RG-improved BTZ BH.

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June, 2014

MEXICUERDAS 2014

Outline

Introduction

- Classical Gravity

$$S_{EH} = \frac{1}{16\pi G_N} \int \sqrt{g}\,(-R + \Lambda)$$

Coupling constants

$$G_N = 6.7x10^{-11}\,\frac{m^3}{kgs^2}, \qquad \Lambda \sim 10^{-35} s^{-2}$$

Valid from $10^{-2} - 10^{28}$ cm

- New phisics?

Extra Dimensions

Dark matter and Energy
Large distance corrections
Extra dimensions

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Problems with GR...

- General Relativity breaks down (Black holes, Big Bang).
    - High energy density.
    - Gravity dominates over the other interactions.
    - Information paradox.
- Not-so-General Relativity $\longrightarrow$ Fundamental Theory.
- Black Holes are the perfect lab to test this theory.

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- QFT aproach:
  - A simple dimensional analisis tell us that

$$[G_N] = 2 - D \longrightarrow g = G_N k^{-2+D}$$

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- Alternatives:
  - String theory
  - Quantum loop Gravity
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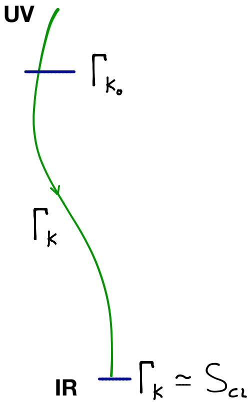
How to renormalize the non-renormalizable?



Renormalizatión Group

Wilson's Idea ['75]

- Couplings run with the energy scale
  In the path integral formulation, the modern implementation

$$Z_k[J] = e^{W_k[J]} = \int D[\phi] e^{-S[\phi]-\Delta S_k[\phi]+J\cdot\phi}$$

- Where $\Delta S_k[\phi] = \frac{1}{2}\int \phi \cdot \mathcal{R}_k(q^2)\cdot\phi$
  - $\lim_{p^2/k^2\to\infty} \mathcal{R}_k(q) \to 0$ we recover the original model
  - $\lim_{p^2/k^2\to 0} \mathcal{R}_k(q) \to \infty$ works as UV regulator
- With its Legendre transformation

$$\Gamma_k[\phi] = -W_k[J] + J\cdot\phi$$

Average Effective Action

R.G. Flow Equations

[Wetterich, '93]

$$\frac{\partial W_k}{\partial \ln k} = -e^{-W_k[J]} \frac{\partial \mathcal{R}_k}{\partial \ln k} \frac{\delta}{\delta J} e^{W_k[J]}$$

ERGE: $$\frac{\partial \Gamma_k}{\partial (\ln k)} = \frac{1}{2} Tr \left[\frac{\partial \mathcal{R}_k}{\partial \ln k} \left(\mathcal{R}_k + \frac{\partial^2 \Gamma_k}{\partial \phi^2} \right)^{-1} \right]$$

$$\frac{\partial \Gamma_k}{\partial (\ln k)} \rightarrow \begin{cases} \beta_\mu = \frac{\partial \mu(k)}{\partial (\ln k)}, \\ \beta_\nu = \frac{\partial \nu(k)}{\partial (\ln k)}, \\ \ldots \end{cases}$$

Fixed Points

- Critical points (ν_*) of the Renormalization Flow satisfy

$$\beta_{\nu_*} = \frac{\partial \nu_*}{\partial(\ln k)} = 0$$

  We call them *Gaussian Fixed Points* if $\nu_* = 0$ and *Non-Gaussian Fixed Points* if $\nu_* \neq 0$
- Renormalizability (UV completion)
  - If $\Gamma_{k\to\infty} = \Gamma_*$, Γ_k qualifies as a fundamental theory

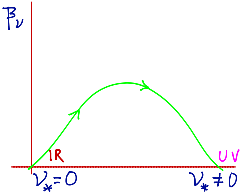


Figure : Asymptotic Safety Scenario

asymptotic safe theories

Although originally proposed by Steven Weinberg ['76] to find a theory of quantum gravity, the idea of a non-Gaussian fixed point providing UV completion has been applied also to other perturbatively nonrenormalizable field theories.

- Sigma non-linear [Brézin, Zinn-Justin]
- Gross-Neveu [Gadewdzki, Kupiainen]
- Sine-Gordon [Gordon]

The Einstein-Hilbert Truncation

If we consider the Einstein-Hilbert truncation as our effective action [M. Reuter]

$$\Gamma_k = \frac{1}{G(k)} \int d^D x \sqrt{g} [-R + \Lambda(k)]$$

we hope to find a system of two coupled beta functions

$$\frac{\partial \Gamma_k}{\partial(\ln k)} \to \begin{cases} \beta_\lambda = \dfrac{\partial \lambda(k)}{\partial(\ln k)}, \\ \beta_g = \dfrac{\partial g(k)}{\partial(\ln k)}. \end{cases}$$

expressed in terms of the adimensional couplings

$$\lambda(k) = \Lambda(k) k^{-2} \qquad g(k) = G(k) k^{-2+D}$$

2+1-Dimensional Asymptotic Safe Gravity

When we consider the Einstein-Hilbert truncation with D=3.
By means of the ERGE ($\frac{\partial \Gamma_k}{\partial \ln k}$) with a particular regulator [Litim,'05] we find the coupled equations

$$\beta_\lambda = a(b + 20g)^{-1}$$
$$\beta_g = gb(b + 20g)^{-1},$$

where

$$a = -16\lambda^3 + 8(2 - 15g)\lambda^2 + 4(12g - 1)\lambda + 120g^2,$$
$$b = 8(\lambda^2 - \lambda - 3g) + 2$$

The phase portrait of the 3D RG

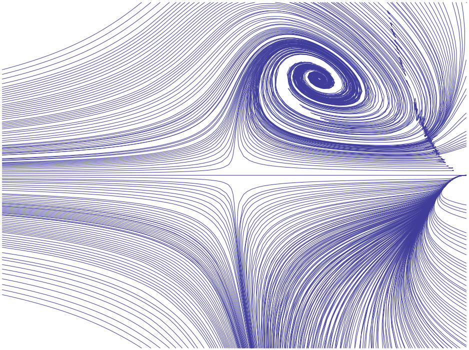


Figure : Full phase portrait of the 3-Dimensional Einstein-Hilbert Renormalización Group.

The phase portrait of the 3D RG

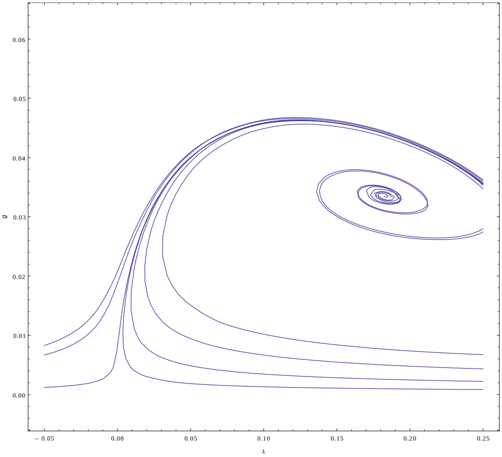


Figure : 3-D Phase portrait when we consider only positive values for g.

We have found GPF and NGFP

- Gaussian:

$$\lambda_* = 0, \quad g_* = 0$$

- NG1:

$$\lambda_* = \frac{1}{2}, \quad g_* = 0$$

- NG2

$$\lambda_* = \frac{\sqrt{3}-1}{4}, \quad g_* = -\frac{15\pi^{3/2}(\sqrt{3}-2)}{64}$$

Analyzing the Fixed Points stability

$$\begin{pmatrix} \dot{\lambda} \\ \dot{g} \end{pmatrix} = A \begin{pmatrix} \lambda \\ g \end{pmatrix}$$

where

$$A = \begin{pmatrix} \frac{\partial \beta_\lambda}{\partial \lambda} & \frac{\partial \beta_\lambda}{\partial g} \\ \frac{\partial \beta_g}{\partial \lambda} & \frac{\partial \beta_g}{\partial g} \end{pmatrix}_*$$

Eigenvalues:

- Gaussian Fixed Point:

$$\alpha_\lambda < 0, \quad \alpha_g > 0 \qquad \text{Saddle point}$$

- NG2:

$$\text{Re } \alpha_\lambda < 0, \quad \text{Re } \alpha_g > 0 \qquad \text{Atractor}$$

- Gaussian Fixed point: $g(k) = 0 = \lambda(k)$

$$G(k) \sim G_N, \quad \Lambda(k) \sim \Lambda$$

Domain of the General Relativity.

- IR fixed point (NG1): $k \to 0$

$$G(k) = g_* k^{-1} > G_N$$

- UV fixed point (NG2): $k \to \infty$

$$G(k) = g_* k^{-1} < G_N$$

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Conclusions

- 3-D *Asymptotic safe* gravity (Einstein-Hilbert truncation)
- In the domain of the Gaussian Fixed point we recover the classical action

$$\Gamma_k = \frac{1}{G(k)} \int d^3x \sqrt{g}(-R + \Lambda(k))$$

$$\Downarrow$$

$$S_{EH} = \frac{1}{G_N} \int d^3x \sqrt{g}(-R + \Lambda)$$

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An analitic (spherical-cow) solution

So far we haven't mentioned anything about the explicit solution for the R.G. (This is not trivial)
So we consider a first approximation expanding the beta functions up to second order in g and λ, i.e.

$$\beta_\lambda = 60g(k)^2 - 2\lambda(k), \tag{1}$$

$$\beta_g = g(k)\left(1 - 10g(k)\right), \tag{2}$$

we can formulate the solutions

$$g_k = \frac{G_N k}{1 + \frac{G_N k}{g^*}}, \tag{3}$$

$$\lambda(k) = \lambda^* + \frac{\Lambda}{k^2} - \frac{4g^*\lambda^*}{G_N k} + \frac{2g^*\lambda^{*2}}{G_N^2 k^2}\log\left(1 + \frac{G_N k}{g^*}\right) + \frac{g^*\lambda^*}{5G_N^2 k^2(1 + \frac{G_N k}{g^*})} \tag{4}$$

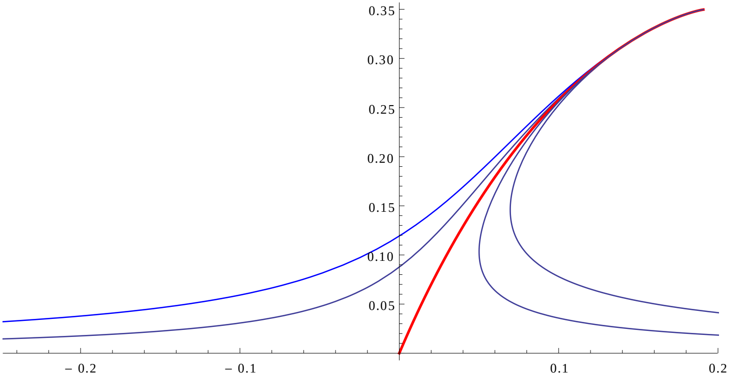

Figure : Trajectories of the aproximate solution for the 3-D Renormalization Group. The red line is known as *directriz*.

BTZ Black Hole

[Bañados, Teitelboim, Zanelli, '92]

- $$S_{\rm EH} = \frac{1}{8\pi G_N}\int d^3x\sqrt{g}(R-\Lambda)$$
- Solution with line element
$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$
- where we define the lapse function
$$f(r) = -8MG_N - \Lambda r^2 + \frac{16G_N^2 J^2}{r^2}.$$
- It has two horizons
$$r_\pm^2 = \frac{8G_N M}{\Lambda}\left[1 \pm \left(1 - \frac{\Lambda J^2}{2M^2}\right)^{1/2}\right]$$

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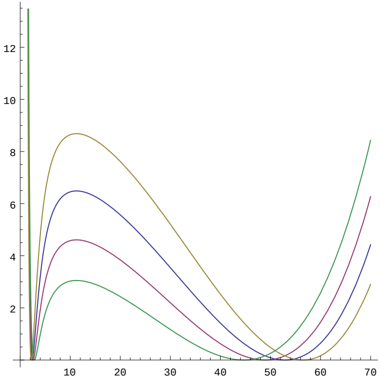

Figure : The classic lapse function $f(r)^2$. Two horizons for various mass values can be observed

R.G. Improved BTZ Black Hole

How to iprove the black hole space-time?

- promoting the constant couplings to running couplings,

$$f_k(r) = -8MG_k - \Lambda_k r^2 + 16\frac{G_k^2 J^2}{r^2}. \tag{5}$$

where we already know $g(k)$ and $\lambda(k)$, but we want the lapse as a function that depends only on r.

We need to know the relationship between momentum and distance. An analysis in dimensions allow us to propose that this relationship is given by

$$k(P) = \frac{\xi}{D(P)} \tag{6}$$

where

$$D(P) = \int_c \sqrt{ds^2}. \tag{7}$$

$$= r\left[\frac{1}{8G_N M}\left(1 - \frac{2G_N J^2}{Mr^2}\right)\right]^{1/2} \tag{8}$$

An expansion on $r \to \infty$ result in $D \sim r/\sqrt{8G_N M}$, so we can fix the parameter ξ as

$$\xi = \sqrt{8G_N M} \tag{9}$$

therefore

$$k(r) = \frac{8G_N M}{r}\left(1 - \frac{2G_N J^2}{Mr^2}\right)^{-1/2} \tag{10}$$

and we have finally the improved lapse function $f_k(r)$.

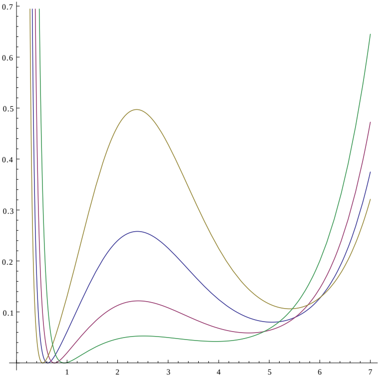

Figure : Radial dependence of the improved lapse. The outer singularity is removed.

Improved BTZ Thermodynamics

The extremal scenario where the two radii tend to

$$r_{\mathrm{Ext}}^2 = \frac{8G_N M}{\Lambda} \tag{11}$$

has been considerated to observe the behavior of the various thermodynamic quantities of the BTZ improved solution.

- Bekenstein-Hawking temperature $T = \frac{f_k'(r_{\mathrm{Ext}})}{4\pi}$
- Pressure

$$P = -\frac{\Lambda_k}{8\pi G_k}$$

- Entropy

$$S = \frac{\pi r_{\mathrm{Ext}}}{2G_k}$$

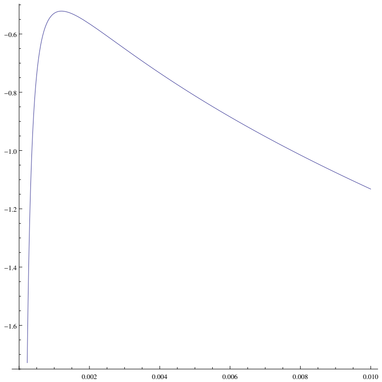

Figure : The improved BTZ Bekenstein-Hawking temperature vs. mass.

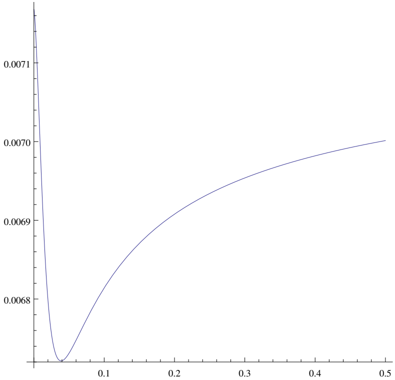

Figure : The improved BTZ pressure.

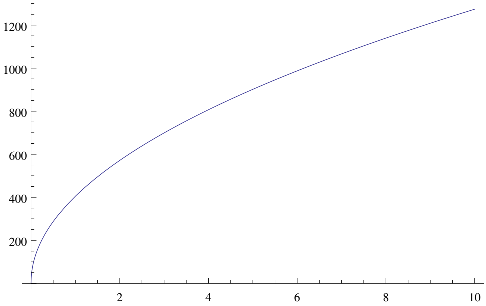

Figure : The improved BTZ entropy.

Summary:

- 2+1 Dimensional Gravity is non-perturvative renormalizable.
- We propose an analitic solution for the RG.
- We have improved the BTZ space-time by means of this solution and the $k - r$ IR identification.
- We have also found improved thermodynamic properties of the extremal black hole.

Thanks