The double copy: Gravity from gluons

Andrés Luna Godoy

University of Glasgow

August 2, 2018

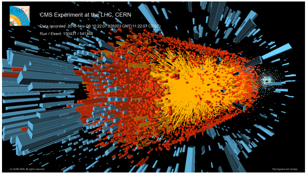
About this talk

- Not stringy enough, but I hope you'll find it cool.
- First part is very sketchy, a story about amplitudes and wine.
- Second part based on work in collaboration with Monteiro, Nicholson, Ochirov, O'Connell and White. The classical double copy.

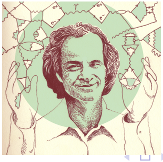
PART I: Amplitudes, the double copy and wine(!)

Scattering amplitudes

- The scattering amplitude determines the probability that a set of incoming particles (an initial state) scatters into some set up of outgoing particles (a final state).
- Necessary to compute cross sections and decay rates (indispensable in collider physics)



- We learn to compute them using Feynman diagrams, but even for relatively simple problems, the number of diagrams is excessive. Need more efficient ways to compute.



Amplitudes conference

Amplitudes workshop in 2009 (Copenhagen)

Hidden Structures in Field Theory Amplitudes 2009

12-14 August 2009
Niels Bohr International Academy
Europe/Copenhagen timezone

Speakers included

Zvi Bern	(UCLA)
James Drummond	(LAPTH, Annecy)
David Dunbar	(Swansea)
Andrew Hodges	(Oxford)
Paul Howe	(King's College, University of London
Gregory Korchemsky	(Orsay & Saclay)
David Kosower	(Saclay)

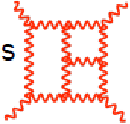
All this in the hope of winning bottles of wine,

Some old opinions

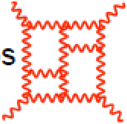
- If certain patterns that emerge should persist in the higher orders of perturbation theory, then $\mathcal{N} = 8$ supergravity in four dimensions would have ultraviolet divergences starting at three loops. Green, Schwarz, Brink, (1982)
- It is therefore very likely that all supergravity theories will diverge at three loops in four dimensions The final word on these issues may have to await further explicit calculations. Marcus, Sagnotti (1985)

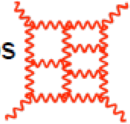
Gravity Feynman diagrams

Suppose we want to compute explicitly...

3 loops  $\sim 10^{20}$ TERMS

No surprise it has never been calculated via Feynman diagrams.

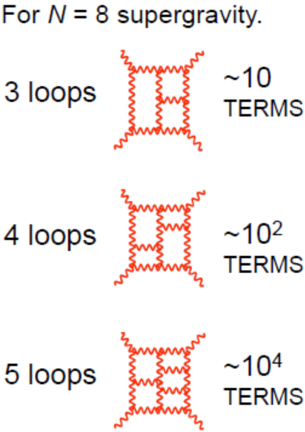
4 loops  $\sim 10^{26}$ TERMS

5 loops  $\sim 10^{31}$ TERMS

More terms than atoms in your brain!

With modern amplitudes methods

Bern, Carrasco, Dixon, Johansson, Roiban, 2008



Scatering Amplitudes: Modern methods

- There are too many Feynman diagrams, need more efficient ways to compute amplitudes.

Scatering Amplitudes: Modern methods

- There are too many Feynman diagrams, need more efficient ways to compute amplitudes.

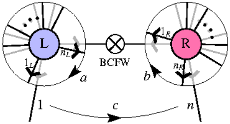


Figure: Recursion relations.

Scatering Amplitudes: Modern methods

- There are too many Feynman diagrams, need more efficient ways to compute amplitudes.

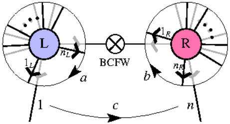


Figure: Recursion relations.

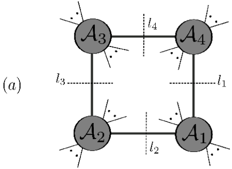


Figure: Generalized unitarity.

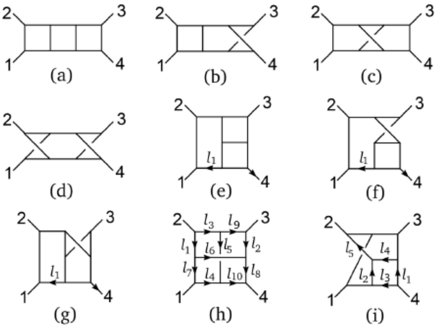
KLT Relations

- The KLT relations, derived in string theory by Kawai, Lewellen and Tye express the n-point tree-level closed string scattering amplitude as a sum over products of n-point open string string partial amplitudes.
- In the limit of infinite tension, the closed string spin-2 states become gravitons and the open-string spin-1 states become gluons.
- The field theory version of KLT then reads:

$$
\begin{aligned}
M_4^{\text{tree}}(1234) &= -s_{12}\, A_4^{\text{tree}}[1234]\, A_4^{\text{tree}}[1243]\,,\\
M_5^{\text{tree}}(12345) &= s_{23}s_{45}\, A_5^{\text{tree}}[12345]\, A_5^{\text{tree}}[13254] + (3 \leftrightarrow 4)\,,\\
M_6^{\text{tree}}(123456) &= -s_{12}s_{45} A_6^{\text{tree}}[123456]\Big(s_{35} A_6^{\text{tree}}[153462] + (s_{34}+s_{35}) A_6^{\text{tree}}[154362]\Big)\\
&\quad + \mathcal{P}(2,3,4)\,.
\end{aligned}
$$

Three loops (and graphs)

$$\frac{(-i)^L}{g^{n-2+2L}}\mathcal{A}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$



Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills	$\mathcal{N}=8$ Supergravity
(a)–(d)	s^2	$[s^2]^2$
(e)–(g)	$s(l_1+k_4)^2$	$[s\,(l_1+k_4)^2]^2$
(h)	$sl_{1,2}^2+tl_{3,4}^2-sl_5^2-tl_6^2-st$	$(sl_{1,2}^2+tl_{3,4}^2-st)^2-s^2(2(l_{1,2}^2-t)+l_5^2)l_5^2-t^2(2(l_{3,4}^2-s)+l_6^2)l_6^2$ $-s^2(2l_1^2l_8^2+2l_2^2l_7^2+l_1^2l_7^2+l_2^2l_8^2)-t^2(2l_3^2l_{10}^2+2l_4^2l_9^2+l_3^2l_9^2+l_4^2l_{10}^2)+2stl_5^2l_6^2$
(i)	$sl_{1,2}^2-tl_{3,4}^2-\frac{1}{3}(s-t)l_5^2$	$(sl_{1,2}^2-tl_{3,4}^2)^2-(s^2l_{1,2}^2+t^2l_{3,4}^2+\frac{1}{3}stu)l_5^2$

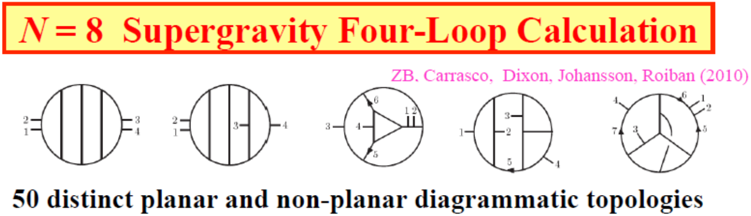
$\mathcal{N}=8$ Sugra is finite to three loops in $D=4$.

Four loops (and wine)

In 2009 Bossard, Howe and Stelle betted that maximal supergravity was likely to diverge at four loops in $\mathrm{D} = 5$ and at five loops in $\mathrm{D} = 4$

Four loops (and wine)

In 2009 Bossard, Howe and Stelle betted that maximal supergravity was likely to diverge at four loops in D = 5 and at five loops in D = 4



UV finite for D = 4 and 5. Actually finite for $D < 11/2$. Very very finite.

Five loops (and wine)

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**

5 loops

Kelly Stelle:
English wine
"It will diverge"

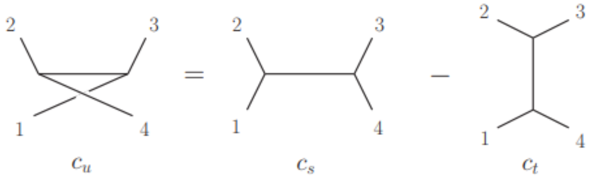
Zvi Bern:
California wine
"It won't diverge"

Seven loops (and wine)

- At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?

Colour kinematics

The graphs contributing to 4-gluon scattering are



Their colour factors satisfy the relation

$$c_u = c_s - c_t,$$

(this is a Jacobi relation), where,

$$c_u \equiv \tilde{f}^{a_4 a_2 b} \tilde{f}^{b a_3 a_1},\ c_s \equiv \tilde{f}^{a_1 a_2 b} \tilde{f}^{b a_3 a_4},\ c_t \equiv \tilde{f}^{a_2 a_3 b} \tilde{f}^{b a_4 a_1}$$

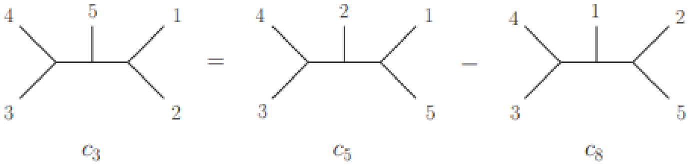
And so do their numerators

$$n_u = n_s - n_t,$$

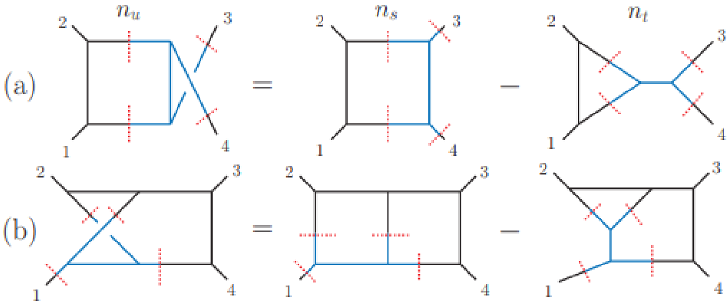
This relation is called colour-kinematics duality.

Colour kinematics

This duality can be extended to higher points



and loop level



The BCJ double copy

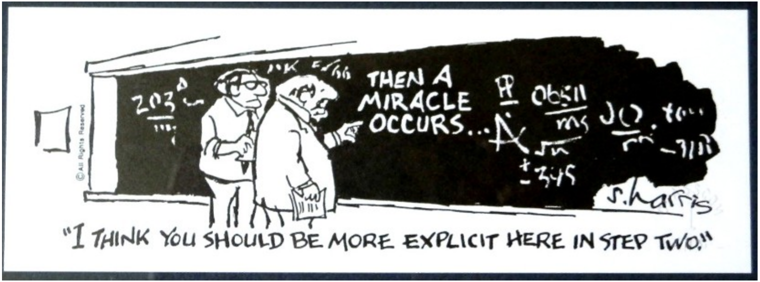
The BCJ double copy relates scattering amplitudes in Yang-Mills theory

$$\mathcal{A}^{\mathrm{YM}} = \sum_{i \in \text{cubic}} \frac{c_i n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}.$$

(assuming they satisfy colour-kinematics duality)

$$c_i = -c_j \Leftrightarrow n_i = -n_j$$
$$c_i + c_j + c_k = 0 \Leftrightarrow n_i + n_j + n_k = 0,$$

THEN A MIRACLE OCCURS...



The BCJ double copy

The BCJ double copy relates scattering amplitudes in Yang-Mills theory

$$\mathcal{A}^{\text{YM}} = \sum_{i \in \text{cubic}} \frac{c_i n_i}{\prod_{\alpha_i} p_{\alpha_i}^2}.$$

(assuming they satisfy colour-kinematics duality)

$$c_i = -c_j \Leftrightarrow n_i = -n_j$$
$$c_i + c_j + c_k = 0 \Leftrightarrow n_i + n_j + n_k = 0,$$

with scattering amplitudes in gravity theory

$$\mathcal{M}^{\text{Gravity}} = \sum_{i \in \text{cubic}} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$

Some comments on the BCJ double copy

- Existence of CK dual representations proven to tree level. Conjectured to loop level.
- Double copy proven if CK dual representation exists.
- Extended to web of theories: String theories, NLSM, DBI, Bi-adjoint scalar theories, etc.

Loops evolution

Colour-kinematic dual representations were found for $\mathcal{N} = 4$ SYM, and subsequently numerators for $\mathcal{N} = 8$ Sugra at:

- 3-loops in 2010
- 4-loops in 2012
- 5-loops* in 2017

Bern and I

Zvi Bern interviewed me for a postdoc in UCLA back in December. An excerpt:

...so, are you doing 6-loops now?

It is in the realm of possibilities. It depends a lot on what happens in the next few months...

What happened...

arXiv.org > hep-th > arXiv:1804.09311

Search or A

(Help | Advance

High Energy Physics - Theory

Ultraviolet Properties of N = 8 Supergravity at Five Loops

Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, Mao Zeng

(Submitted on 25 Apr 2018)

We use the recently developed generalized double-copy construction to obtain an improved representation of the five-loop four-point integrand of $N = 8$ supergravity whose leading ultraviolet behavior we analyze using state of the art loop-integral expansion and reduction methods. We find that the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5$, corresponding to a $D^8 R^4$ counterterm. This ultraviolet behavior stands in contrast to the cases of four-dimensional $N = 4$ supergravity at three loops and $N = 5$ supergravity at four loops whose improved ultraviolet behavior demonstrates enhanced cancellations beyond implications from standard-symmetry considerations. We express this $D_c = 24/5$ divergence in terms of two relatively simple positive-

What happened...

arXiv.org > hep-th > arXiv:1804.09311

Search or A

(Help | Advance

High Energy Physics - Theory

Ultraviolet Properties of N = 8 Supergravity at Five Loops

Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, Mao Zeng

(Submitted on 25 Apr 2018)

We use the recently developed generalized double-copy construction to obtain an improved representation of the five-loop four-point integrand of $N = 8$ supergravity whose leading ultraviolet behavior we analyze using state of the art loop-integral expansion and reduction methods. We find that the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5$, corresponding to a $D^8 R^4$ counterterm. This ultraviolet behavior stands in contrast to the cases of four-dimensional $N = 4$ supergravity at three loops and $N = 5$ supergravity at four loops whose improved ultraviolet behavior demonstrates enhanced cancellations beyond implications from standard-symmetry considerations. We express this $D_c = 24/5$ divergence in terms of two relatively simple positive-

What happened...

arXiv.org > hep-th > arXiv:1804.09311

Search or A

(Help | Advance

High Energy Physics - Theory

Ultraviolet Properties of N = 8 Supergravity at Five Loops

Zvi Bern, John Joseph Carrasco, Wei-Ming Chen, Alex Edison, Henrik Johansson, Julio Parra-Martinez, Radu Roiban, Mao Zeng

(Submitted on 25 Apr 2018)

We use the recently developed generalized double-copy construction to obtain an improved representation of the five-loop four-point integrand of $N = 8$ supergravity whose leading ultraviolet behavior we analyze using state of the art loop-integral expansion and reduction methods. We find that the five-loop critical dimension where ultraviolet divergences first occur is $D_c = 24/5$, corresponding to a $D^8 R^4$ counterterm. This ultraviolet behavior stands in contrast to the cases of four-dimensional $N = 4$ supergravity at three loops and $N = 5$ supergravity at four loops whose improved ultraviolet behavior demonstrates enhanced cancellations beyond implications from standard-symmetry considerations. We express this $D_c = 24/5$ divergence in terms of two relatively simple positive-

5 loops

- **At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?**

Kelly Stelle: English wine "It will diverge"

Zvi Bern: California wine "It won't diverge"

Bern and I

Zvi Bern interviewed me for a postdoc in UCLA back in December. An excerpt:

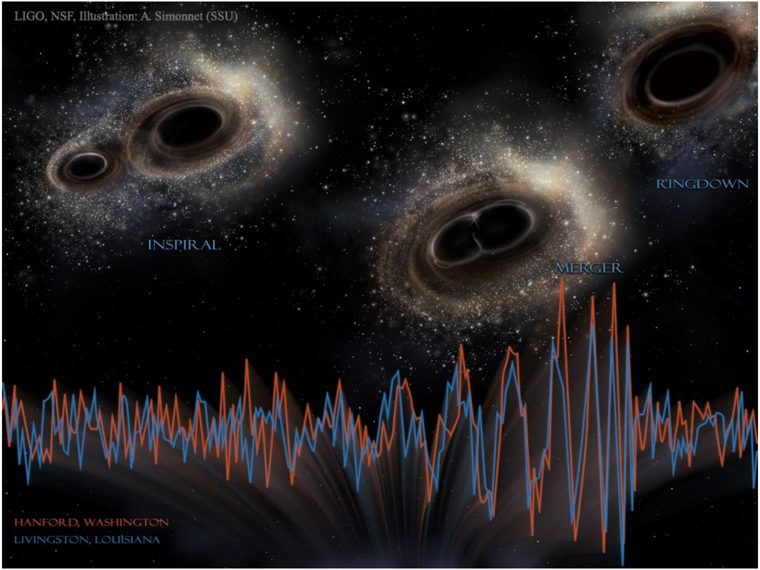
...so, are you doing 6-loops now?

It is in the realm of possibilities. It depends a lot on what happens in the next few months.
Anyway, we would want you to focus on black holes scattering now...

Part II: Black holes and revelations

Loops are certainly hard!
Can we see any classical manifestations of the double copy?

How 'bout them black holes?



Black holes and the double copy

- Although the double copy is a purely perturbative statement, efforts have been made to extend it in a non perturbative way

- Relation of black hole solutions (Kerr-Schild) and abelian (abelianised) gauge fields (Monteiro, O'Connell, White 2014)

Kerr-Schild metrics

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \kappa h_{\mu\nu}, \\ &= \eta_{\mu\nu} + \kappa k_\mu k_\nu \phi, \end{aligned}$$

Abelian (anelianised) gauge fields

$$A^a_\mu = c^a \phi k_\mu,$$

- Kerr-Schild solutions satisfy the conditions

$$k_\mu \eta^{\mu\nu} k_\nu = 0 = k_\mu g^{\mu\nu} k_\nu, \qquad k^\mu \partial_\mu k_\nu = 0,$$

so the Ricci tensor is linear

$$R^\mu{}_\nu = \frac{1}{2}\left(\partial^\mu \partial_\alpha (\phi k^\alpha k_\nu) + \partial_\nu \partial^\alpha (\phi k_\alpha k^\mu) - \partial^2 (\phi k^\mu k_\nu)\right).$$

Black holes and the double copy

- If we consider the stationary limit and the condition $k^0 = 1$, some components of the Ricci tensor take the form

$$R^0{}_0 = \frac{1}{2}\partial_i\partial^i\phi,$$
$$R^i{}_0 = -\frac{1}{2}\partial_j\left[\partial^i(\phi k^j) - \partial^j(\phi k^i)\right].$$

- We define the vector field

$$A^a_\mu = c^a\phi k_\mu,$$

- It's easy to show that the vacuum Einstein equations imply Maxwell equations

$$\partial_\mu F^{\mu\nu} = \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = 0.$$

Black holes and the double copy

- Relation of black hole solutions (Kerr-Schild) and abelian (abelianised) gauge fields

Kerr-Schild metrics

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \\ = \eta_{\mu\nu} + \kappa k_\mu k_\nu \phi,$$

Abelian (abelianised) gauge fields

$$A^a_\mu = c^a \phi k_\mu,$$

- BCJ double copy

Gravity

$$\mathcal{M}^{\text{Gravity}} = \sum_{i \in \text{cubic}} \frac{n_i \tilde{n}_i}{\prod_{\alpha_i} p^2_{\alpha_i}}$$

Gauge theory

$$\mathcal{A}^{\text{YM}} = \sum_{i \in \text{cubic}} \frac{c_i n_i}{\prod_{\alpha_i} p^2_{\alpha_i}}.$$

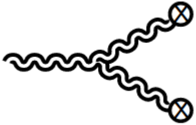
- First examples: Schwarzschild and Kerr black holes (+higher dimension generalizations), 2014.

Extensions to KS double copy

- Taub-NUT (AL, Monteiro, O'Connell, White 2015)
- Energy conditions (Ridgway, Wise 2015)
- Curved Kerr-Schild spacetimes (Bahjat-Abbas, AL, White 2017, Carrillo-Gonzalez, Penco, Trodden 2017)
- Accelerating black holes: Related bremsstrahlung radiation from accelerated point charges (AL, Monteiro, Nicholson, O'Connell, White 2016)
- Double field theory (Lee, 2018)
- Can this be used to study gravitational (waves) radiation?

Perturbative static solutions

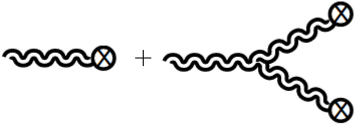
Duff (1973) computed the gravitational field generated by (static) point particles using Feynman diagrams

$h =$  $+$  $+ \dots$

The three-graviton vertex Feynman rule is given by

$$\begin{aligned}
V_3 = \operatorname{sym}\Big[&-\frac{1}{4}P(p\cdot q\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}) - \frac{1}{4}P(p^\sigma p^\tau \eta^{\mu\nu}\eta^{\rho\lambda}) + \frac{1}{4}P(p\cdot q\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}) \\
&+ \frac{1}{2}P(p\cdot q\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}) + P(p^\sigma p^\lambda \eta^{\mu\nu}\eta^{\tau\rho}) - \frac{1}{2}P(p^\tau q^\mu \eta^{\nu\sigma}\eta^{\rho\lambda}) - \frac{1}{2}P(p^\rho q^\lambda \eta^{\mu\sigma}\eta^{\nu\tau}) \\
&+ \frac{1}{2}P(p^\rho p^\lambda \eta^{\mu\sigma}\eta^{\nu\tau}) + P(p^\sigma q^\lambda \eta^{\tau\mu}\eta^{\nu\rho}) + P(p^\sigma q^\mu \eta^{\tau\rho}\eta^{\lambda\nu}) - P(p\cdot q\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big].
\end{aligned}$$

Perturbative static solutions

$h =$  $+ \ldots$

- Results in

$$g^{00} = -1 - \frac{2GM}{r} - \frac{2G^2M^2}{r^2} + \mathcal{O}(G^3),$$
$$g^{ij} = \left(1 - \frac{2GM}{r} + \frac{3G^2M^2}{r^2}\right)\eta^{ij} - \frac{G^2M^2}{r^2}\frac{x^ix^j}{r^2} + \mathcal{O}(G^3),$$

which shows a perfect agreement with Schwarzschild metric.

- Repeat this exercise with double copy methods instead of Einstein-Hilbert.

Perturbative static solutions

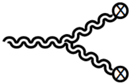
Using double copy

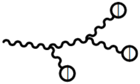
- Obtain the double copy of the field generated by colour charged point particle (AL, Monteiro, Nicholson, Ochirov, O'Connell, Westerberg, White 2016).

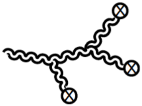
$\partial^2 A^{(0)}_\mu = J_\mu \Leftrightarrow$ 

$\partial^2 H^{(0)}_{\mu\nu} = -\frac{\kappa}{2} T_{\mu\nu} \Leftrightarrow$ 

$\partial^2 A^{(1)}_\mu \propto (A^{(0)})^2 \Leftrightarrow$ 

$\partial^2 H^{(1)}_{\mu\nu} \propto (H^{(0)})^2 \Leftrightarrow$ 

$\partial^2 A^{(2)}_\mu \propto (A^{(0)})^3 \Leftrightarrow$ 

$\partial^2 H^{(2)}_{\mu\nu} \propto (H^{(0)})^3 \Leftrightarrow$ 

- Natural field $H_{\mu\nu}$, the "product" graviton

Simplest source

Consider as source a stationary mass at origin $T^{\mu\nu} = M u^\mu u^\nu \delta(\vec{x})$

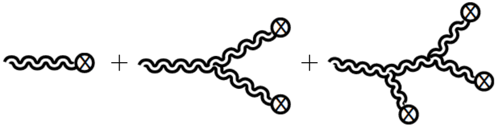
$$H^{(0)}_{\mu\nu} = \frac{\kappa}{2}\frac{M}{4\pi r} u_\mu u_\nu, \qquad \text{with } \; u_\mu = (1, \mathbf{0}),$$

Now, using the double copied vertex we compute the first order

$$H^{(1)\mu\nu} = -\frac{1}{4p^2}\left[V_3^{\mathrm{YM}} \otimes V_3^{\mathrm{YM}}\right]^{\mu\nu}{}_{\mu_2\nu_2,\mu_3\nu_3} H^{(0)\mu_2\nu_2} H^{(0)\mu_3\nu_3}$$

And analogously, we can go to the next order.

$H^{\mu\nu} =$



$+ \dots$

can be easily computed to yield

$$H^{\mu\nu} = \left(\frac{\kappa}{2}\right)\frac{M}{4\pi r} u^\mu u^\nu - \left(\frac{\kappa}{2}\right)^3 \frac{M^2}{4}\frac{1}{(4\pi r)^2} k^\mu k^\nu - \left(\frac{\kappa}{2}\right)^5 \frac{M^3}{6}\frac{1}{(4\pi r)^3} u^\mu u^\nu + \dots$$

What did we compute?

What is our theory of gravity? $\mathcal{N}=0$ Supergravity

$$S=\int d^dx\sqrt{-g}\left[\frac{2}{\kappa^2}R-\frac{1}{2(d-2)}\partial^\mu\phi\partial_\mu\phi-\frac{1}{6}e^{-2\kappa\phi/d-2}H^{\lambda\mu\nu}H_{\lambda\mu\nu}\right],$$

Need a map between product graviton and component fields

$$H_{\mu\nu}=H_{\mu\nu}(\mathfrak{h}_{\mu\nu},B_{\mu\nu},\phi),$$

To first order, it turns out to be

$$H^{(0)}_{\mu\nu}(x)=\mathfrak{h}^{(0)}_{\mu\nu}(x)+B^{(0)}_{\mu\nu}(x)+P^q_{\mu\nu}\phi^{(0)}(x)$$

To higher order, it is necessary to correct it

$$H^{(i)}_{\mu\nu}(x)=\mathfrak{h}^{(i)}_{\mu\nu}(x)+B^{(i)}_{\mu\nu}(x)+P^q_{\mu\nu}\phi^{(i)}(x)+\mathcal{T}^{(i)}_{\mu\nu}.$$

with a transformation function, which represents a map to BCJ coordinates.

Component fields solution

The product graviton is given by

$$H^{\mu\nu} = \left(\frac{\kappa}{2}\right) \frac{M}{4\pi r} u^\mu u^\nu - \left(\frac{\kappa}{2}\right)^3 \frac{M^2}{4} \frac{1}{(4\pi r)^2} k^\mu k^\nu + \dots$$

After computing the transformation function

$$\mathcal{T}^{(1)\mu\nu}(x) = -\left(\frac{\kappa}{2}\right)^2 \left[3u^\mu u^\nu + 2k^\mu k^\nu + 2P_q^{\mu\nu}\right] \frac{M^2}{4(4\pi r)^2}.$$

We can read the component fields

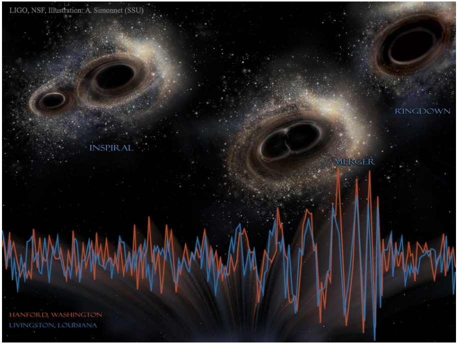
$$\begin{aligned}
\mathfrak{h}_{\mu\nu} &= \frac{\kappa}{2} \frac{M}{4\pi r} u_\mu u_\nu + \left(\frac{\kappa}{2}\right)^3 \frac{M^2}{4(4\pi r)^2} \left(3u_\mu u_\nu + k_\mu k_\nu\right) + \mathcal{O}(\kappa^5), \\
\phi &= -\frac{\kappa}{2} \frac{M}{4\pi r} + \mathcal{O}(\kappa^5),
\end{aligned}$$

They correspond to the first terms in an expansion of the JNW solution metric and dilaton

$$\begin{aligned}
ds^2 &= -\left(1 - \frac{\rho_0}{\rho}\right)^\gamma dt^2 + \left(1 - \frac{\rho_0}{\rho}\right)^{-\gamma} d\rho^2 + \left(1 - \frac{\rho_0}{\rho}\right)^{1-\gamma} \rho^2 d\Omega^2, \\
\phi &= \frac{\kappa}{2} \frac{Y}{4\pi\rho_0} \log\left(1 - \frac{\rho_0}{\rho}\right).
\end{aligned}$$

Scattering in gravity

- Ultimate objective: apply to binary problem



- Need to consider non-static sources

Classical scattering

Breakthrough by Goldberger, Ridgway (2016)

$$D^\mu F^a_{\mu\nu} = g \sum_i \int d\tau c^a_i(\tau) \frac{dx^\nu_i(\tau)}{d\tau} \delta^{(d)}(x - x_i(\tau)),$$

$$m_i \frac{d^2 x^\mu_i(\tau)}{d\tau^2} = g F^{a\mu\nu} c^a_i(\tau) \frac{dx_{i\nu}(\tau)}{d\tau},$$

$$\frac{dc^a_i(\tau)}{d\tau} = g f^{abc} \frac{dx^\mu_i}{d\tau} A^b_\mu(x_i(\tau))\, c^c_i(\tau).$$

$$x^\mu_i(\tau) = x^{(0)\mu}_i(\tau) + x^{(1)\mu}_i(\tau) + \cdots,$$
$$c^a_i(\tau) = c^{(0)a}_i + c^{(1)a}_i(\tau) + \cdots,$$
$$A^a_\mu(x) = A^{(0)a}_\mu(x) + A^{(1)a}_\mu(x) + \cdots.$$

$$x^\mu_i(\tau) \simeq b^\mu_i + v^\mu_i \tau,$$
$$c^a_i(\tau) \simeq c^{(0)a}_i.$$

Classical Scattering

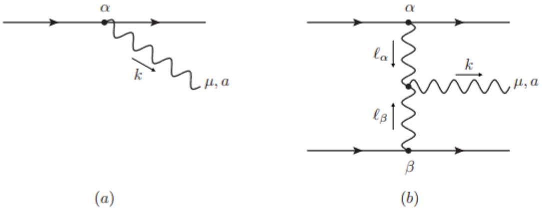


FIG. 1: Leading order Feynman diagrams for the perturbative expansion of $\tilde{J}^{\mu}_{a}(k)$.

Figure from Goldberger-Ridgway. In our notation

$$k^2 A^{(1)a\mu}(k) = g^3 \int \text{đ}q_1 \text{đ}q_2 \delta\!\!\!{}^-(k - q_1 - q_2) \delta\!\!\!{}^-(q_1 \cdot v_1) e^{iq_1 \cdot b_1} \delta\!\!\!{}^-(q_2 \cdot v_2) e^{iq_2 \cdot b_2}$$

$$\times \left\{ \frac{c_1^{(0)a}}{m_1} \frac{c_1^{(0)} \cdot c_2^{(0)}}{k \cdot v_1 \; q_2^2} \left[-v_1 \cdot v_2 \left(q_2^\mu - \frac{k \cdot q_2}{k \cdot v_1} v_1^\mu \right) + k \cdot v_1 \; v_2^\mu - k \cdot v_2 \; v_1^\mu \right] \right.$$

$$\left. + \frac{i f^{abc} c_1^b c_2^c}{q_1^2 q_2^2} \left[2k \cdot v_2 \; v_1^\mu - v_1 \cdot v_2 \; q_1^\mu + v_1 \cdot v_2 \frac{q_1^2}{k \cdot v_1} v_1^\mu \right] + (1 \leftrightarrow 2) \right\}.$$

Classical Scattering

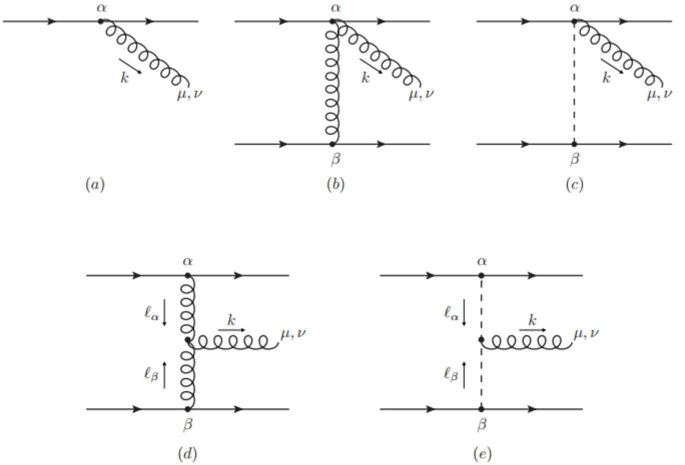


FIG. 2: Leading order Feynman diagrams in the perturbative expansion of $\tilde{T}^{\mu\nu}(k)$.

Figure from Goldberger-Ridgway

Classical Scattering

Perform the double copy procedure

$$A_\mu^{(1)a}(x) \to H_{\mu\nu}^{(1)}(x)$$

using the replacements

$$c_i^{(0)a} \to m_i v_i^\mu,$$
$$if^{abc} \to \frac{1}{2}\Gamma^{\mu\nu\rho}(q_1, q_2, q_3).$$

This results in

$$k^2 H^{(1)\mu\nu}(k) = -\frac{m_1 m_2}{8 m_{\rm pl}^{3(d-2)/2}} \int \text{đ} q_1 \text{đ} q_2 \delta\!\!\!{}^{-}(k-q_1-q_2)\delta\!\!\!{}^{-}(q_1\cdot v_1)e^{iq_1\cdot b_1}\delta\!\!\!{}^{-}(q_2\cdot v_2)e^{iq_2\cdot b_2}\times$$
$$\left[\frac{P_{12}^\mu P_{12}^\nu}{q_1^2 q_2^2} + \frac{v_1\cdot v_2}{2q_1^2q_2^2}\left(Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu\right) + \frac{(v_1\cdot v_2)^2}{4}\left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k\cdot v_1)^2(k\cdot v_2)^2}\right)\right].$$

$$P_{12}^\mu \equiv k\cdot v_1\ v_2^\mu - k\cdot v_2\ v_1^\mu,$$
$$Q_{12}^\mu \equiv (q_1-q_2)^\mu - \frac{q_1^2}{k\cdot v_1}v_1^\mu + \frac{q_2^2}{k\cdot v_2}v_2^\mu,$$

Classical scattering

- The result is interpreted as the radiation from two JNW singularities scattering off each other.

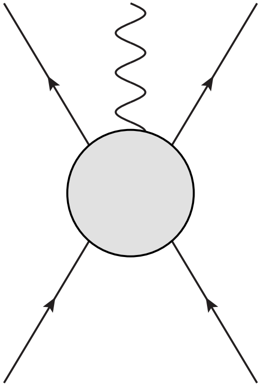
- Several extensions of this formalism just last year (2017):
  Biadjoint scalars: Goldberger, Prabhu, Thompson.
  Bound states: Goldberger, Ridgway.
  Spinning particles: Goldberger, Li, Prabhu
  Einstein-Yang-Mills: Chester.

- Validity of replacement? Will hold to higher orders?

- Can it be understood in terms of scattering amplitudes?

Inelastic scattering of massive scalars

- What amplitude should we consider? The inelastic scalar scattering with gluon production.



Inelastic scattering of massive scalars

- Consider a gauge field A_μ^a coupled to two different massive scalars Φ_i.
- The Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F^{\mu\nu}F_{\mu\nu} + \sum_i \left[(D_\mu\Phi_i)^\dagger(D^\mu\Phi_i) - m_i^2|\Phi|^2\right].$$

- The full amplitude takes the form

$$\mathcal{A} = \frac{n_A c_A}{d_A} + \frac{n_B c_B}{d_B} + \frac{n_C c_C}{d_C} + \frac{n_D c_D}{d_D} + \frac{n_E c_E}{d_E},$$

Inelastic scattering of massive scalars

Cubic topologies

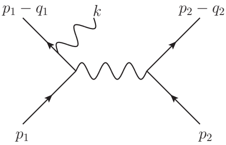


Diagram A

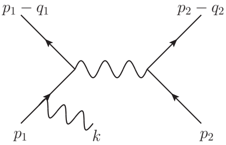


Diagram B

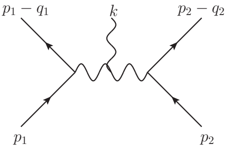


Diagram C

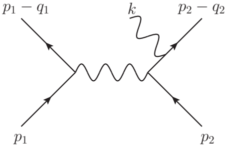


Diagram D

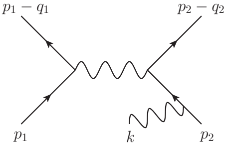


Diagram E

The five cubic diagrams contributing to the amplitude

Inelastic scattering of massive scalars

The amplitude in gauge theory is given by

$$\mathcal{A} = \frac{n_A c_A}{d_A} + \frac{n_B c_B}{d_B} + \frac{n_C c_C}{d_C} + \frac{n_D c_D}{d_D} + \frac{n_E c_E}{d_E}.$$

The numerators take the explicit form

$$\begin{aligned}
n_A &= (2p_1 + q_2) \cdot (2p_2 - q_2)\, \epsilon \cdot (2p_1 + 2q_2) - (2p_1 \cdot q_2 + q_2^2)\, \epsilon \cdot (2p_2 - q_2), \\
n_B &= (2p_1 - k - q_1) \cdot (2p_2 - q_2)\, 2\epsilon \cdot p_1 + 2p_1 \cdot k\, \epsilon \cdot (2p_2 - q_2), \\
n_C &= (2p_1 - q_1)^\mu (2p_2 - q_2)^\rho \left[(k + q_2)_\mu \eta_{\nu\rho} + (q_1 - q_2)_\nu \eta_{\rho\mu} - (k + q_1)_\rho \eta_{\mu\nu}\right] \epsilon^\nu, \\
n_D &= (2p_1 - q_1) \cdot (2p_2 + q_1)\, \epsilon \cdot (2p_2 + 2q_1) - (2p_2 \cdot q_1 + q_1^2)\, \epsilon \cdot (2p_1 - q_1), \\
n_E &= (2p_1 - q_1) \cdot (2p_2 - k - q_2)\, 2\epsilon \cdot p_2 + 2p_2 \cdot k\, \epsilon \cdot (2p_1 - q_1).
\end{aligned}$$

So they satisfy colour-kinematics duality

$$n_A - n_B = n_C; \quad n_D - n_E = n_C.$$

We can build a gravity amplitude as a double copy

$$\mathcal{M} = \frac{n_A n_A}{d_A} + \frac{n_B n_B}{d_B} + \frac{n_C n_C}{d_C} + \frac{n_D n_D}{d_D} + \frac{n_E n_E}{d_E},$$

Large mass limit / Classical scattering

- To obtain the classical limit, we take the first order in a large mass expansion.

- In order to do this, we express the momenta of the incoming particles in terms of proper velocities, so $p_i^\mu = m_i v_i^\mu$.

- The result is

$$\mathcal{M}_{\text{cl}} = 16 m_1^2 m_2^2 \, \epsilon_\mu \epsilon_\nu \left[4 \frac{P_{12}^\mu P_{12}^\nu}{q_1^2 q_2^2} + 2 \frac{v_1 \cdot v_2}{q_1^2 q_2^2} \left(Q_{12}^\mu P_{12}^\nu + Q_{12}^\nu P_{12}^\mu \right) + (v_1 \cdot v_2)^2 \left(\frac{Q_{12}^\mu Q_{12}^\nu}{q_1^2 q_2^2} - \frac{P_{12}^\mu P_{12}^\nu}{(k \cdot v_1)^2 (k \cdot v_2)^2} \right) \right],$$

which reproduces the worldline formalism result by Goldberger and Ridgway

Cool recent papers

arXiv.org > hep-th > arXiv:1806.07388

High Energy Physics - Theory

Gravitational Radiation from Color-Kinematics Duality

Chia-Hsien Shen

(Submitted on 19 Jun 2018)

We perturbatively calculate classical radiation in Yang-Mills theory and dilaton gravity, to next-to-leading order in couplings.

arXiv.org > hep-th > arXiv:1807.09859

High Energy Physics - Theory

Effective action of dilaton gravity as the classical double copy of Yang-Mills theory

Jan Plefka, Jan Steinhoff, Wadim Wormsbecher

(Submitted on 25 Jul 2018)

We compute the classical effective action of color charges moving along worldlines by integrating out the Yang-Mills gauge field to next-to leading order

Recap+Outlook

- The double copy relates scattering amplitudes in Yang-Mills and gravity. It was discovered when investigating UV divergences of supergravities.
- Their ideas have been extended to classical systems, including black hole solutions and point masses using the worldline formalism.
- It is useful to understand them in terms of amplitudes, so the relation to BCJ double copy is manifest, although colour-kinematics duality has now been observed also in the worldline formalism.
- Promising results to relate with post Newtonian (PN) formalism open up the possibility of applying to binary systems.

Thank you.

(...just five years ago...)