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Exact Ghost-Free Bigravitational Waves

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This talk is about a relevant modification of gravity which is a natural consequence of the path to build QFTs:

Wigner theorem : quantum states = irred. representations of system symmetries

For relativistic systems

irred. repres. of Poincaré group are labeled by

<u>massless:</u>	<u>massive:</u>
– helicity (spin projection along momentum)	– spin – mass

Self-interactions are included respecting symmetries and consistency (no-ghost)

free: $\longrightarrow$ self-interacting:

massless spin-2 particle

Fierz-Pauli theory $\longrightarrow$ ¡General Relativity!

massive spin-2 particle

massive Fierz-Pauli theory $\longrightarrow$?

The answer was provided by

[deRham, Gabadaze, Tolley, 1011.1232]

which define ghost-free massive gravity:

$$S[g] = \frac{1}{2\kappa_g}\int d^4x\sqrt{-g}\,R[g] - \frac{m^2}{\kappa}\int d^4x\sqrt{-g}\,U[g,f].$$

- To avoid Boulware-Deser ghost requires a new background metric $f_{\mu\nu}$.
- To build the self-interaction it is needed the square root of the product of both metrics:

$$(\gamma^2)^\mu{}_\nu = \gamma^\mu{}_\alpha \gamma^\alpha{}_\nu \equiv g^{\mu\alpha} f_{\alpha\nu}.$$

- The self-interaction potential is a precise four-degree polynomial on the components of the square root matrix:

$$U[g,f] = \sum_{k=0}^{4} b_k U_k(\gamma),$$

where the functions U_k have degree k and are defined by

$$U_k(\gamma) \equiv \sum_{A_1<\cdots<A_k} \lambda_{A_1}\cdots\lambda_{A_k} = f_k([\gamma],\ldots,[\gamma^k]),$$

λ_A $(A=0,1,2,3)$: eingenvalues of $\gamma^\mu{}_\nu$

and $[\gamma^k] \equiv tr(\gamma^k)$.

The b_k are coupling constant.

It was later proved by [Hassan, Rosen, 1109.3515] that the reference metric $f_{\mu\nu}$ can be promoted to a dynamical one without spoiling unitarity, which defines ghost-free bigravity

$$S[g,f] = \frac{1}{2\kappa_g}\int d^4x\sqrt{-g}\,R[g] - \frac{m^2}{\kappa}\int d^4x\sqrt{-g}\,U[g,f]$$

$$+\frac{1}{2\kappa_f}\int d^4x\sqrt{-f}\,R[f] .$$

- Its uniqueness is an open issue.

- U_0 and U_4 represent cosmological constants for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively.

This action gives rise to a pair of Einstein equations

$$G^{\mu}{}_{\nu} - \frac{m^2 \kappa_g}{\kappa} V^{\mu}{}_{\nu} = 0, \qquad G^{\mu}{}_{\nu} - \frac{m^2 \kappa_f}{\kappa} V^{\mu}{}_{\nu} = 0.$$

Where $V^{\mu}{}_{\nu}$ and $V^{\mu}{}_{\nu}$ are the interaction terms.

- Non-trivial configurations, beyond spherical symmetry are hard to find due to the difficulty to build the square root matrix.

However, it was recently shown in

[Higuita-Borja, Mendez-Zavaleta, A-B, 1511.01108]

that if both metrics allow a Kerr-Schild representation

$$g_{\mu\nu} = g^{0}_{\mu\nu} - F_1 K_\mu K_\nu ,$$

$$f_{\mu\nu} = c^2\left(g^{0}_{\mu\nu} - F_2 K_\mu K_\nu\right),$$

from any seed metric $g^{0}_{\mu\nu}$ with null vector field K_μ

then the square root interaction matrix is easily given

$$(\gamma^2)^\mu{}_\nu = \left(g_0^{\mu\alpha} + F_1 K^\mu K^\alpha\right) c^2 \left(g^0_{\alpha\nu} - F_2 K_\alpha K_\nu\right)$$

$$= c^2\left[\delta^\mu_\nu - \left(F_2 - F_1\right) K^\mu K_\nu\right].$$

nil potent contrib.

From square root and higher series

$$\gamma^{\mu}{}_{\nu} = c^{2}\left[\delta^{\mu}{}_{\nu} - \frac{1}{2}(F_2 - F_1)K^{\mu}K_{\nu}\right],$$

$$(\gamma^{n})^{\mu}{}_{\nu} = c^{n}\left[\delta^{\mu}{}_{\nu} - \frac{n}{2}(F_2 - F_1)K^{\mu}K_{\nu}\right],$$

and the related interaction terms are straightforward

$$V^{\mu}{}_{\nu} = P_1\delta^{\mu}{}_{\nu} - \frac{cP_0}{2}(F_2 - F_1)K^{\mu}K_{\nu},$$

$$V^{\mu}{}_{\nu} = \frac{P_2}{c^3}\delta^{\mu}{}_{\nu} - \frac{P_0}{2c^3}(F_2 - F_1)K^{\mu}K_{\nu}.$$

where P_0, P_1, P_2 are linear combinations of coupling constants.

We shall exploit this fact to characterize exact gravitational waves that allow this represent.

Concretely, the case of AdS waves

$$ds^2 = \frac{l^2}{y^2}\left(-F(u,y,x)\,du^2 - 2\,du\,dv + dy^2 + dx^2\right).$$

– This is the single type N spacetime allowing a Killing field in presence of a cosmological constant.

Studied by [Siklos, 85] in General Relativity.

Using the Kerr-Schild machinery bigravity equations are decoupled to

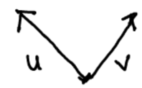
$$\Delta_S \mathcal{F} = 0,$$

(exact massless excitation
on AdS

$$\Delta_S \mathcal{H} - \frac{\ell^2 \hat{m}^2}{y^2}\mathcal{H} = 0,$$

(exact massive excitation)

Both invariant along advanced light-cone direction v:



where

$$\mathcal{F} = \frac{\kappa_f}{\kappa} F_1 + \frac{c^2 \kappa_g}{\kappa} F_2,$$

$$\mathcal{H} = F_2 - F_1,$$

$$\hat{m}^2 = \frac{(\kappa_f + c^2 \kappa_g) P_0}{c\kappa} m^2,$$

and

$$\Delta_S = \Delta_L - \frac{2}{y}\partial_y$$

$$\Delta_L = \partial_x^2 + \partial_y^2$$

are the Siklos operator and the Laplacian.

The massless equation was already solved by [Siklos, 85]. Using the complex wafefront coordinate $z = x + iy$, the Siklos operator becomes a particular case of Euler-Darboux operators

$$\frac{1}{4}\Delta_S = \partial^2_{z\bar{z}} - \frac{1}{z-\bar{z}}\partial_{\bar{z}} + \frac{1}{z-\bar{z}}\partial_z = E_{-1,-1}$$

whose solutions are known. Consequently

$$\mathcal{F}(u, z, \bar{z}) = U(-1,-1) \quad \xleftarrow{\text{E-D sol.}}$$

$$= y^2 \partial_y \left(\frac{w(u,z) + \overline{w(u,z)}}{y} \right),$$

$w(u,z)$ is arbitrary and holomorphic in z.

The massive case can be also characterized by E-D operators after the redefinition

$$H(u,z,\bar{z}) = \left(\frac{z-\bar{z}}{2i}\right)^{\rho} h(u,z,\bar{z}) \; ;$$

$$\rho = \frac{3}{2} \pm l\sqrt{\hat{m}^2 - m_{BF}^2} \; ;$$

$$m_{BF}^2 \equiv -\frac{9}{4l^2}$$ (Breitenlohner-Freedman mass)

$$E_{\rho-1,\rho-1}(h) = \left(\partial^2_{z\bar{z}} + \frac{\rho-1}{z-\bar{z}}\partial_{\bar{z}} - \frac{\rho-1}{z-\bar{z}}\partial_z\right)h = 0.$$

The general solution above the B-F bound $\tilde{m}^2 > m^2_{BF}$ is

$$\mathcal{H}(u,z,\bar{z}) = \left(\frac{z-\bar{z}}{2i}\right)^{3-\rho} \int_0^1 \varphi\left(u, z+(\bar{z}-z)t\right) t^{1-\rho}(1-t)^{1-\rho}\,dt$$

$$+ \left(\frac{z-\bar{z}}{2i}\right)^{\rho} \int_0^1 \psi\left(u, z+(\bar{z}-z)t\right) t^{\rho-2}(1-t)^{\rho-2}\,dt,$$

where φ, ψ arbitrary real functions.

- The Breintenlohner-Freedman bound can be saturated taking a nontrivial limit $\tilde{m}^2 \to m^2_{BF}$

- Any matter source can be incorporated using the classic Riemann method.