F-Theory: at the interface of geometry and partilce physics

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Based on **arXiv:1708.09452**, **arXiv: 1503.02068**, **arXiv:1408.4808**
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UNIVERSIDAD DE GUANAJUATO

Mexicuerdas 2018, CINVESTAV, August 3, 2018

Plan

- Introduction and motivation
- What is F-theory?
- Elliptic Fibrations
- Toric Fibers
- Some realistic models
- Conclusions and Prospects

Introduction and Motivation

- Besides the insights it provides on the quantum nature of gravity, string theory can be also used for model building in particle physics and cosmology.
- Perhaps the most striking feature of string pheno, in contrast to field theory pheno is that the low energy particle spectrum is fixed (for good or bad) and no particles are added by hand.
- Stringy particle models can be completed to include gravity (landscape) whereas puere field theory particle models may or may not (swampland).

...,Vafa'05,...

- String pheno is concerned with compactifications of 10D string theory (11D M-theory) to 4 dimensions, in a consistent limit where gravity is decoupled.

Introduction and Motivation

- If we want some remainder SUSY in 4D, this implies focusing on particular classes of manifolds: Calabi-Yau (CY) threefolds, (elliptically fibered) fourfolds, $SU(3)$ structure manifolds, $SU(4)$ structure, $Spin(7)$ manifolds, G_2 manifolds.
- Different approaches involve diferent compactifications:
  - Heterotic $E_8 \times E_8$ ($SO(32)$) on CY threefolds, or orbifold limits

    Font, Ibanez, Nilles, Quevedo, Buchmüller, Vaudrevange, Ramos-Sanchez, Anderson, gray, Lukas...see Saúl's talk.
  - Intersecting D6 branes in IIA, compactified on CY thereefolds with $\mathbb{Z}_2$ involutions.

    Aldazabal, Cvetic, Ibanez, Quevedo, Uranga, Marchesano...
  - IIB with fractional D3 branes at singularities.

    Aldazabal, Cvetic, Ibanez, Quevedo, Uranga, Marchesano, García-Etxerbarría,...
  - M-theory on G_2 manifolds, F-theory on elliptically fiebered CY fourfolds

    Acharya, Kane, Vafa, Beasley, Heckman, Schäfer-Nameki, Wiegand, Cvetic, Anderson, Gray...

F-theory

A glimpse to the type IIB theory: A zoo of objects!

- Closed strings (gravity information: g_{MN}, B_{MN}, ϕ)
- A bunch of potentials C_0, C_2, C_4,...
- open strings ending on "branes" (gauge information A_μ, chiral matter).
- effective description: IIB supergravity (in the Einstein frame)

$$S_{10}^E \supset \int d^{10}x\sqrt{-g^E}\left[R^E + \frac{|d\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{2\mathrm{Im}\,\tau}\right] + \dots\,, \tag{1}$$

with

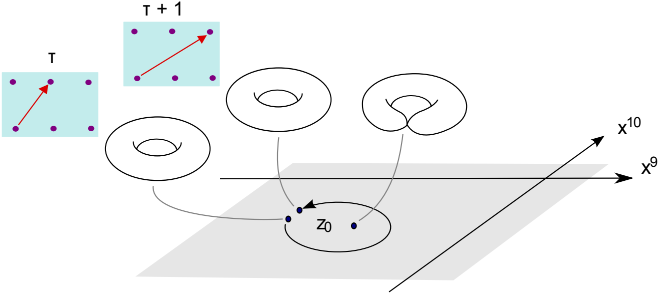
$$\tau = C_0 + \mathrm{i}e^{-\phi}\,, \quad G_3 = F_3 - \tau dB_2\,. \tag{2}$$

Then one can show that the action S_{10} is $SL(2,\mathbb{R})$ invariant.

F-theory

In fact, the $SL(2,\mathbb{R})$ is broken by non-perturbative effects down to $SL(2,\mathbb{Z})$ (the modular symmetry of the torus).

Type IIB in the presence of a D7-brane



- Think of the axio-dilaton as the complex structure of an auxiliary torus.
- D7-branes are located at places where the torus pinches off.

Vafa'96

F-theory

The F-theory conjecture: Type IIB theory with 7-branes on B_n is described by F-theory on an $n+1$-fold Y_{n+1}, which is an elliptic fibration over B_n.

$$\pi : \; Y_{n+1} \longrightarrow B_n \, ,$$

such that for an arbitrary point $b \in B_n$, $\pi^{-1}(b)$ gives us a torus. Similarly, there must be a way to cut the base out of Y_{n+1}, since B_n is the physical internal space $\rightarrow$ The elliptic fibration must have a holomorphic section.

$$\sigma_0 : \; B_n \longrightarrow Y_{n+1} \, , \quad \pi \circ \sigma_0 = id_{B_n}$$

- What is the particle spectrum in F-theory?
- What are the interactions?

Elliptic Fibrations

Elliptic fibrations with a section are described in terms of the Weierstraß form

$$P_W = y^2 - x^3 - fxz^4 - gz^6 = 0\,,$$

with $[x : y : z]$ homogeneous coordinates in $\mathbb{P}[2, 3, 1]$. Note that P_W contains the point $[0 : 0 : 1]$, this point is where the section σ_0 intersects the fiber. We get an elliptic fibration when we set f and g to be suitable functions of the base B_n. The fiber becomes singular whenever

$$P_W = 0 \quad \text{and} \quad dP_W = 0\,,$$

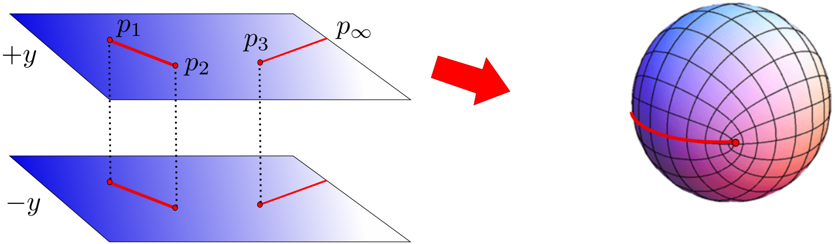
This is captured by the vanishing the discriminant

$$\Delta = 4f^3 + 27g^2\,. \tag{3}$$

Elliptic Fibrations

How come $y^2 = x^3 + fxz^4 + gz^6$ in $\mathbb{P}[2,3,1]$ describes a torus?

- Since $[x : y : z] \sim [\lambda^2 x : \lambda^3 y : \lambda z]$, $z = 1$ defines a point.
- Setting $z = 1$ we get $y^2 = (x - p_1)(x - p_2)(x - p_3)$, defines two Riemann sheets $\pm y$.



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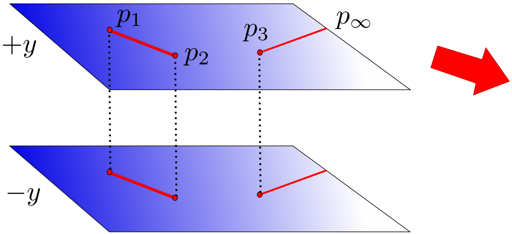

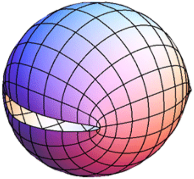

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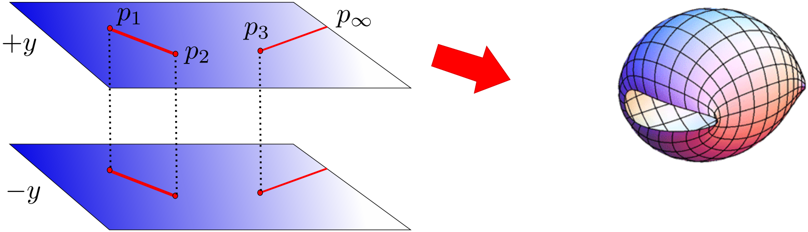

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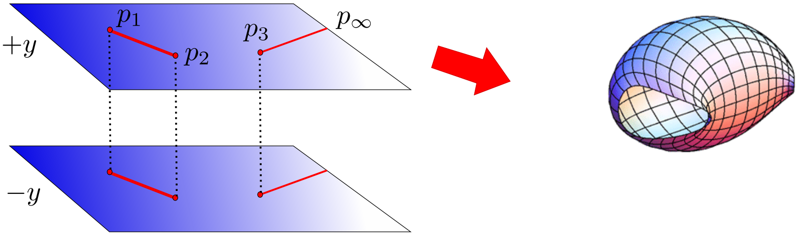

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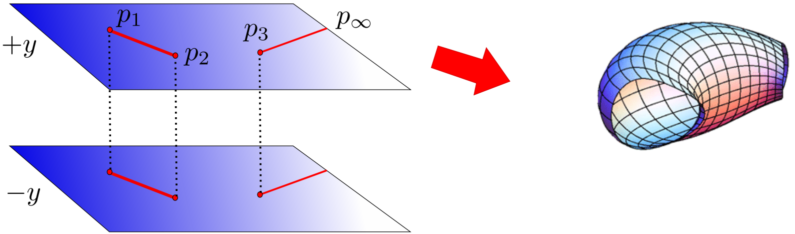

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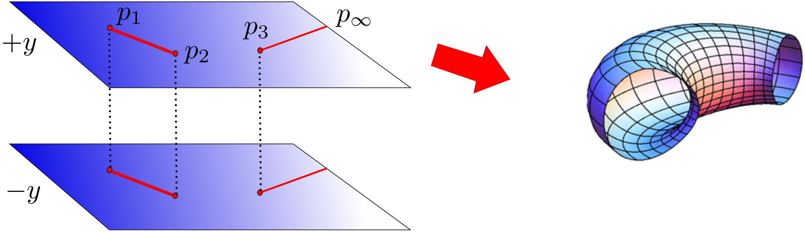

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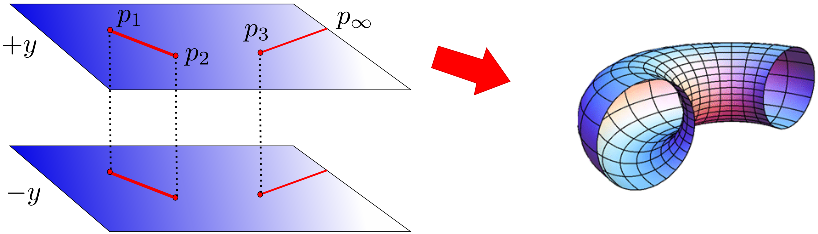

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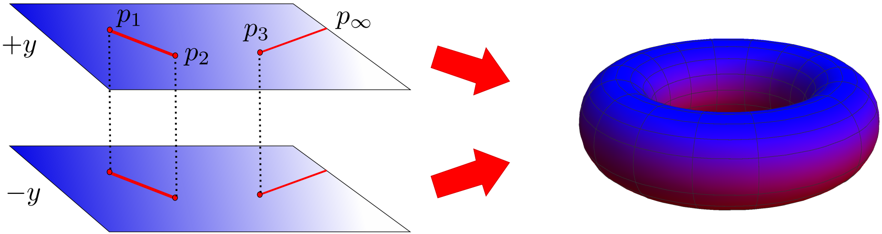
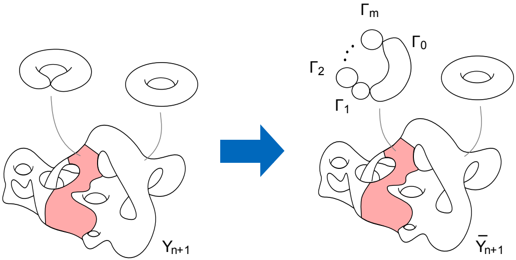


F-theory for model building

- We are interested in four dimensional theories exhibiting $\mathcal{N} = 1$ supersymmetry. Hence the fourfold Y_{n+1} must be Calabi-Yau.

Vafa'96, Donagi, Wijnjolt'11

- F-theory does not have a fundamental description but it can be accessed via some dualities in connection to M-theory. This enables us to understand how the gauge symmetry, particles and interactions arise.



- Desingularization of the elliptic fiber introduces additional $\mathbb{P}^1$s in the fiber. From reducing the M-theory three form C_3 we get vectors in 4D

$$A_i = \int_{\Gamma_i} C_3 \, .$$

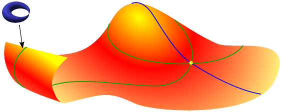
F-theory for model building

- Non-Abelian enhancements occur when we shrink the $\mathbb{P}^1$s to zero, as M2 branes wrapping them become massless. The gauge symmetry depends on the intersection pattern of the $\mathbb{P}^1$s. They follow the pattern of the ADE classification.

Kodaira'63, Bershadsky *et. al.*'96

$\rightarrow E_6, E_8$ are possible gauge symmetries!

- The fiber could degenerate further at given loci or points on the surface S. The additional degrees of freedom correspond to matter or interaction points, respectively.



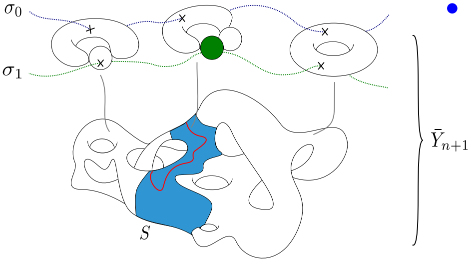
F-theory for model building

A disclaimer: Abelian gauge symmetries

- These do not allow for a local description as they relate to extra sections of the elliptic fibration.

Morrison, Park'10

- These extra sections need not to be holomorphic.



- The U(1) generator

$$s(\sigma_I) = \sigma_I - \sigma_0 + \sum_{\mathfrak{g}} (\sigma_I \cdot \Gamma_i)(C^{-1}(\mathfrak{g}))_{ij} D_j + \ldots$$

Number of U(1) s fixed by the rank of the Mordell-Weil group.

F-theory for model building

Why did we choose F-theory for model building?

- In contrast to (perturbative) type IIB, F-theory allows us to realize exceptional groups.

$$\mathrm{SU}(5) \subset \mathrm{SO}(10) \subset E6 \subset E8\,,$$

  $\rightarrow$ The presence of an underlying exceptional group is one of the reasons for the success of the heterotic landscape.

  Buchmüller'06

- To a certain extent, the particle physics is concentrated on the surface of degeneration.
- U(1) symmetries can help the phenomenology to remain "in shape".
- With regards to the stabilization of the moduli, it is expected to proceed as in the perturbative picture.

Toric hypersurfaces as fibers for F-theory

$\mathbb{P}^2, \mathbb{P}^1 \times \mathbb{P}^1, \mathbb{P}[1,1,2], \mathbb{P}[2,3,1]$ are examples of 2D toric varieties. There are 16 of these, with the others being blow ups of the first three.
We study the F-theory compactifications whose fiber is cut out from any of these varieties

- Each toric variety has an associated reflexive polyhedron F_i in a lattice $N = \mathbb{Z}^2$, and a dual polyhedron F_i^* in the dual lattice $M = N^*$.

$$F_i^* = \{q \in M \,|\, \langle y, q \rangle \geq -1, \forall y \in F_i\}\,.$$

- To each $v_k \in F_i$ $k = 1, ..., m$ we associate a coord. $x_k \in \mathbb{C}$. The toric vty. assciated to F_i is given by

$$\mathbb{P}_{F_i} = \{x_k \sim \prod_{a=1}^{m-2} \lambda_a^{\ell_k^{(a)}} x_k \,|\, \underline{x} \notin \mathrm{SR}\,, \lambda_a \in \mathbb{C}^*\}$$

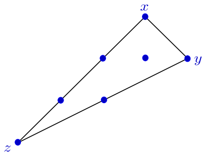
with $\ell_k^{(a)}$ stemming from $m-2$ linear equivalences: $\sum_{k=1}^m \ell_k^{(a)} v_k = 0$.

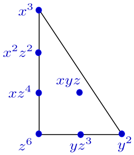
Toric hypersurfaces as fibers for F-theory

Every toric variety $\mathbb{P}_{F_i}$ has an associated Calabi-Yau hypersurface: a genus one curve $\mathcal{C}_{F_i}$. This curve is obtained by the Batyrev construction as the vanishing of the polynomial

$$p_{F_i} = \sum_{q \in F_i^*} a_q \prod_k x_k^{\langle v_k, q \rangle + 1} \,.$$

For example, in F_{10}: $z + 2y + 3z = 0$



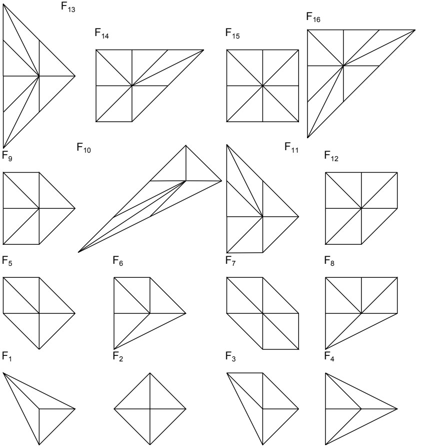


$$\begin{aligned} p_{F_{10}} = & y^2 + a_1 xyz + a_2 yz^3 - x^3 \\ & - b_1 x^2 z^2 - b_2 xz^4 - b_3 z^6 \end{aligned}$$

which upon redefinitions for y and z can be recast to the WSF.

Same procedure can be applied to all of the 16 2D toric vties to find different tori, all birrationally equivalent to the WSF, but with more explicit properties

Toric hypersurfaces as fibers for F-theory



Kreuzer, Skarke'97

- Toric Mordell-Weil (MW) group

  Braun, Grimm, Keitel'13

- Further specializations leading to $SU(5)$ factors

  Borchman, Mayrhofer, Palti, Weigand'13
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- Fibers with torsional MW

  Mayrhofer, Morrison, Till, Weigand'14

- Fibrations with multisections

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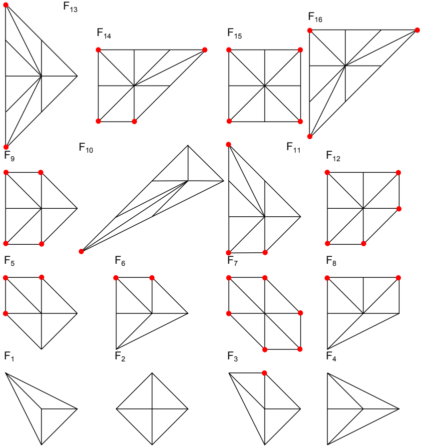

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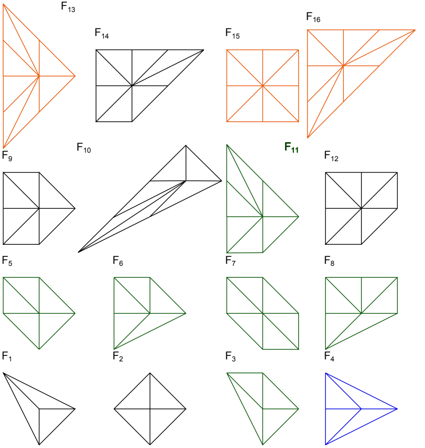

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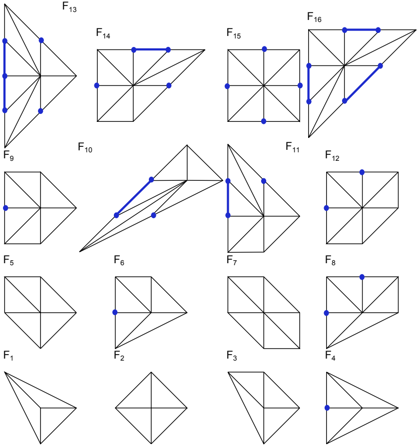


- Usually in p_{F_i} one excludes coordinates v_i interior to edges since their associated divisors do not intersect $\mathcal{C}_{F_i}$. However, in the fibration these divisors resolve singularities, and become Cartan divisors.
- We aim at finding the spectra in all of these cases (we focus on threefolds)
- Can we relate between models by means of Higgs mechanisms?

Toric hypersurfaces as fibers for F-theory

Constructing toric hypersurface fibrations

- In a fibration of $\mathcal{C}_{F_i}$ over a base B, the coeffs. in p_{F_i} as well as the coods. x_k must be promoted to sections of suitable line bundles in the base

$$\begin{array}{ccc} \mathbb{P}_{F_i} & \rightarrow & \mathbb{P}^B_{F_i}(D,\tilde{D}) \\ & & \downarrow \\ & & B \end{array}$$

  so that the structure is parameterized by two divisors D and $\tilde{D}$.

- Next we cut $\mathcal{C}_{F_i}$ out of $\mathbb{P}^B_{F_i}(D,\tilde{D})$,

$$\begin{array}{ccc} \mathcal{C}_{F_i} & \rightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array}$$

  where x_{F_i} is a CY as long as p_{F_i} belongs to the anti-canonical bundle $K^{-1}_{\mathbb{P}^B_{F_i}(D,\tilde{D})}$.

Tools and strategies for model building

G_4-**flux:**

- To construct the (chirality inducing) G_4 flux we have to work out $H_V^{(2,2)}(X_{F_i})$.
- G_4 is subjected to a quantization condition and must allow for a positive (integer) tadpole cancelling number of D3 branes

Witten'96, Sethi, Vafa, Witten'96, Gukov, Vafa, Witten'99

$$G_4 + \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z}) \ , \quad \frac{\chi(X)}{24} - \frac{1}{2}\int_X G_4 \wedge G_4 = n_{\mathrm{D3}} \in \mathbb{N} \ .$$

- There are also constraints on certain CS terms $\Theta_{AB} = \int_X G_4 \wedge D_A \wedge D_B$

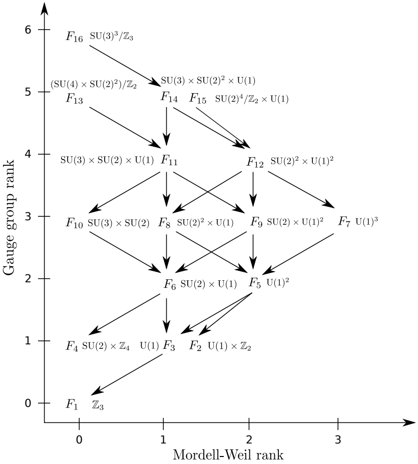
Marsano, Schäfer-Nameki'11, Grimm, Hayashi'12, Cvetič, Grimm, Klevers'12

...we do not have an integral basis to check flux quantization. Instead we demand $\Theta_{AB} \in \mathbb{Z}/2$.

Intrilligator, Jockers, Mayr, Morrison, Plesser'12

- The chiralities: $\chi(\mathbf{R}) = n(\mathbf{R}) - n(\bar{\mathbf{R}}) = \int_{\mathcal{C}^w_{\mathbf{R}}} G_4$.

A Higgsing network

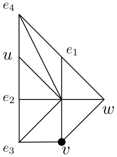


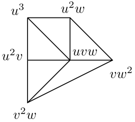
We could relate among all the six dimensional field theories studied by means of a Higgs mechanism.

The miracles

- Network is symmetric with respect to Gauge group rank=3. Gauge group ranks of the model and its dual add up to six.
- There is a kind of duality relating the Mordell-Weil torsion and the discrete symmetries.

An MSSM-like model





- **sections:** $S_0 \;:\; v = 0$, $S_1 \;:\; e_4 = 0$
- **Non Abelian factors:**
  - SU(3) at $\{s_9 = 0\}$
  - SU(2) at $\{s_3 = 0\}$

The fiber is cut by the cubic

$$\begin{aligned} p_{F_{11}} = \; & s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 \\ & + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 uvw + s_9 e_1 v w^2 \end{aligned}$$

as $\{p_{F_{11}} = 0\}$. In the fibration, the div. classes of the s_i are written in terms of two divisors $\mathcal{S}_7$, $\mathcal{S}_9$.

For the specific case of base $\mathbb{P}^3$ one can write these in terms of the hyperplane class H_B.

$$\mathcal{S}_7 = n_7 H_B \,, \quad \mathcal{S}_9 = n_9 H_B \,, \quad [K_B^{-1}] = 4 H_B \,.$$

...with (n_7, n_9) being constrained by *effectiveness.*

An MSSM-like model

- **at codimension two:**

Representation	Locus
$(\mathbf{3},\mathbf{2})_{1/6}$	$V(I_{(1)}) := \{s_3 = s_9 = 0\}$
$(\mathbf{1},\mathbf{2})_{-1/2}$	$V(I_{(2)}) := \{s_3 = s_2 s_5^2 + s_1(s_1 s_9 - s_5 s_6) = 0\}$
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$V(I_{(3)}) := \{s_5 = s_9 = 0\}$
$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$V(I_{(4)}) := \{s_9 = s_3 s_5^2 + s_6(s_1 s_6 - s_2 s_5) = 0\}$
$(\mathbf{1},\mathbf{1})_{1}$	$V(I_{(5)}) := \{s_1 = s_5 = 0\}$

- **at codimension three:**

Yukawa	Locus
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\bar{\mathbf{3}},\mathbf{1})_{-2/3} \cdot \overline{(\mathbf{1},\mathbf{2})_{-1/2}}$	$s_3 = s_5 = s_9 = 0$
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\mathbf{1},\mathbf{2})_{-1/2}$	$s_3 = s_9 = 0 = s_1 s_6 - s_2 s_5$
$(\bar{\mathbf{3}},\mathbf{1})_{-2/3} \cdot \overline{(\bar{\mathbf{3}},\mathbf{1})_{1/3}} \cdot (\mathbf{1},\mathbf{1})_{1}$	$s_1 = s_5 = s_9 = 0$
$(\mathbf{3},\mathbf{2})_{1/6} \cdot (\mathbf{3},\mathbf{2})_{1/6} \cdot \overline{(\bar{\mathbf{3}},\mathbf{1})_{1/3}}$	$s_3 = s_9 = s_6 = 0$
$(\mathbf{1},\mathbf{2})_{-1/2} \cdot (\mathbf{1},\mathbf{2})_{-1/2} \cdot (\mathbf{1},\mathbf{1})_{1}$	$s_1 = s_5 = s_3 = 0$
$(\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\bar{\mathbf{3}},\mathbf{1})_{1/3} \cdot (\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$s_5 = s_6 = s_9$

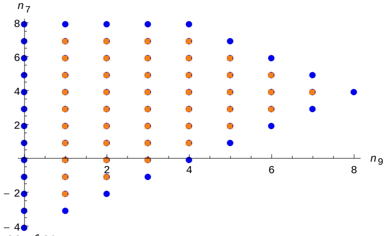
An MSSM-like model

The flux is of the form

$$G_4 = a_6 H_B \cdot \sigma(\hat{s}_1) - a_7 \left[\tilde{S}_0^2 + (20n_7 - n_7^2 + 8n_9 - n_7 n_9 - 92)H_B^2\right] ,$$

Flux solutions could break the hypercharge generator. Only flux solutions preserving a family structure allowed.

$$a_6 = -\frac{6a_7(n_7-8)(4+n_7-n_9)}{3n_7+n_9-36} = \frac{6(\sharp\text{Families})}{n_9(4+n_7-n_9)} .$$



For each (n_7, n_9) one has to check for $\sharp$Families which give half-integral CS terms and allows for tadpole cancelation.

An MSSM-like model

The smallest ♯Families for a positive n_{D3}:

$n_7 \setminus n_9$	1	2	3	4	5	6	7
7	—	(27; 16)	—	—			
6	—	(12; 81)	(21; 42)	—	—		
5	—	—	(12; 57)	(30; 8)	—	**(3; 46)**	
4	(42; 4)	—	(30; 32)	—	—	—	—
3	—	(21; 72)	—	—	—	(15; 30)	
2	(45; 16)	(24; 79)	(21; 66)	(24; 44)	**(3; 64)**		
1	—	—	—	—			
0	—	—	(12; 112)				
-1	(36; 91)	(33; 74)					
-2	—						

- Two strata for which one gets a three family solution as a posibility.
- Not much to say about the vector-like sector from which the Higgses emerge.

Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H_u, H_d
$(\mathbf{3}, \mathbf{2})_{1/6}$	$(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	$(\mathbf{1}, \mathbf{2})_{-1/2}$	$(\mathbf{1}, \mathbf{1})_{1}$	$(\mathbf{1}, \mathbf{2})_{\pm 1/2}$

the following couplings will be induced

$$\mathcal{W} \supset Y^u_{i,j} Q_i \overline{u}_j H_u + Y^d_{i,j} Q_i \overline{d}_j H_d + Y^L_{i,j} \overline{e}_i \overline{L}_j H_d$$
$$\lambda^{(0)}_{i,j,k} Q_i \overline{d}_j L_k + \lambda^{(1)}_{i,j,k} \overline{e}_i L_j L_k + \lambda^{(2)}_{i,j,k} \overline{u}_i \overline{d}_j \overline{d}_k + \mu H_u H_d$$

The Sen limit of an MSSM

Sen'96

Make the coord. change $s = x - b_2 z^2/3$, with $b_2 = \frac{s_6^2}{4} - s_2 s_9$, such that the section S_1 touches the fiber at $s = 0$. The WSF reads

$$y^2 = s^3 + b_2 s^2 z^2 + 2 b_4 s z^4 + b_6 z^6 \,,$$

with

$$b_4 = \frac{s_3 s_9}{2}\left(-\frac{s_5 s_6}{2} + s_1 s_9\right)\,, \quad b_6 = \frac{s_3^2 s_5^2 s_9^2}{4}\,.$$

setting the scalings $s_1 \to \epsilon s_1$, $s_5 \to \epsilon s_5$, one gets $b_2 \to b_2$, $b_4 \to \epsilon b_4$, $b_6 \to \epsilon^2 b_6$.

- Gauge symmetry SU(3) $\times$ SU(2) $\times$ U(1) for any ϵ.
- For $\epsilon = 0$ the fiber degenerates globally. Better study a resolved case

$$y^2 = s^3 \lambda + \tilde{b}_2 s^2 z^2 + 2 \tilde{b}_4 t s z^4 + \tilde{b}_6 t^2 z^6$$

  with $s \mapsto s\lambda$, $y \mapsto y\lambda$ and $\epsilon \mapsto t\lambda$.

The Sen limit of an MSSM

- At $\lambda = t = 0$

$$X_3 : \quad \xi^2 = \frac{s_6^2}{4} - s_2 s_9 \, ;$$

with $\xi = y/sz$.

- In the limit $\lambda \to 0$, the equation

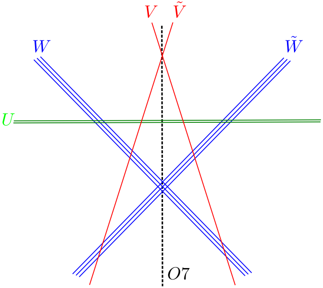
$$W_E \; : \quad y^2 = \tilde{b}_2 s^2 z^2 + 2\tilde{b}_4 tsz^4 + \tilde{b}_6 t^2 z^6$$

defines a $\mathbb{P}^1$ fibration degenerating whenever:

$$\Delta_E = \frac{s_3^2 s_9^3}{4} \left(s_1 s_5 s_6 - s_2 s_5^2 - s_1^2 s_9 \right) = 0 \, .$$

- Non Abelian symmetry is preserved in the Sen limit. Remaining piece responsible for the U(1).

The Sen limit of an MSSM



The curve $(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$ splits at leading order in ϵ

$$\{s_9\,, \epsilon^2\, s_3 s_5^2 + \epsilon s_6(s_1 s_6 - s_2 s_5)\}\,,$$

the split curves are $(\overline{\mathbf{3}}, \mathbf{1})_{(+1,-1)}$ at $\{s_9,\ (s_1 s_6 - s_2 s_5)\}$ and $(\overline{\mathbf{3}}, \mathbf{1})_{(0,2)}$ at $\{s_9,\ s_6\}$.

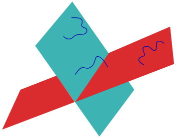
Hypercharge

$$Q_Y = \frac{1}{6}(3Q_V + Q_W)\,,$$

Representation	
$(\mathbf{3}, \mathbf{2})_{(0,1)}$	WU
$(\bar{\mathbf{3}}, \mathbf{1})_{(-1,-1)}$	$\tilde{W}V$
$(\bar{\mathbf{3}}, \mathbf{1})_{(1,-1)}$	$\tilde{W}\tilde{V}$
$(\bar{\mathbf{3}}, \mathbf{1})_{(0,2)}$	$W\tilde{W}$
$(\mathbf{1}, \mathbf{2})_{(-1,0)}$	UV
$(\mathbf{1}, \mathbf{1})_{(2,0)}$	$V\tilde{V}$

And beyond?

- How to extend the model to forbid dangerous operators? Easiest way is to include discrete symmetries.
- Gravity does not like global symmetries. Discrete symmetry must come from a Higgsed $U(1)$. A discrete $\mathbb{Z}_N$ symmetry can be obtained by giving a VEV to a field with charge N under the $U(1)$.
- In type IIB $U(1)$ symmetries are localized, and correspond to (3,5,9) 7-branes. The U(1) charges of matter arise from Chan-Paton indices of strings stretching between branes.



typical charges under Chan-Paton factors are 0, 1, -1 and 2. What about higher U(1) charges? Big challenge in F-theory compactifications. Maximum charge obtained so so far is $q = 4$.

Raghuram'17

And beyond?

- We constructed globally defined models of particle physics that have
  - three chiral families
  - the exact MSSM, Pati-Salam and Trinification gauge group and matter content
  - the anomalies and n_{D3} tadpoles canceled
- No distinction between Leptons and H_d, no control over the μ term in the models.
- Vector-like sector and real reps. not visible.
- Constructions beyond this must invoke additional symmetries, these constructions require concrete fiber realisations with suitable singularity structure. Can there be a stringy reestriction on the highest possible $U(1)$ charge (order of the discrete symmetry)?

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Gracias!