Stealth configurations in scalar-tensor theories

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- Stealth configurations
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Introduction

- The search for alternatives to general relativity has been an active research field, motivated both by the open questions remaining at the frontiers of gravitation, cosmology, and particle physics; such as the dark energy and the hierarchy problems on one side and by applications of the AdS/CFT correspondence on the other.
- One approach to modify standard general relativity is to couple it non-minimally with matter fields.
- The case of a scalar field non-minimally coupled to gravity has been widely studied.

Introduction

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- One approach to modify standard general relativity is to couple it non-minimally with matter fields.
- The case of a scalar field non-minimally coupled to gravity has been widely studied.

One consequence of the non-minimal coupling is the existence of

stealth configurations

(non-trivial solutions for matter fields without back-reaction)

Stealth configurations

- It is well known from General Relativity that in the absence of matter sources, the maximally symmetric solution of the Einstein equations, without cosmological constant term, is the Minkowski spacetime.

$$G_{\mu\nu} = \kappa T_{\mu\nu}.$$

absence of matter sources $\Rightarrow$ $T_{\mu\nu} = 0$ $\Rightarrow$ $G_{\mu\nu} = 0$ $\Rightarrow$ Minkowski spacetime

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$
$$\eta_{\mu\nu} = diag(-1, 1, 1, \cdots, 1)$$

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- Does Minkowski spacetime imply the absence of matter?

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Flat Minkowski spacetime does not always imply that spacetime is devoid of matter

Stealth: Minkowski spacetime and $\zeta\Phi^2 R$ coupling

- It was shown that if the matter source is a real scalar field non-minimally coupled to gravity, the footprint of the non-minimal coupling persists, even if the gravitational field is switched off.

$$S[g_{\mu\nu},\Phi] = \int dx^D \sqrt{-g}\left[\frac{R}{2\kappa} - \frac{1}{2}\nabla^\mu\Phi\nabla_\mu\Phi - U(\Phi) - \zeta\Phi^2 R\right].$$

$\zeta = 0$:

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - \eta_{\mu\nu}\left(\tfrac{1}{2}\partial^\beta\Phi\partial_\beta\Phi + U\right),$$

Minkowski spacetime ... absence of matter

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu \quad\Rightarrow\quad G_{\mu\nu} = 0 \quad\Rightarrow\quad T_{\mu\nu} = 0 \quad\Rightarrow\quad \Phi = constant$$

$\zeta \neq 0$:

$$T_{\mu\nu} = \partial_\mu\Phi\partial_\nu\Phi - \eta_{\mu\nu}\left(\tfrac{1}{2}\partial^\beta\Phi\partial_\beta\Phi + U\right) + \zeta(\eta_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\Phi,$$

Minkowski spacetime ... stealth configuration

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu \quad\Rightarrow\quad G_{\mu\nu} = 0 \quad\Rightarrow\quad T_{\mu\nu} = 0 \quad\Rightarrow$$

$$\Phi = \sigma^{\frac{2\zeta}{4\zeta-1}}, \qquad \zeta \neq \tfrac{1}{4}$$

$$\Phi = e^\sigma, \qquad \zeta = \tfrac{1}{4}$$

$$\sigma = \tfrac{\alpha}{2}x_\mu x^\mu + A_\mu x^\mu + \sigma_0.$$

[E. Ayón-Beato, C. Martinez, R. Troncoso, J. Zanelli, Phys. Rev. D 71 104037 (2005)]

Stealth: Minkowski spacetime and $\zeta\Phi^2 R$ coupling

- Requiring the existence of non-trivial solutions for which the modified energy-momentum tensor vanishes, fixes the form of the allowed self-interaction potential of the scalar field.

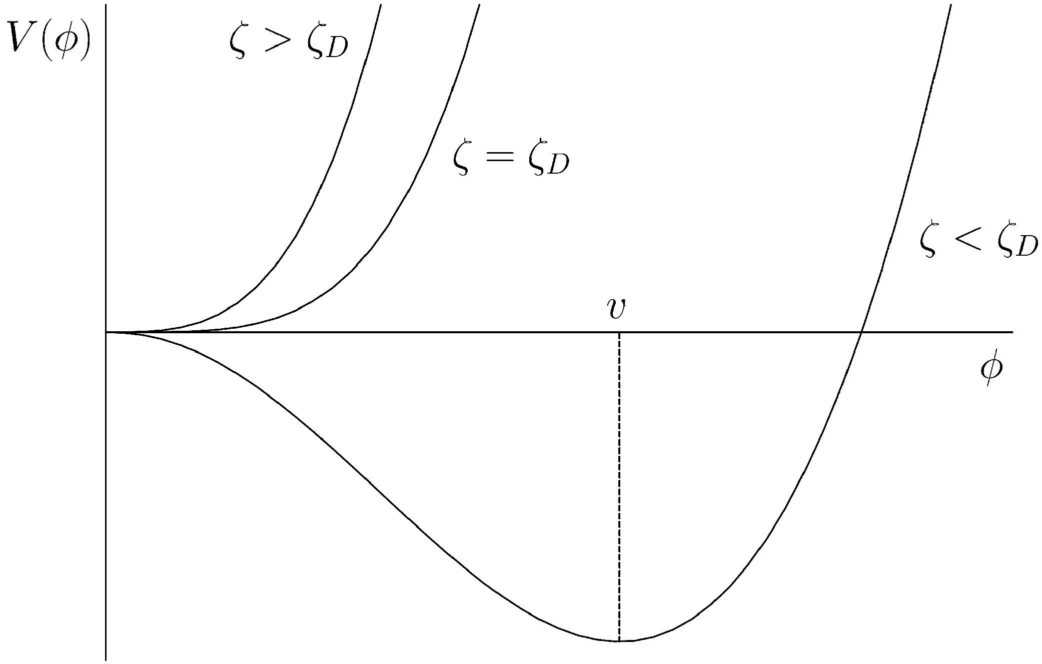

Figure 1: $U(\Phi)$

$$U(\Phi) = \begin{cases} \frac{2\zeta^2}{(1-4\zeta)^2}\left(\lambda_1 \Phi^{\frac{1-2\zeta}{\zeta}} + 8(D-1)(\zeta-\zeta_D)\lambda_2 \Phi^{\frac{1}{2\zeta}}\right), & \zeta \neq \frac{1}{4} \\ \frac{\lambda_2}{2}\Phi^2\left(2\ln\Phi + \frac{\lambda_1}{\lambda_2} + D - 1\right), & \zeta = \frac{1}{4} \end{cases}$$

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Stealth: Scalar-Gauss-Bonnet gravity

- We want to study the existence of stealth configurations for the theory described by the following action

$$S[g_{\mu\nu},\Phi]=\int dx^D\sqrt{-g}\left[\frac{R-2\Lambda}{2\kappa}-\frac{1}{2}\nabla^\mu\Phi\nabla_\mu\Phi-U(\Phi)+\zeta\Phi\mathcal{G}\right].$$

Gauss-Bonnet term:

$$\mathcal{G}=R^{\alpha\beta\mu\nu}R_{\alpha\beta\mu\nu}-4R^{\mu\nu}R_{\mu\nu}+R^2.$$

Equations of motion:

$$G_{\mu\nu}+\Lambda g_{\mu\nu}=\kappa T_{\mu\nu},$$

$$\Box\Phi=-\zeta\mathcal{G}+U'(\Phi),$$

Energy-momentum tensor:

$$T_{\mu\nu}=\nabla_\mu\Phi\nabla_\nu\Phi-g_{\mu\nu}\left(\frac{1}{2}\nabla^\beta\Phi\nabla_\beta\Phi+U\right)+T^{GB}_{\mu\nu}.$$

$$T^{GB}_{\mu\nu}=4\zeta\big[4R_{\beta(\mu}\nabla^\beta\nabla_{\nu)}\Phi+g_{\mu\nu}(R\Box\Phi-2R^{\beta\gamma}\nabla_\beta\nabla_\gamma\Phi)+2R_{\mu\beta\nu\gamma}\nabla^\beta\nabla^\gamma\Phi-R\nabla_{(\mu}\nabla_{\nu)}\Phi$$
$$-2R_{\mu\nu}\Box\Phi\big]-\Phi\big[\frac{1}{4}g_{\mu\nu}\mathcal{G}-R_{\alpha\beta\sigma\mu}R^{\alpha\beta\sigma}{}_{\nu}-RR_{\mu\nu}+2R^{\alpha}{}_{\mu}R_{\alpha\nu}+2R_{\sigma\mu\alpha\nu}R^{\alpha\sigma}\big].$$

(A)dS spacetime

- We will study the existence of stealth configurations in a constant curvature spacetime

$$
\begin{aligned}
R_{\mu\beta\nu\gamma} &= k(g_{\mu\nu}g_{\beta\gamma}-g_{\mu\gamma}g_{\beta\nu}) \\
R_{\mu\nu} &= k(D-1)g_{\mu\nu}, \\
R &= kD(D-1),
\end{aligned}
\qquad \Rightarrow \qquad G_{\mu\nu}+\Lambda g_{\mu\nu}=0, \qquad \Lambda=\frac{k}{2}(D-1)(D-2).
$$

- We use the Riemann coordinates $x^\mu=(t,x^i)$, $i=1,2,\ldots,D-1$, in terms of which the metric looks conformally flat:

$$
ds^2=\Omega^2\eta_{\mu\nu}dx^\mu dx^\nu, \qquad \Omega=\frac{1}{1+\frac{k}{4}\eta_{\alpha\beta}x^\alpha x^\beta}.
$$

- The stealth configurations must satisfy the conditions

$$
T_{\mu\nu}=0.
$$

Stealth conditions

- After redefining the scalar field Φ in terms of a new coordinate dependent function $\sigma = \sigma(x^\mu)$,

$$\Phi = \beta \ln(\Omega\sigma), \qquad \beta = 4\zeta k(D-2)(D-3).$$

the constraints can be written as

$$\begin{aligned}
T_{\mu\nu} &= \frac{\beta^2}{\sigma}\partial^2_{\mu\nu}\sigma = 0, \qquad \mu \neq \nu, \\
T_{tt} + T_{ii} &= \frac{\beta^2}{\sigma}\left(\partial^2_{tt}\sigma + \partial^2_{ii}\sigma\right) = 0, \qquad \text{(no sum)}, \\
T^\mu_\mu &= -D\Omega^2 U + \frac{\beta}{4}kD(D-1)(D-4)\Phi + \beta^2\eta^{\alpha\beta}\left\{-(D-1)\left(\frac{1}{\sigma}\partial^2_{\alpha\beta}\sigma - \Omega\partial^2_{\alpha\beta}\Omega^{-1}\right)\right. \\
&\left. + \frac{D}{2}\left[\frac{1}{\sigma^2}\partial_\alpha\sigma\partial_\beta\sigma + 2(D-2)\frac{\Omega}{\sigma}\partial_\alpha\sigma\partial_\beta\Omega^{-1} - (2D-3)\Omega^2\partial_\alpha\Omega^{-1}\partial_\beta\Omega^{-1}\right]\right\} = 0.
\end{aligned}$$

- The off-diagonal constraints determine the complete separability of the the function σ, that is,

$$\sigma = T(t) + X^1(x^1) + \cdots + X^{D-1}(x^{D-1}).$$

- It follows from the combination of the diagonal terms that the functions T, X^i must satisfy

$$-\frac{d^2T}{dt^2} = \frac{d^2X^1}{d(x^1)^2} = \cdots = \frac{d^2X^{D-1}}{d(x^{D-1})^2} = \alpha,$$

where α is a constant.

Solutions

- The general form of the solution is

$$\Phi = \beta \ln(\Omega\sigma), \qquad \sigma = \frac{\alpha}{2} x_\mu x^\mu + A_\mu x^\mu + \sigma_0,$$

$$V = \frac{\beta^2}{2}\left[k(2D-3) - 2(D-2)\lambda_1 e^{-\frac{1}{\beta}\Phi} + \lambda_2 e^{-\frac{2}{\beta}\Phi}\right] + \frac{k}{4}\beta(D-1)(D-4)\Phi.$$

- There are stealth configurations if

$$D > 3 \qquad \text{and} \qquad k \neq 0 \qquad\qquad \beta = 4\zeta k(D-2)(D-3) \neq 0.$$

- The integration constants α, A_μ, σ_0, are related to the coupling constants of the potential λ_1 and λ_2

$$\lambda_1 = \alpha + \frac{k}{2}\sigma_0, \qquad\qquad \lambda_2 = A_\mu A^\mu - 2\alpha\sigma_0,$$

being $A^\mu = \eta_{\mu\nu} A^\nu$.

Final Remarks

- We have studied stealth configurations in a modified gravity model, with a linear coupling of the Gauss-Bonnet invariant with scalar field, in (A)dS spacetime.
- These results can be generalized to $\zeta f(\Phi)\mathcal{G}$ coupling case.
- Stealth configurations have been studied in different contexts:
  - *Stealth scalar field overflying a 2 + 1 black hole,* Eloy Ayón-Beato, Cristián Martínez, Jorge Zanelli, Gen. Relativ. Gravit. 38(1), 145 (2006).
  - *Conformal stealth for any standard cosmology,* Eloy Ayón-Beato, Alberto A. García,P. Isaac Ramírez-Baca, César A. Terrero-Escalante , Phys, Rev. D 88, 063523 (2013).
  - *Rotating AdS black hole stealth solution in $D = 3$ dimensions,* Mokhtar Hassaïne , Phys, Rev. D 89, 044009 (2014).
  - *Stealth configurations in vector-tensor theories of gravity,* Javier Chagoya, Gianmassimo Tasinato, JCAP 89, 046 (2018).