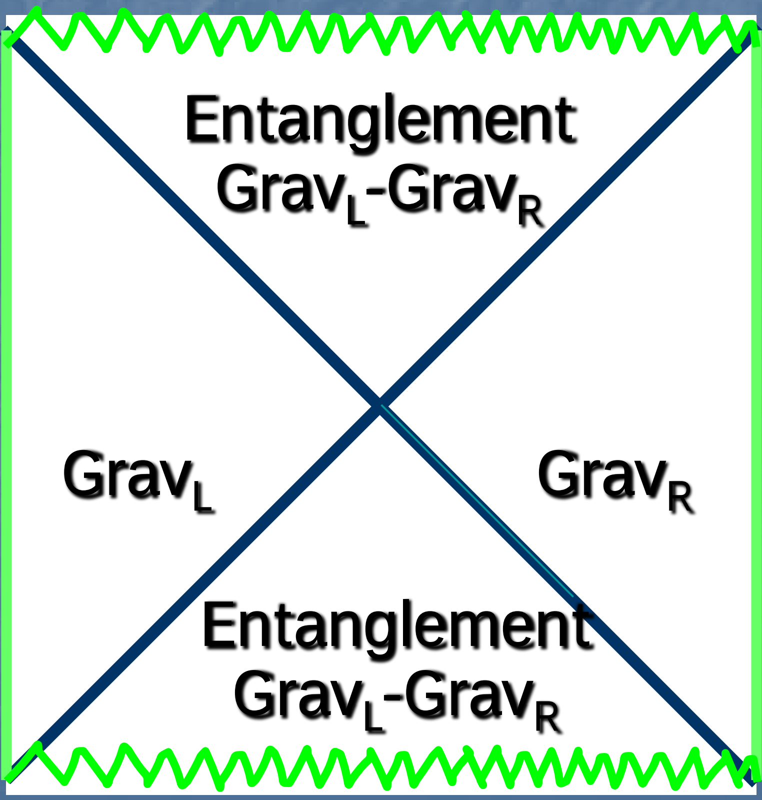
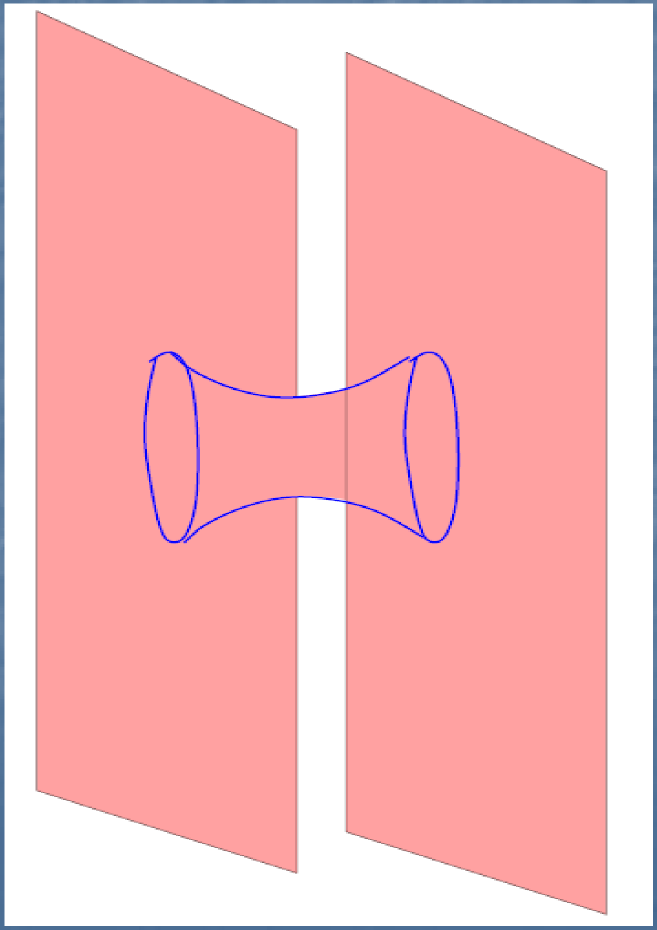




ER=EPR

Maldacena and Susskind examined various other situations, and highlighted a number of parallels between **entanglement** (à la Einstein-Podolsky-Rosen) and nontraversable **wormholes** (Einstein-Rosen bridges)

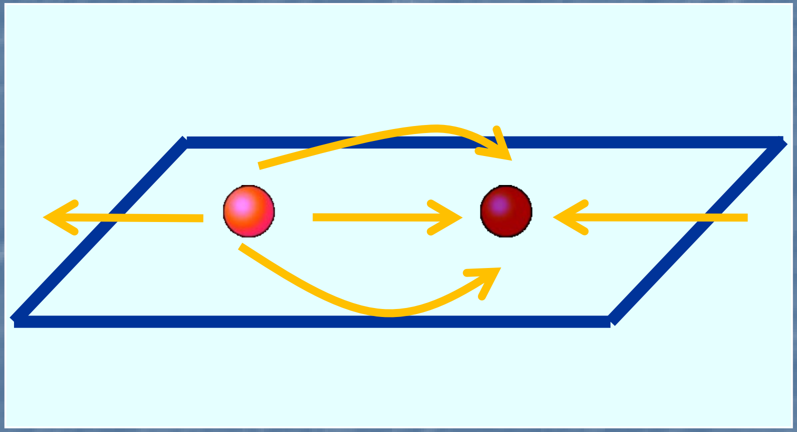
Conjecture: in a quantum theory of gravity, 2 entangled subsytems (even a single EPR pair) are connected by a (generically highly quantum) wormhole

[Maldacena,Susskind]

Related work: [Van Raamsdonk; Swingle]

Holographic EPR Pair

Jensen and Karch provided evidence for ER=EPR by considering quark-antiquark pair in AdS/CFT:



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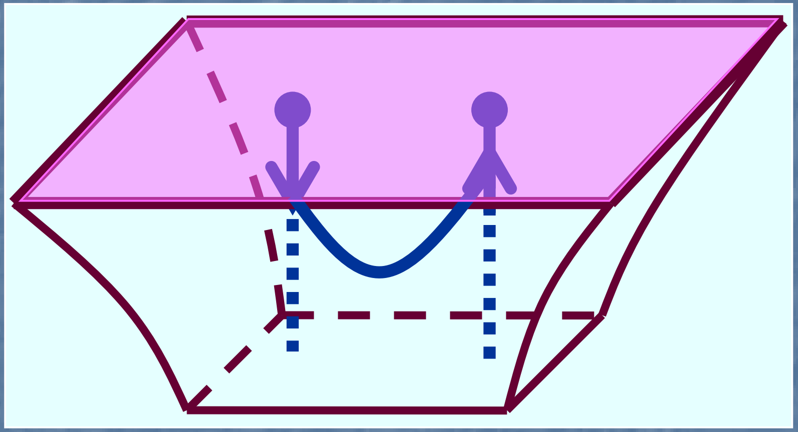
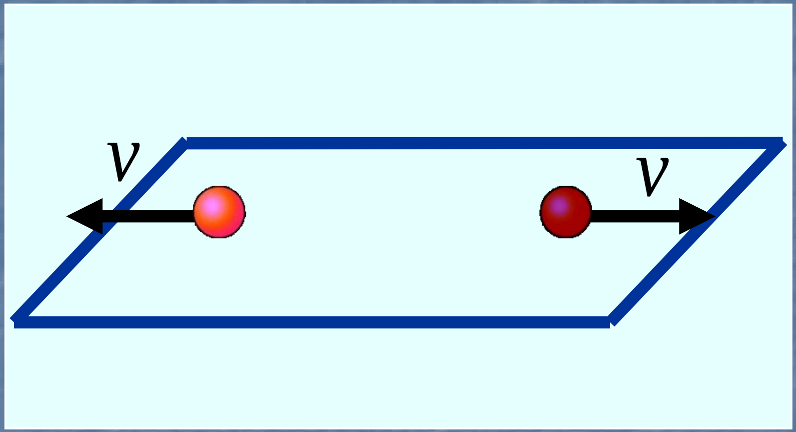


Quark-Antiquark in vacuum = U-shaped string in pure AdS

Color singlet, therefore **entangled**

Pair in Uniform Acceleration



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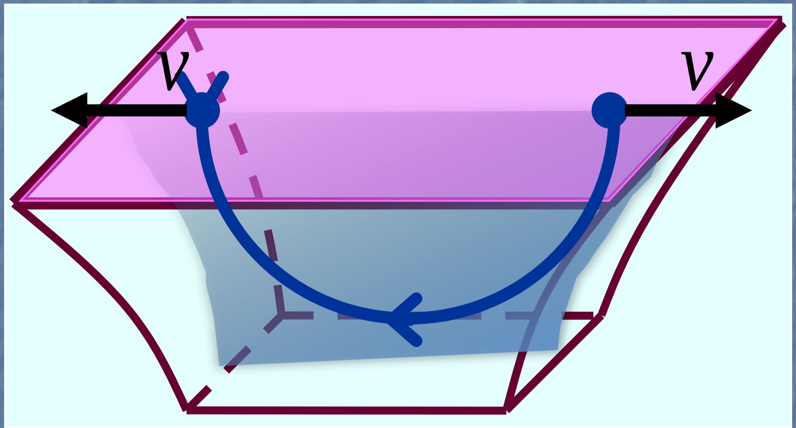
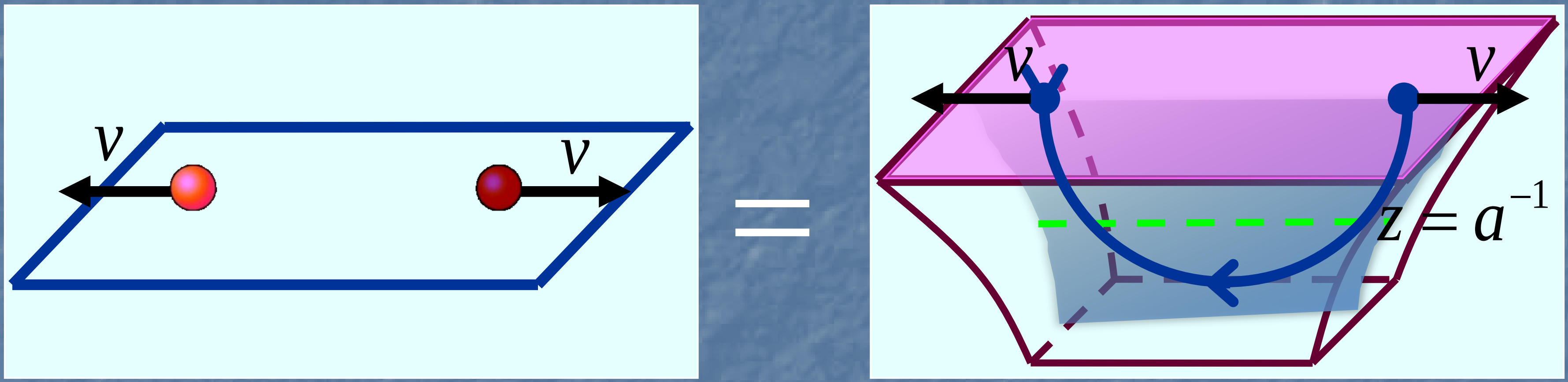


Jensen and Karch considered case where quark and antiquark undergo back-to-back **uniform acceleration**,

$$x_{\bar{q}}(t) = -\sqrt{a^{-2}+t^2}\,, \qquad x_q(t) = \sqrt{a^{-2}+t^2}$$

Pair in Uniform Acceleration



The dual string is known to simply take the shape of a shrinking/growing semicircle,

$$X(t,z) = \pm\sqrt{a^{-2} + t^2 - z^2}$$

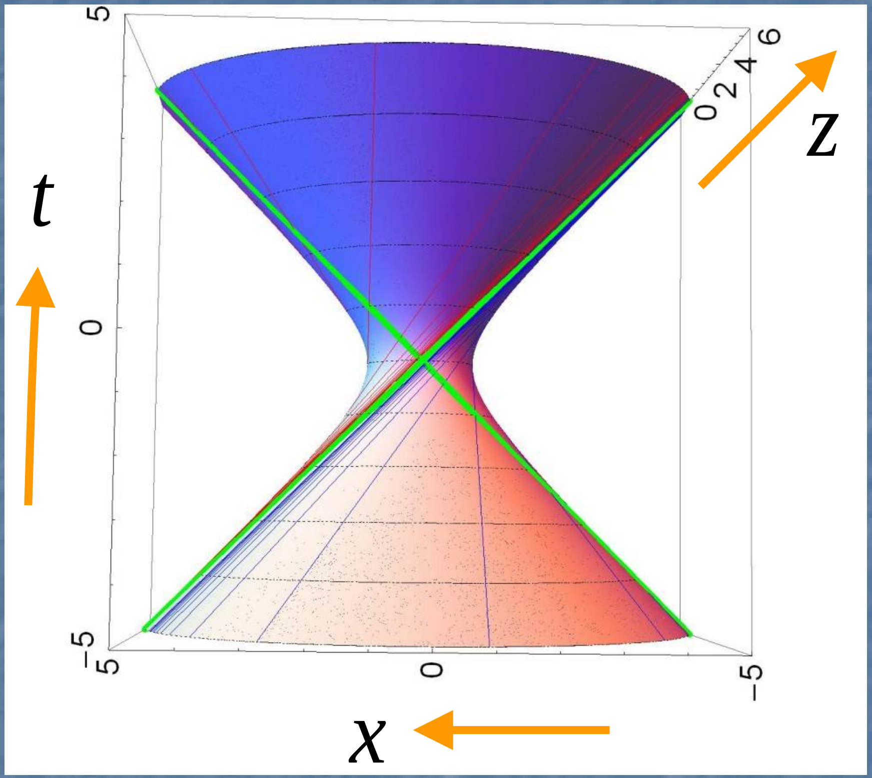
[Xiao]

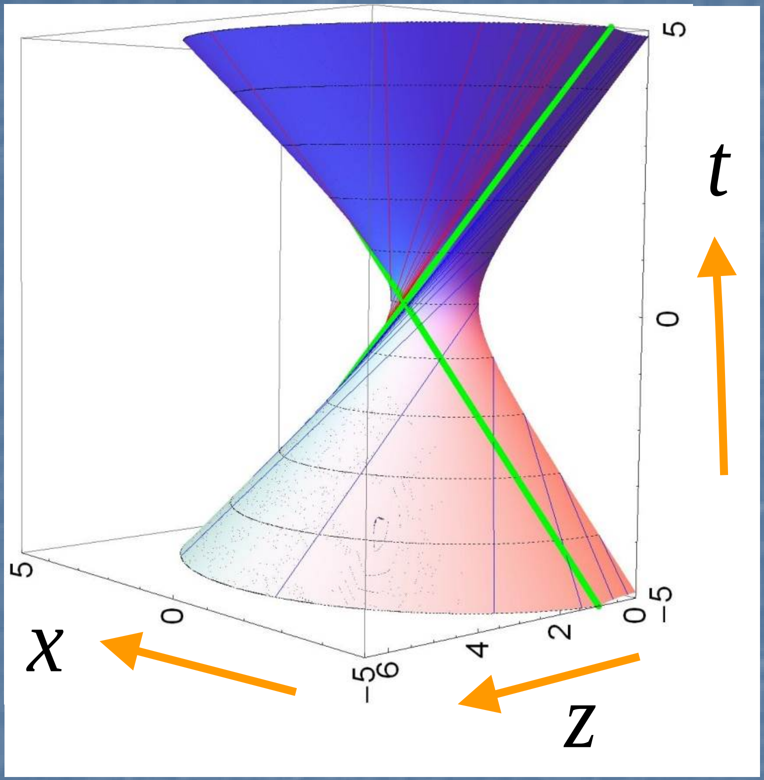
and has a **double-sided worldsheet horizon** at

$$z = a^{-1}$$

There is a **worldsheet BH**, even though no spacetime BH

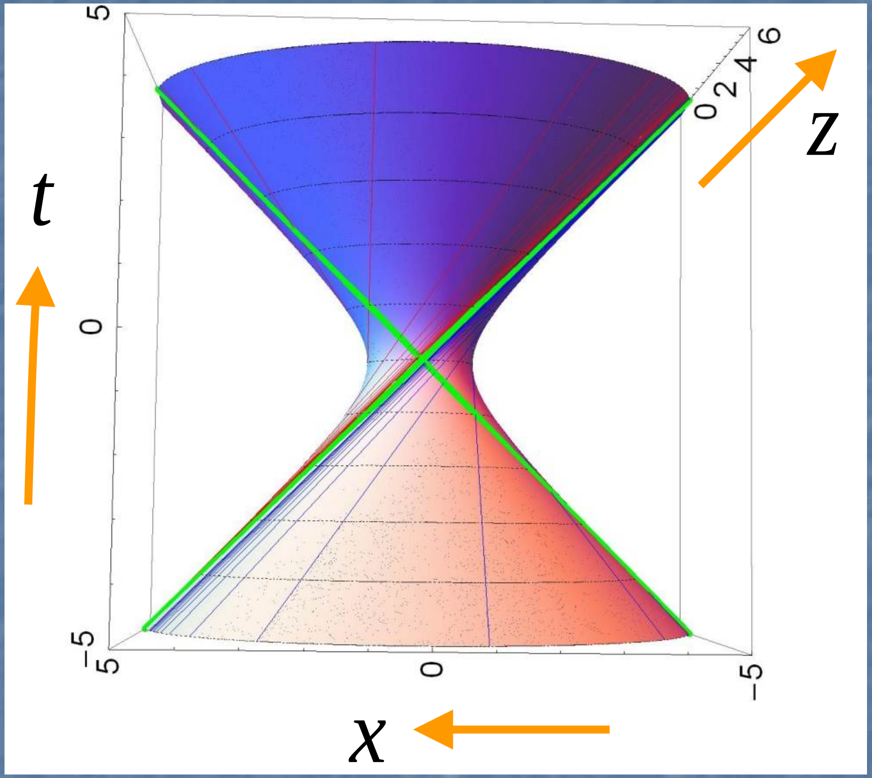
Jensen and Karch noticed that this implies that string dual to accelerating $q\bar{q}$ has a **worldsheet wormhole**!

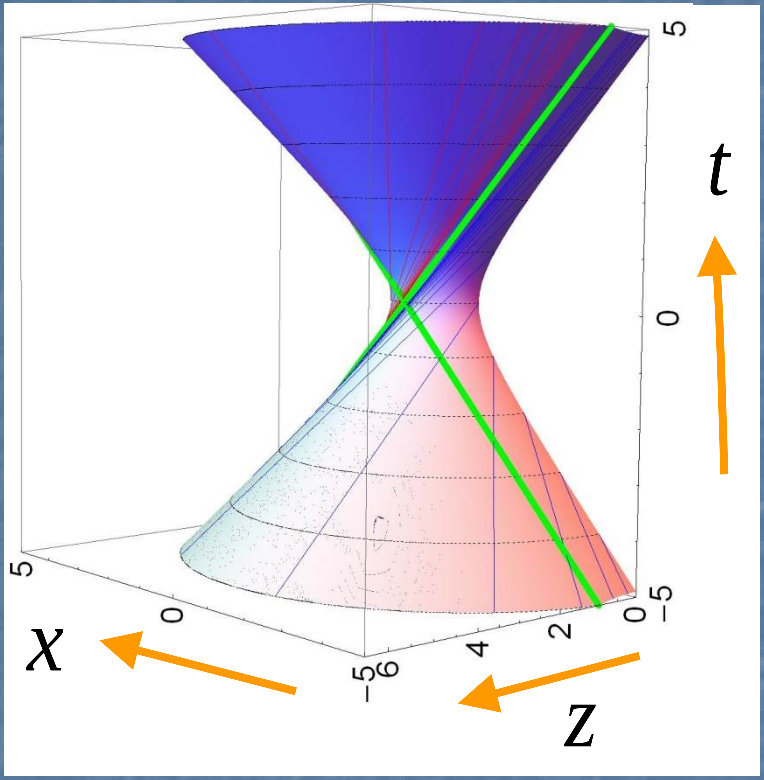




Causal structure is the same as in eternal BH

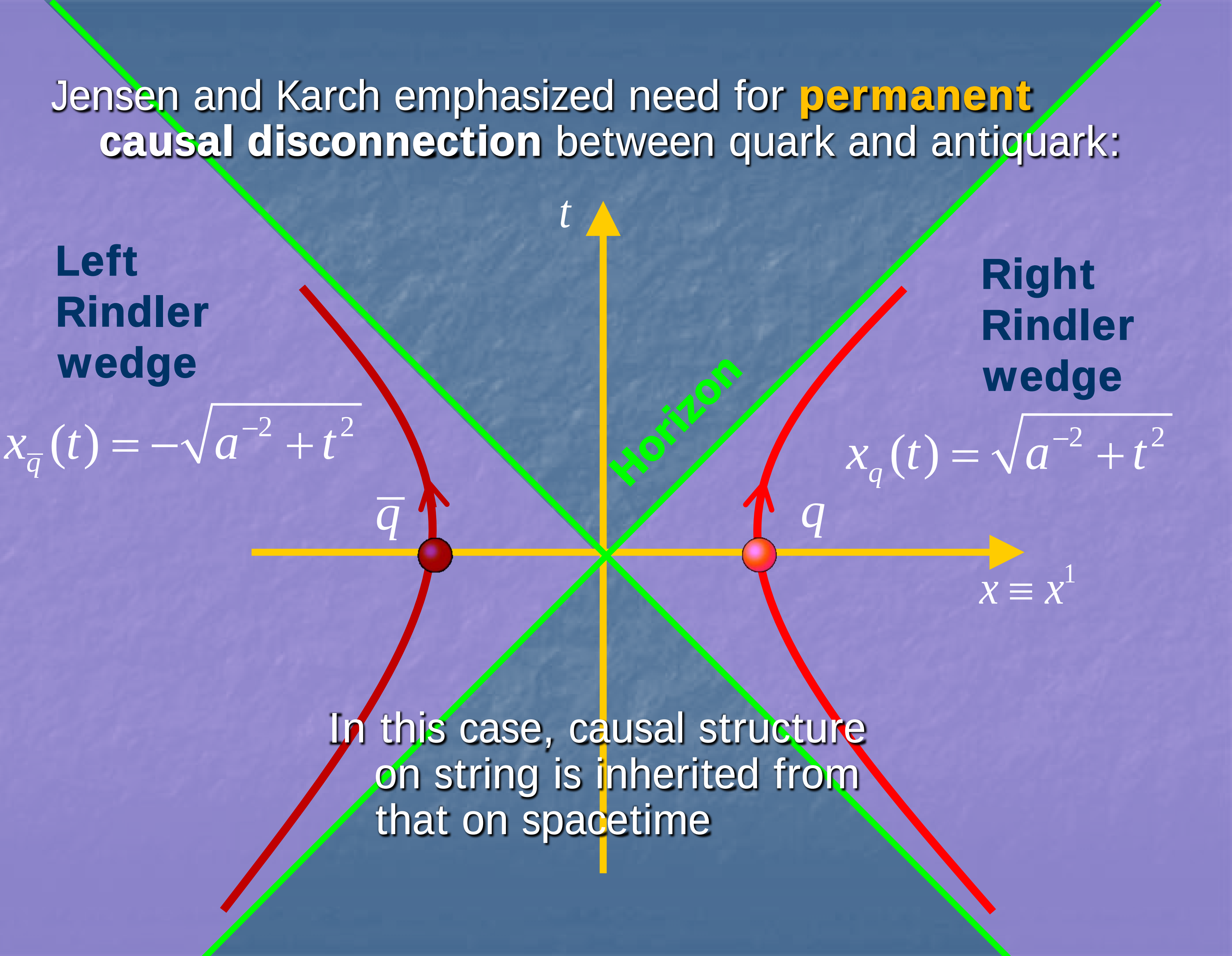
Jensen and Karch noticed that this implies that string dual to accelerating $q\bar{q}$ has a **worldsheet wormhole**!





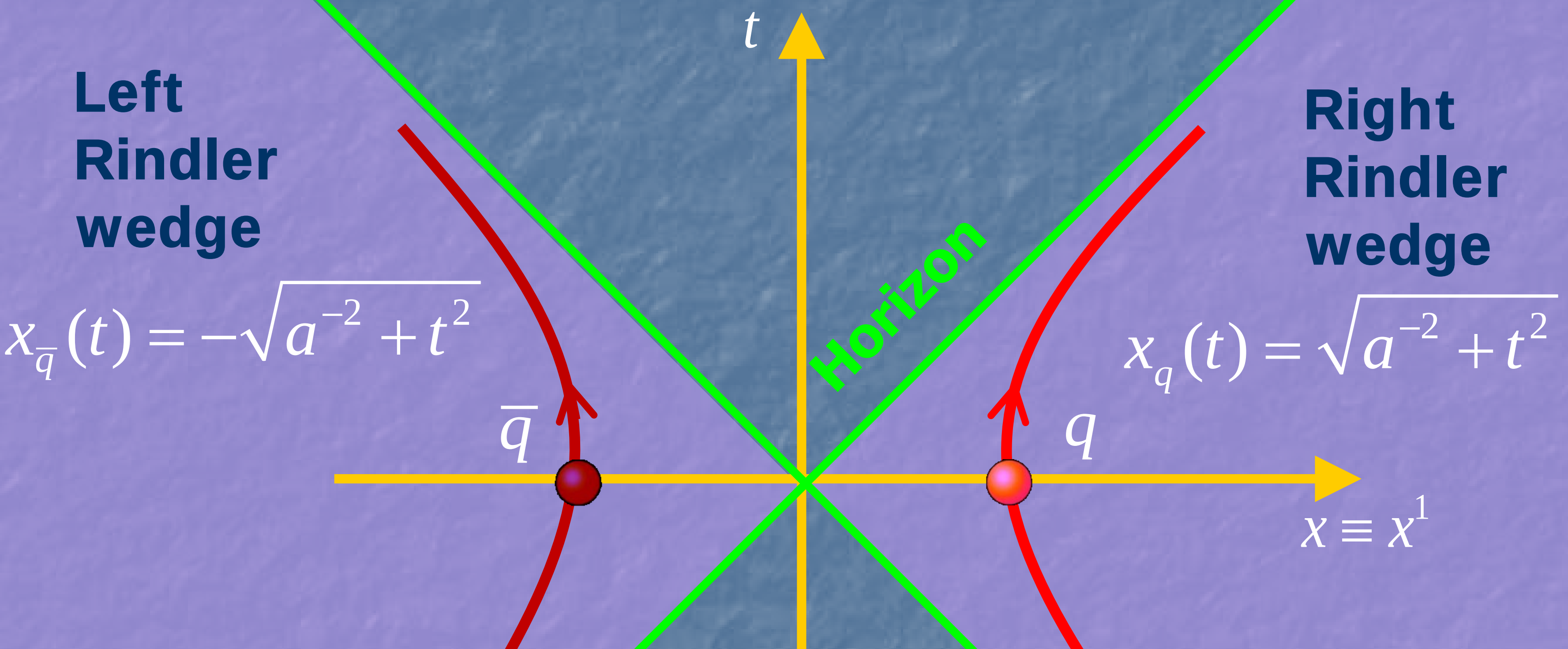
This provides **evidence for ER=EPR**: we see that entanglement of a single pair in CFT is geometrized into (worldsheet) wormhole in AdS side

Jensen and Karch emphasized need for **permanent causal disconnection** between quark and antiquark:



In this case, causal structure on string is inherited from that on spacetime

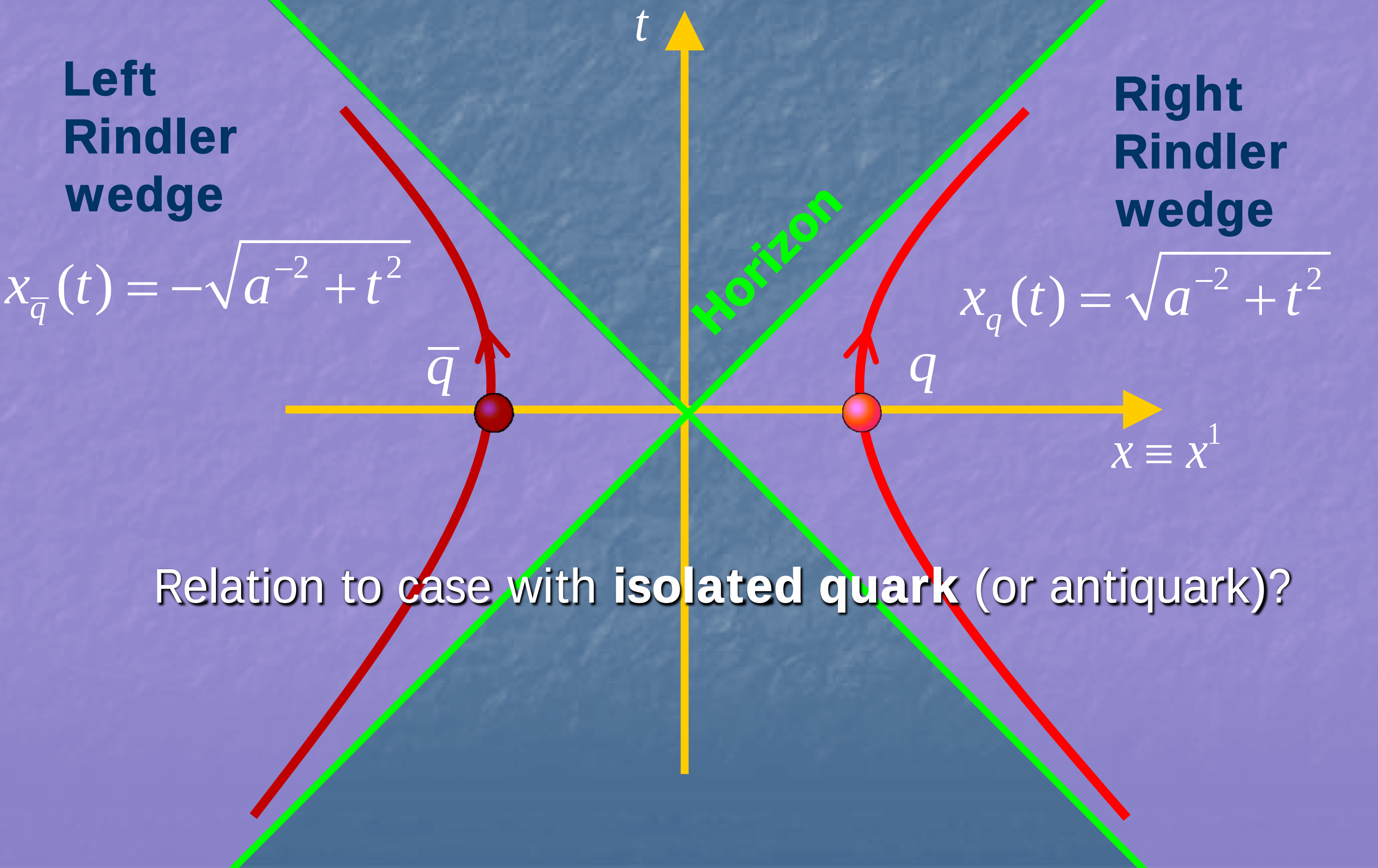
Jensen and Karch emphasized need for **permanent causal disconnection** between quark and antiquark:



But if this were all, we'd obtain a ws wormhole only for **asymptotically uniformly accelerated** motions

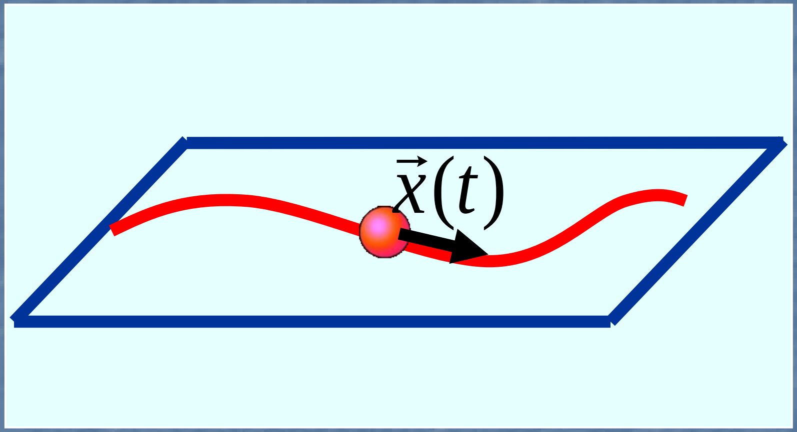
EPR don't require such a restriction
What happens for other motions?

For asympt. uniformly accelerated motion, the quark and antiquark can't influence one another



Relation to case with **isolated quark** (or antiquark)?

Isolated Quark



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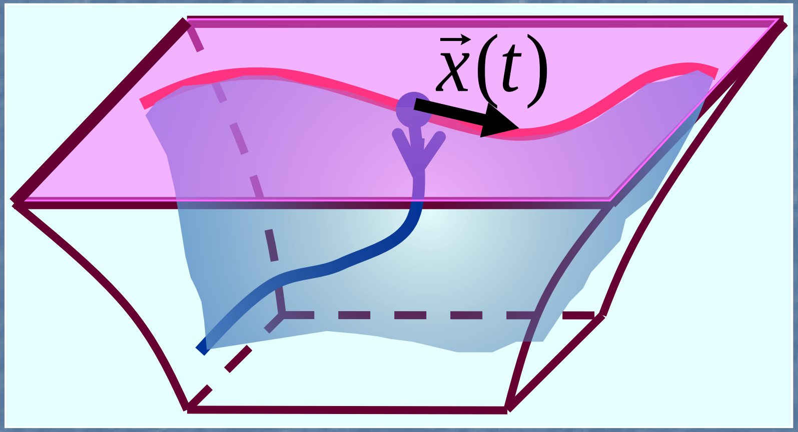


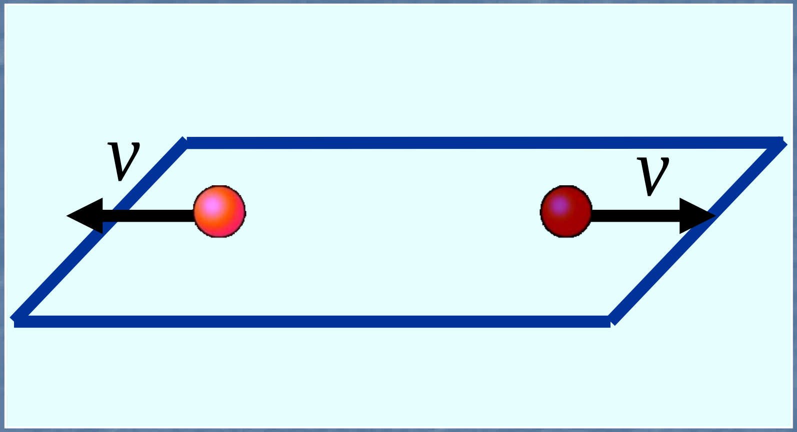
Solution for **arbitrary** timelike quark trajectory $\vec{x}(t)$

$$\vec{X}(t,z) = z\frac{\vec{v}(t_{\text{ret}})}{\sqrt{1-\vec{v}(t_{\text{ret}})^2}} + \vec{x}(t_{\text{ret}})$$

[Mikhailov]

$$t = z\frac{1}{\sqrt{1-\vec{v}(t_{\text{ret}})^2}} + t_{\text{ret}}$$

Accelerating Quark-Antiquark



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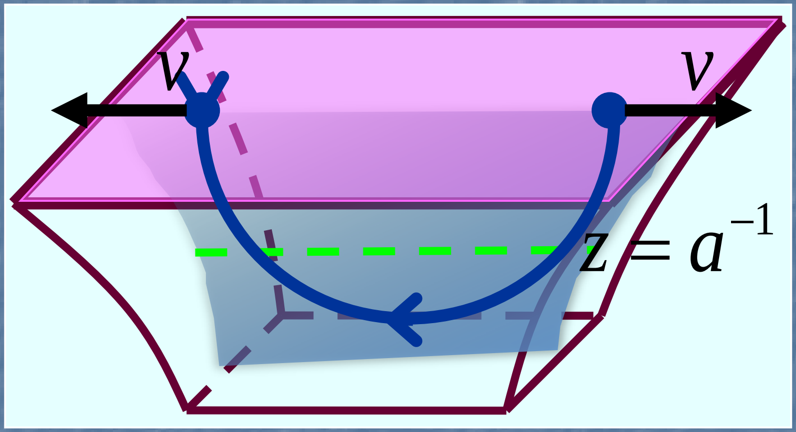
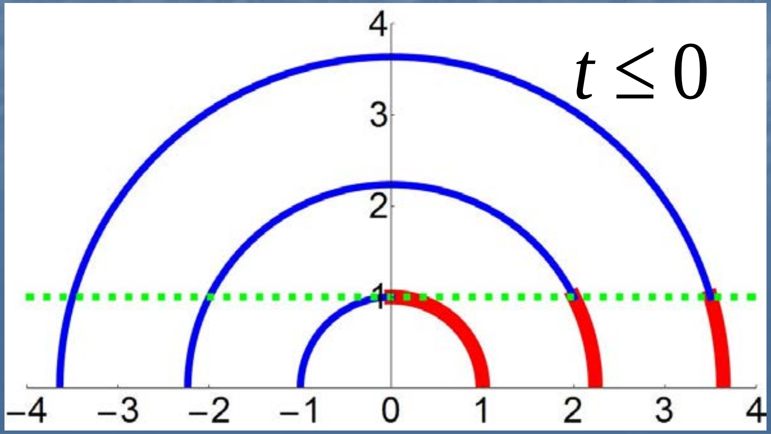
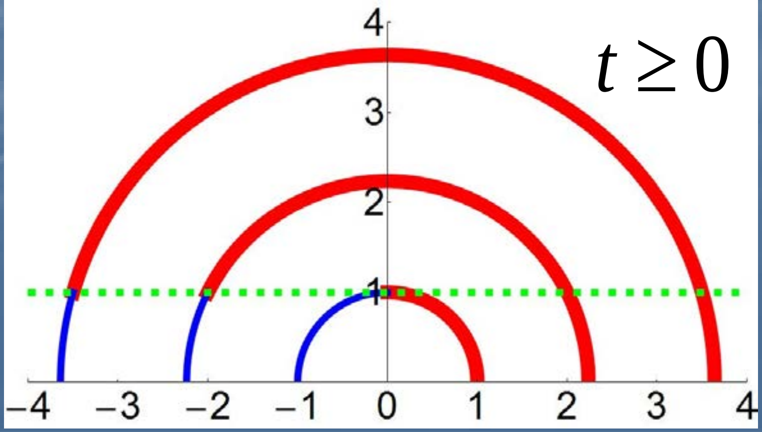


Xiao's solution for the **pair** is assembled by pasting together a retarded and an advanced version of Mikhailov's solution for **isolated** quark!

[García,AG,Pulido]





Accelerating Quark-Antiquark

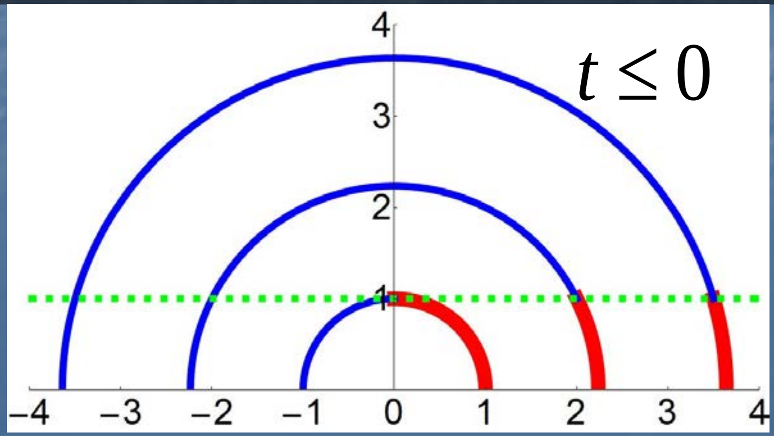
Generalizing, we can construct an **infinite class of analytic quark-antiquark** solutions, by pasting together any

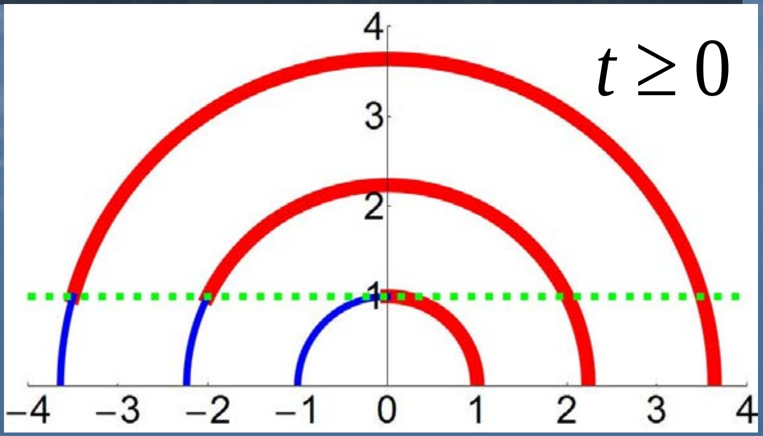
retarded solution for a quark that was uniformly accelerated **in the remote past**

with any

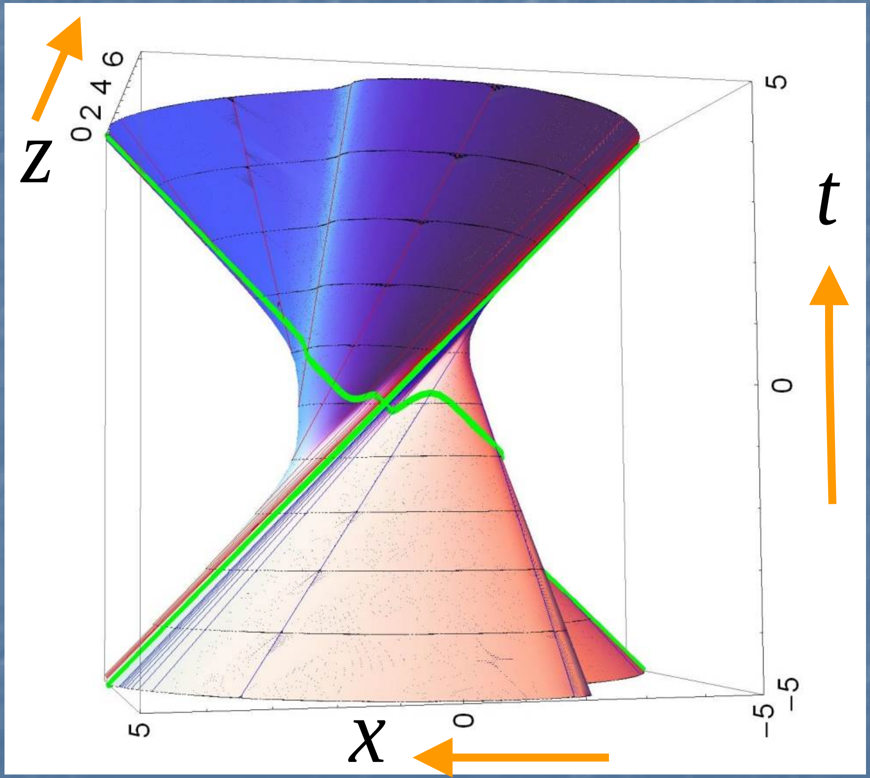
advanced solution for an antiquark that will be uniformly accelerated **in the remote future**

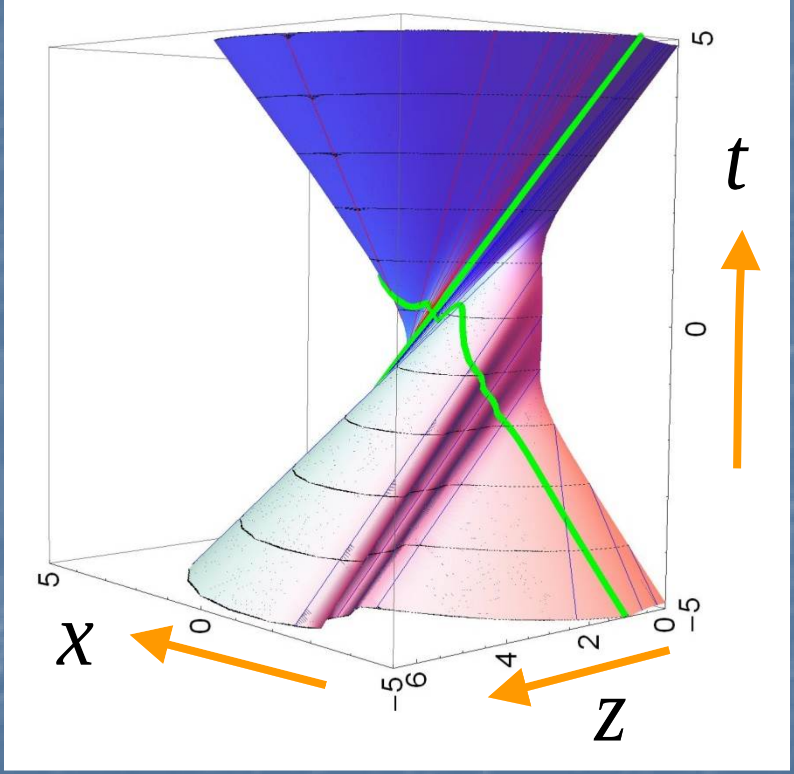
[Chernicoff,AG,Pedraza]





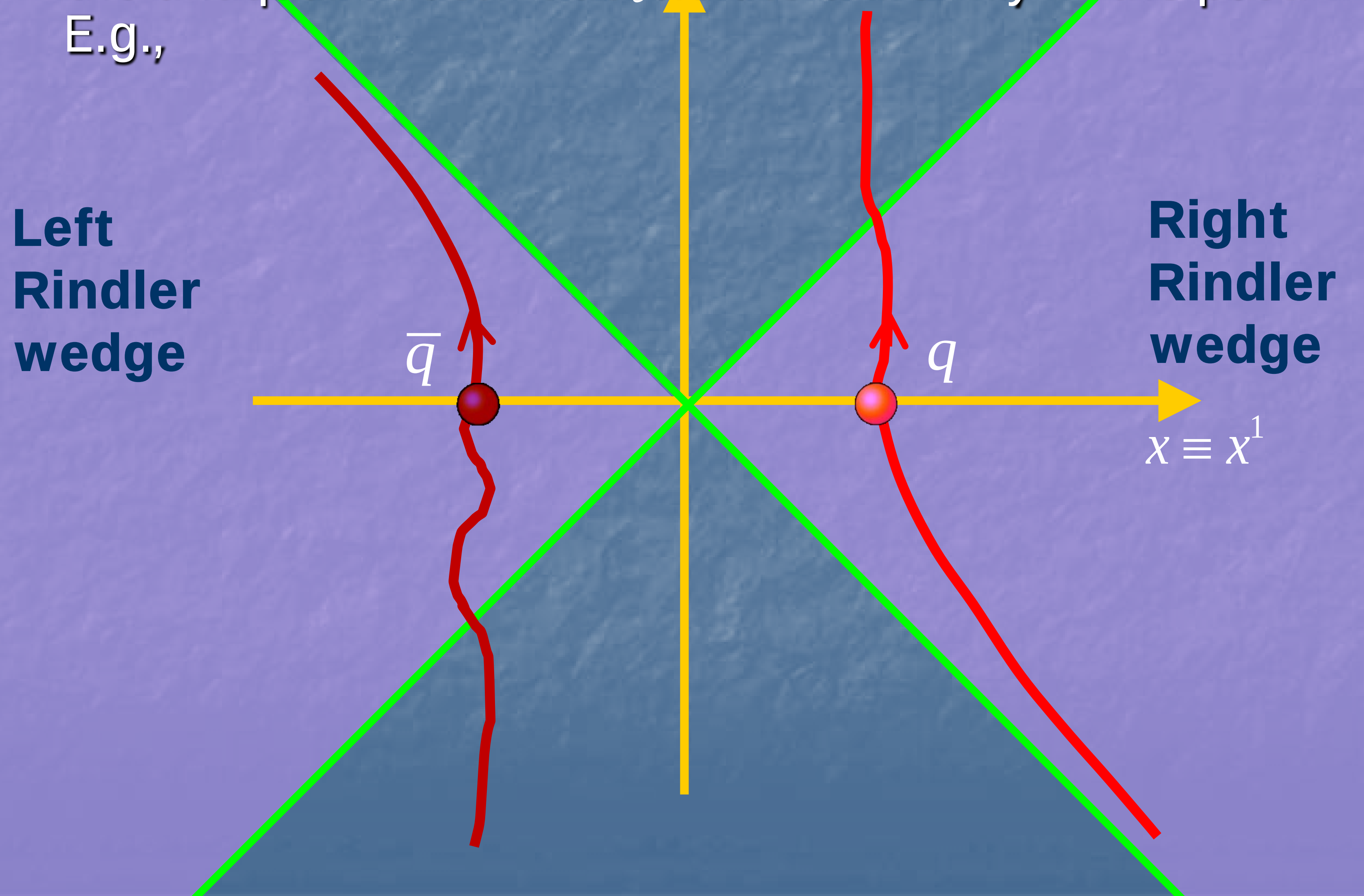
This includes all solutions envisioned by Jensen and Karch, where quark and antiquark asymptote to uniform acceleration BOTH in the past and in the future. E.g.



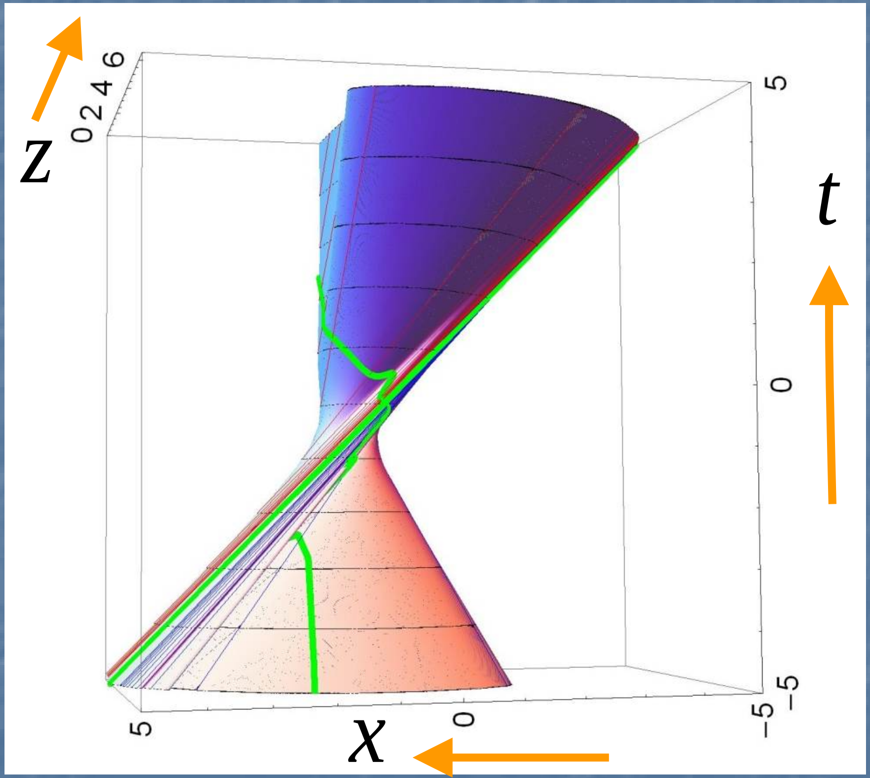


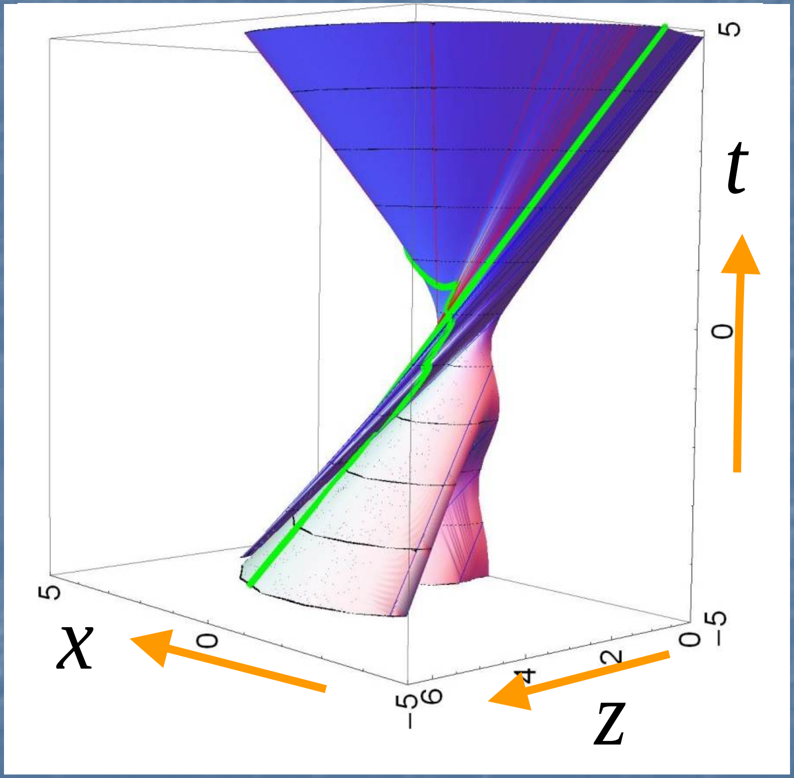
As expected, we get a worldsheet wormhole, inherited from spacetime causal structure

But our analytic solutions also include cases where the quark doesn't accelerate uniformly in the future, and/or the antiquark doesn't accelerate uniformly in the past. E.g.,



But our analytic solutions also include cases where the quark doesn't accelerate uniformly in the future, and/or the antiquark doesn't accelerate uniformly in the past. E.g.,





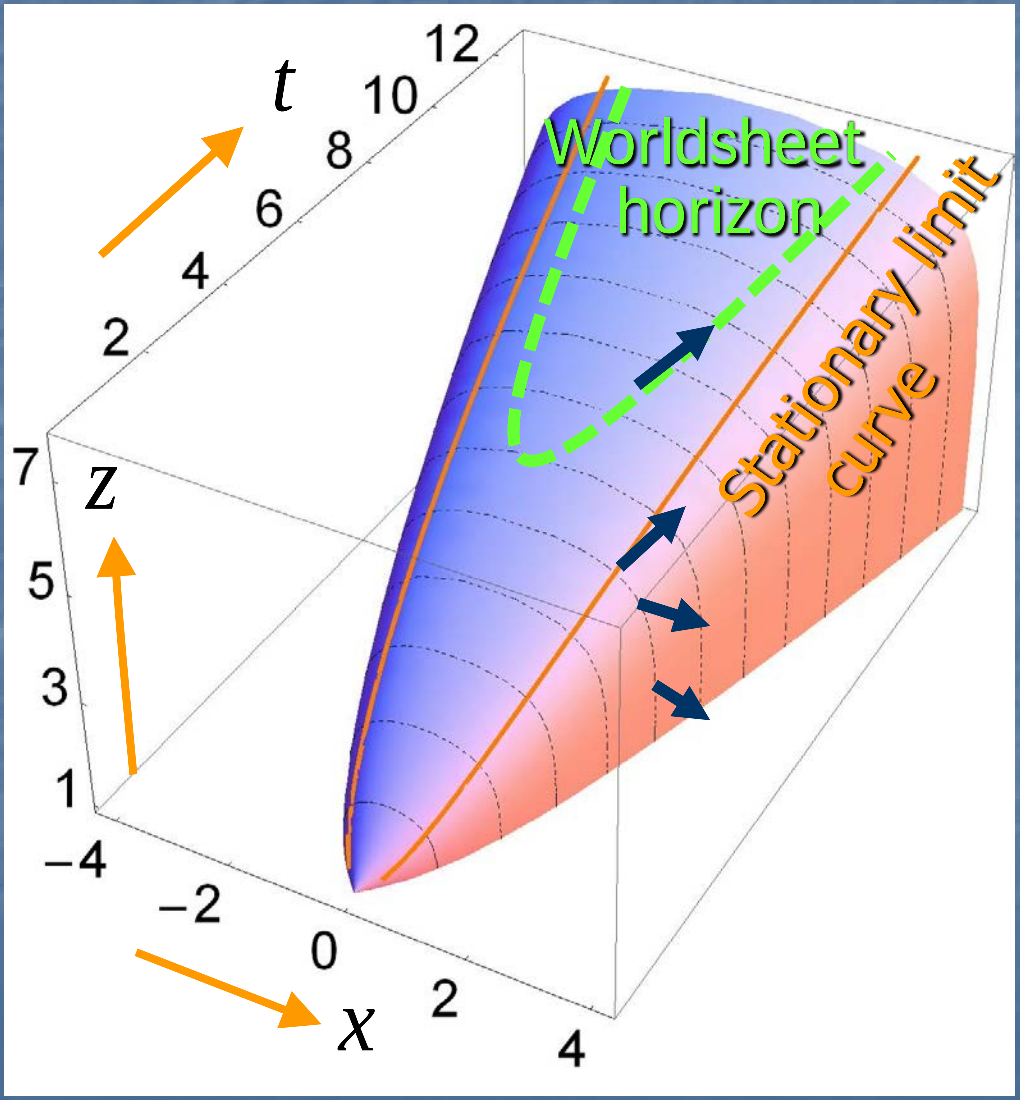
Now, signals CAN travel from antiquark to quark in CFT, but we **still get a wormhole** on the string worldsheet

One can examine many other examples: pair production, asymptotically nonforced, rotating, oscillating, etc.

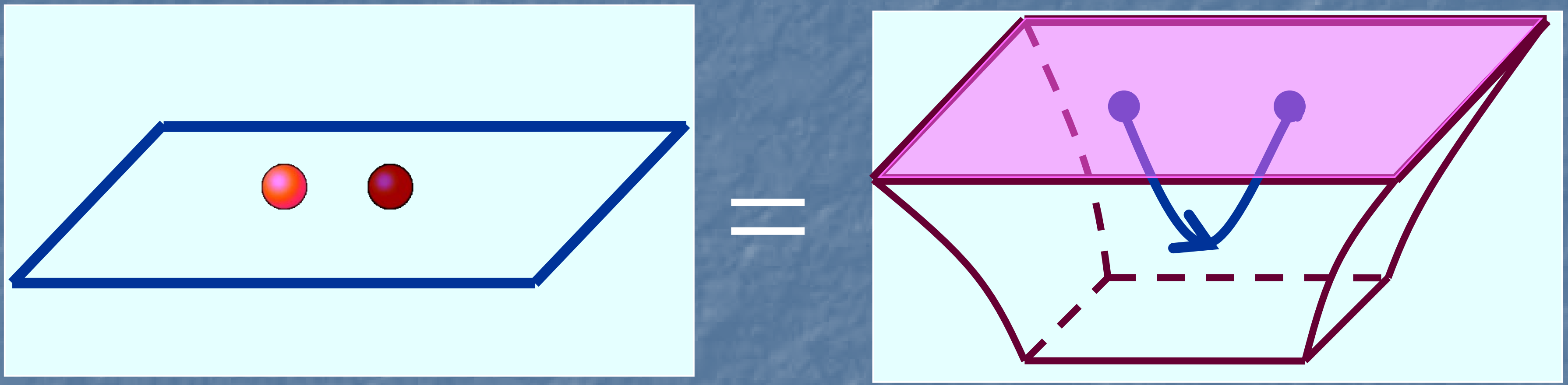
Bottom line:

We get a worldsheet wormhole iff the pair radiates

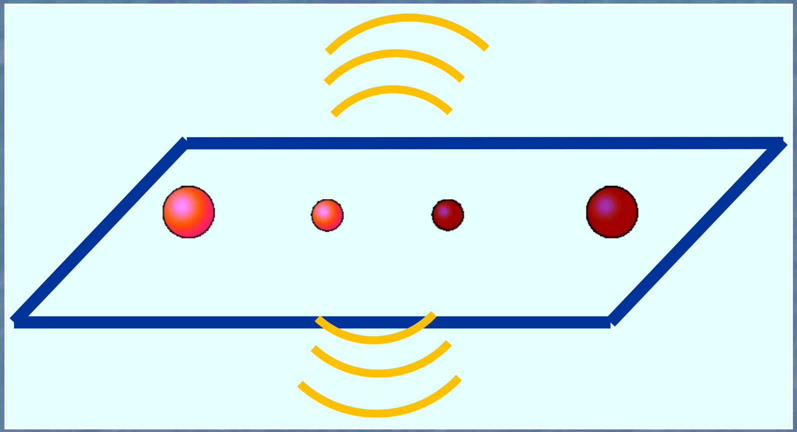
In the $N_c \gg 1, \lambda \gg 1$ regime, **pair emits radiation iff it becomes unbound**



A good example is to start with the pair at rest



A good example is to start with the pair at rest, move the quark & antiquark apart, injecting enough energy to unbind them, and then hold them still again



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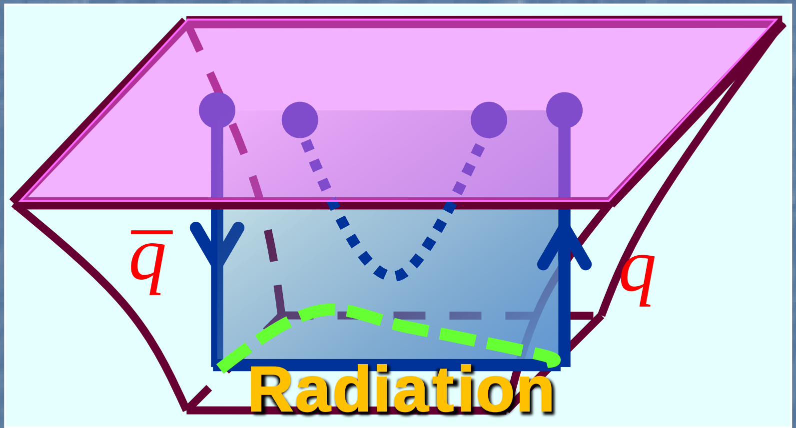


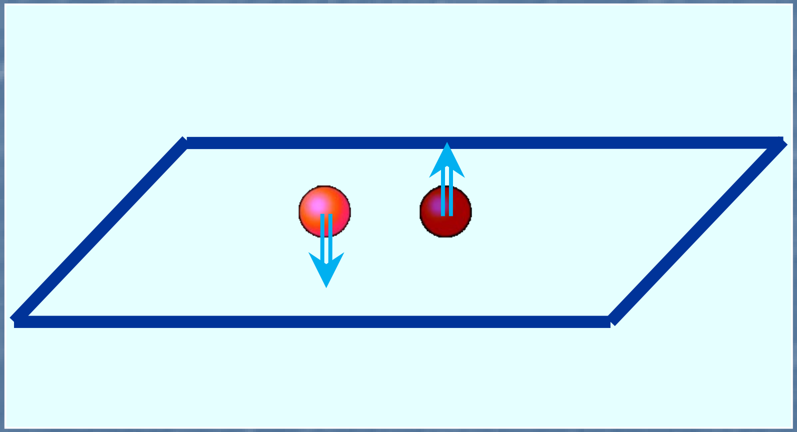
A double-sided worldsheet horizon (BH) appears, associated with energy loss via gluonic radiation [Chernicoff, AG]

So we see that a **wormhole** can form on string even when the **quark and antiquark remain in (bidirectional) causal contact in the CFT**

String wormhole signifies that quark can’t signal antiquark **via perturbations of their gluonic field** (gluonic field is a nonlinear medium, and gives rise to refraction index)

Holographic Pairs: Lessons for ER=EPR

- Worldsheet **wormholes** (always nontraversable) appear not only for uniform acceleration, but for rather **generic trajectories** —consistent with ER=EPR
- Wormhole forms iff **radiation** is emitted by the pair
- Our system is then much more complicated than a pair! (Analogous to spin-entangled e^+e^- which emit photons)



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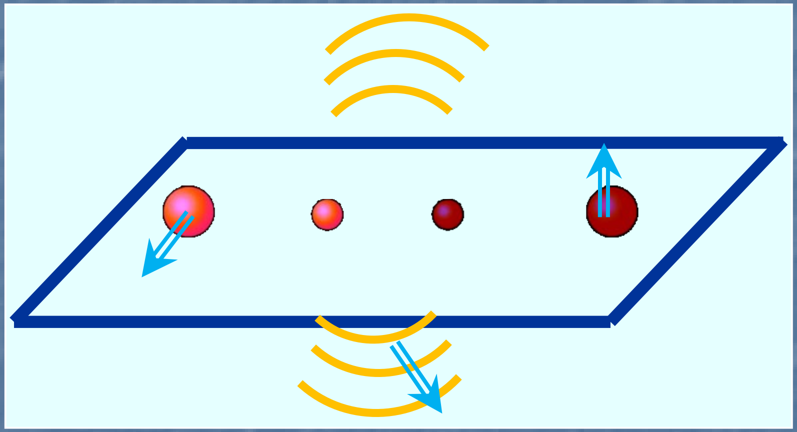

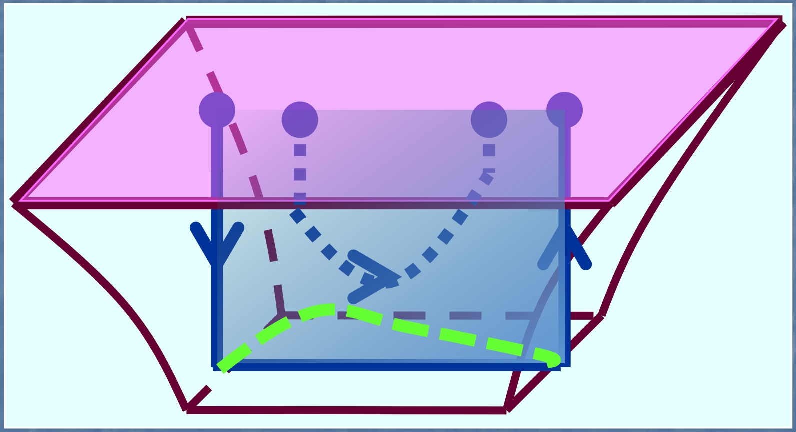

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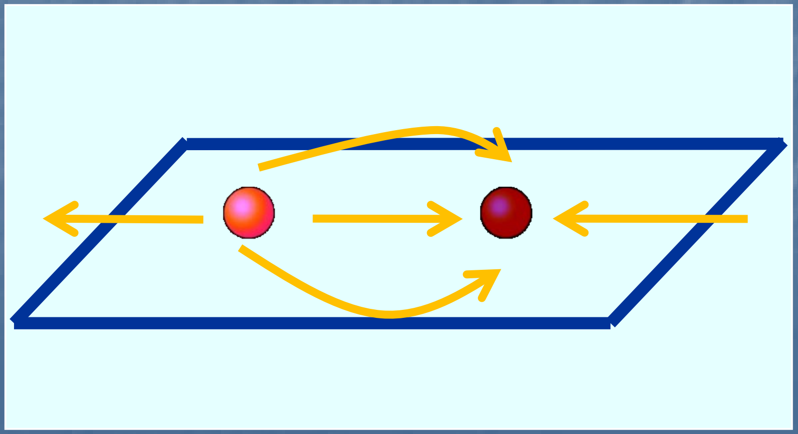

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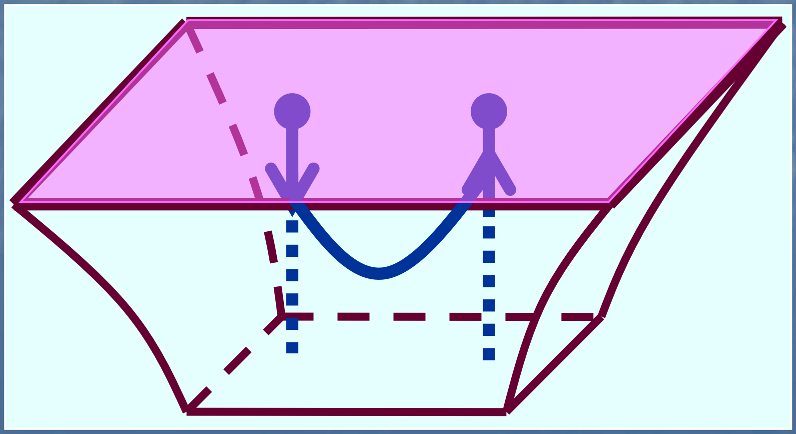

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This is worldsheet analog of spacetime w/o BH (e.g., pure AdS, or Minkowski): there IS entanglement between adjacent regions, but no INTRINSIC separation

(Equivalently, the only 'wormhole' is that between causal development of arbitrarily selected regions)

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- Whether string endpoints are in **causal contact across spacetime** is DISTINCT from contact **along string**

  (signals across spacetime vs. fluctuations in gluonic field jointly sourced by quark and antiquark)

Holographic Pairs: Lessons for ER=EPR

Notice this gives us a better understanding of what ER=EPR means in the context of the holographic correspondence:

Maldacena-Susskind

Entanglement in Gravity ⟷ Wormhole

AdS/CFT

Jensen-Karch

Entanglement in CFT

??

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AdS/CFT AdS/CFT

Entanglement in CFT ⟷ Gluonic ‘Wormhole’ (refraction index allows correlations but no signaling)