Non Abelian T-dualities in Gauged Linear Sigma Models

Nana Cabo Bizet

University of Guanajuato

MEXICUERDAS 2018
August 3.

Based on: JHEP04(2018)054. N.G.Cabo Bizet, A.Martínez Merino, L.Pando Zayas and R.Santos Silva, and some work in progress.

1. Motivation and Summary
2. $(2,2)$ Gauged Linear Sigma Models in 2d
3. Non-Abelian T-duality in the GLSM
4. GLSM with an SU(2) global symmetry
5. Direction $V = |V| n_a \sigma_a$
6. Dualization of a GLSM leading to a Grassmanian
7. Conclusions

Motivation

T-Duality is a $R \to \alpha'/R$ symmetry of string theory which exchanges winding modes with momentum modes.

Mirror symmetry: Exchanges Kähler deformations with complex deformations on dual manifolds. It is connected to T-duality [Morrison, Plesser, 95][Strominger, Yau, Zaslow, 96].

The Gauged Linear Sigma Models (GLSMs) [Witten, 93] in 2D with $(2, 2)$ SUSY are a powerful tool to describe strings propagating in a Kähler manifold. Under the renormalization group GLSMs have infrared points with conformal invariance: strings NLSM.

[Hori, Vafa, 00] developed a proof of mirror symmetry for supersymmetric sigma models on Kähler manifolds, based on studying Abelian T-dual models of the GLSMs.

Motivation

We study non-Abelian T-duality [Buscher, 88][de la Ossa, Quevedo, 93][Giveon, Rocek, 94] in GLSMs. For that we develop a dualization procedure that can be generalized to the case where there are non-Abelian global symmetries in the GLSM.

We aim to explore the connections between GLSM non Abelian T-dualities and symmetries between target manifolds of string theory. For example to explore mirror symmetry on determinantal CY varieties [Jockers,Kumar,Lapan,Morrison,Romo, 12].

Non-abelian GLSMs [Hori,Tong,07] provide more general ambient spaces than toric varities. CY manifolds in these spaces are related to determinantal, Pfaffian and more general non-complete intersection varieties.

Motivation

[Căldăraru,Sharpe,Knapp, 17] studied Grassmanian embeddings and Pfaffian CYs related by dualities [Hori,13]. Those are [Seiberg,94] dualities in GLSMs.

Recently [Gu,Sharpe,18] proposed an abelian Ansatz to relate non abelian mirrors, looking at models on the the topologically twisted A theory and B theory. They also comment on the hypothesis that non-Abelian T-duality is related to those mirrors.

It is estimated that there is a far bigger class of non-complete intersections CYs [Tonoli, 01] than of complete intersections CYs [Kreuzer, Skarke,00].

Summary

We construct the Abelian T-duality on a 2D $\mathcal{N}=2$ U(1) GLSM as a gauging of a global U(1) symmetry and the addition of Lagrange multipliers. This procedure in contrast to [Hori, Vafa, 00] allows a generalization to the non-Abelian case.

We describe the non-Abelian T-duality on the 2D $\mathcal{N}=2$ U(1) GLSM by gauging global non-Abelian symmetries.

We obtain the e.o.m. to describe the dual model. We solve them for the SU(2) group for a choice of vector superfield obtaining the dual model with a generated twisted superpotential, and the dual vacuum manifold of the GLSM.

Non perturbative corrections: Making an ansatz for the instantons, the effective potential for the U(1) gauge field on the original theory matches the one of the dual theory.

$(2,2)$ Gauged Linear Sigma Models in 2d

The reduction to 2d of $\mathcal{N} = 1$ 4d supersymmetric $U(1)$ gauge field theory gives rise to a $\mathcal{N} = 2$ (2,2) 2d theory.

The $\mathcal{N} = 1$ superfield language is employed to describe chiral superfields (csf) $\bar{D}_{\dot{\alpha}}\Phi = 0$, antichiral superfields (acsf) $D_{\alpha}\bar{\Phi} = 0$ and vector superfields (vsf) $V^{\dagger} = V$.

In the duality twisted chiral superfields (twisted-csf) arise. The twisted-csf satisfy the constraint $D_{+}X = \bar{D}_{-}X = 0$ and the twisted-acsf satisfy $\bar{D}_{+}\bar{X} = D_{-}\bar{X} = 0$.

$(2,2)$ Gauged Linear Sigma Models in 2d

The Lagrangian of a GLSM with gauge group $U(1)$ with vsf V_0 and N csfs Φ_i with charge Q_i can be written as [Hori, Vafa, 00]

$$L_0 = \int d^4\theta \left(\sum_{i=1}^{N} \bar{\Phi}_i e^{2Q_i V_0} \Phi_i - \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 \right) - \frac{1}{2} \int d^2\tilde{\theta} t \Sigma_0 + c.c., \qquad (1)$$

where $t = r - i\theta$.

The parameters are the $U(1)$ gauge coupling e, the Fayet-Iliopoulos(FI) term r and the Theta angle θ.

The twisted field strength for V_0 is given by $\Sigma_0 = \frac{1}{2}\bar{D}_+ D_- V_0$.

$(2,2)$ Gauged linear sigma models in 2d

Generically there are $N-1$ global $U(1)$ symmetries in those models.

We implement T-Duality of a $(2,2)$ GLSM with gauge group $U(1)$ with vsf V_0 and 2 csfs $\Phi_{1,2}$ with charges Q_1 and Q_2.

There is $\boxed{1 \text{ global } U(1)}$: the phase rotations of Φ_1 and Φ_2 modulo the $U(1)$ gauge transformations. This is gauged to obtain the vector field V and we add a Lagrange multiplier.

The T-dualization is implemented with respect to Φ_1 as

$$\begin{aligned} L_1 &= \int d^4\theta \left(\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV} \Phi_1 + \bar{\Phi}_2 e^{2Q_{0,2} V_0} \Phi_2 + \Psi\Sigma + \bar{\Psi}\bar{\Sigma} - \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 \right) \quad (2) \\ &+ \frac{1}{2} \left(- \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right). \end{aligned}$$

Abelian T-duality in the GLSM

Integrating the Lagrange multipliers Ψ and $\bar{\Psi}$ one obtains the original model.

Dual model: We integrate V to obtain $\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV}\Phi_1 = (\Lambda + \bar{\Lambda})/(2Q)$, with $\Lambda = \bar{D}_+ D_- \Psi$. The dual Lagrangian reads

$$
\begin{aligned}
L_1 &= \int d^4\theta \left(-\frac{\Lambda + \bar{\Lambda}}{2Q} \ln \left(\frac{\Lambda + \bar{\Lambda}}{2Q} \right) + \bar{\Phi}_2 e^{2Q_{0,2} V_0} \Phi_2 \right) \\
&+ \frac{1}{2} \left(\int d^2\tilde{\theta} (\Lambda Q_0 / Q - t) \Sigma_0 + \int d^2 \bar{\tilde{\theta}} (\bar{\Lambda} Q_0 / Q - \bar{t}) \bar{\Sigma}_0 \right), \\
&- \int d^4\theta \left(\frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 \right).
\end{aligned} \tag{3}
$$

Mirror symmetry: The csf Φ_1 is exchanged by the twisted csf Λ [Hori, Vafa, 00].

Multiple Abelian T-dualities in the GLSM

As mentioned we develop an alternative procedure to [Hori, Vafa, 00] dualization, which can be generalized to non-Abelian global symmetries.

Consider first a system of N chiral superfields and M $U(1)$ gauge symmetries. We perform the dualization procedure along the direction of the global Abelian symmetries.

One requires an extra set of M spectators chiral fields in order to have $U(1)^N$ global symmetries not contained in the $U(1)^M$ gauge symmetry.

The result of dualization are precisely the dual models in [Hori, Vafa, 00].

Multiple Abelian T-dualities in the GLSM

The chiral superfields Φ_i have charge $\hat{Q}_i$ under the gauged symmetry $U(1)_i$ and charges $Q_{i,a}$ under the original $U(1)_a$ gauge symmetries of the GLSM.
The transformation parameters with respect to the gauge symmetry and the gauged symmetry read Λ_i and $\hat{\Lambda}_a$:

$$\begin{aligned} U(1)_i &: \quad \Phi_i \to e^{i\hat{Q}_i\Lambda_i}\Phi_i, \;\; \forall_{j\neq i}\Phi_j \to \Phi_j, i = 1..N \\ U(1)_a &: \quad \Phi_i \to e^{i\hat{Q}_{a,i}\hat{\Lambda}_a}\Phi_i, a = 1...M \end{aligned} \tag{4}$$

We start with a Lagrangian where the global symmetries have been gauged, and Lagrange multiplier fields have been added:

$$\begin{aligned} L &= \int d^4\theta \left(\sum_{i=1}^{N} \bar{\Phi}_i e^{\sum_a 2Q_{i,a}V_a + 2\hat{Q}_i V_i}\Phi_i - \sum_a \frac{1}{2e_a^2}\bar{\Sigma}_a\Sigma_a + \sum_{i=1}^{N}(\Psi_i\Sigma_i + \bar{\Psi}_i\bar{\Sigma}_i) \right) \\ &+ \int d^4\theta \left(\sum_{k=N+1}^{M+N} \bar{\Phi}_k e^{\sum_a 2Q_{k,a}V_a}\Phi_k \right) - \sum_a \frac{1}{2}\int d^2\tilde{\theta}\, t_a \Sigma_a + c.c., \end{aligned}$$

Multiple Abelian T-dualities in the GLSM

The equation of motion obtained by varying the action with respect to V_i as $\frac{\delta S}{\delta V_i} = 0$ is given by

$$\bar{\Phi}_i e^{\sum_a 2Q_{ia}V_a + 2\hat{Q}_i V_i} \Phi_i = \frac{\Lambda_i + \bar{\Lambda}_i}{2\hat{Q}_i},$$

with $\Lambda_i = \frac{1}{2}\bar{D}_+ D_- \Psi_i$. Solving for V_i the dual Lagrangian reads

$$\begin{aligned} L_{dual} &= \int d^4\theta \left(-\sum_{i=1}^{N} \frac{\Lambda_i + \bar{\Lambda}_i}{2\hat{Q}_i} \ln \frac{\Lambda_i + \bar{\Lambda}_i}{2\hat{Q}_i} + \sum_{k=N+1}^{M+N} \bar{\Phi}_k e^{\sum_a 2Q_{k,a}V_a} \Phi_k - \sum_a \frac{1}{2e_a^2} \bar{\Sigma}_a \Sigma_a \right) \\ &+ \sum_a \frac{1}{2} \int d^2\tilde{\theta} \left(\sum_{i=1}^{N} \Lambda_i Q_{ia}/\hat{Q}_i - t_a \right) \Sigma_a + h.c. \end{aligned}$$

Including the instanton contribution the Lagrangian also leads to the [Hori, Vafa, 00] results for Mirrors.
The dual theory is obtained by taking the vacua where the vev of the scalar components of Φ_k and Σ_a are 0 and the field strength $v_{03,a}$ is integrated.

Non Abelian T-duality

Now we take a dualization procedure further:

The GLSM action can have $\boxed{\text{non-Abelian global symmetries}}$. We gauge these symmetries to describe non Abelian T-dual models.

$\boxed{\text{Local non-Abelian transformations}}$ for superfields are required. Consider a non Abelian group with generators T_A, a chiral superfield Φ transforms as $\Phi' = e^{i\Lambda}\Phi$, $\Lambda = \Lambda^A T_A$, $\bar{D}_{\dot{\alpha}}\Lambda = 0$, with Λ a csf in the adjoint of the group.

A vsf transforms as $e^{V'} = e^{i\bar{\Lambda}} e^{V} e^{-i\Lambda}$, $V = V^A T_A$. The twisted chiral gauge field strength $\Sigma = \frac{1}{2}\{\bar{\mathcal{D}}_+, \mathcal{D}_-\}$ and its conjugate transform as

$$\Sigma \to e^{i\Lambda}\Sigma e^{-i\Lambda},\ \bar{\Sigma} \to e^{i\bar{\Lambda}}\bar{\Sigma} e^{-i\bar{\Lambda}}. \tag{5}$$

Non Abelian T-duality

Consider a GLSM with Abelian gauge group $U(1)$, N chiral superfields $\Phi_{k,i}$ with charges Q_k, $\sum_k n_k = N$, $i = 1, ..., n_k$ where n_k is the number of superfields with charge n_k.

There is a non-Abelian global symmetry G that can be gauged to obtain the vsf V. By adding a Lagrange multiplier Ψ one gets a new action.

$$
\begin{aligned}
L_2 &= \int d^4\theta \left(\sum_k \bar{\Phi}_{k,i} (e^{2Q_k V_0 + V})_{ij} \Phi_{k,j} + \mathrm{tr}(\Psi\Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma}) \right). \\
&- \int d^4\theta \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 + \frac{1}{2} \left(- \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right). \qquad (6)
\end{aligned}
$$

$\boxed{\text{Integrating out } \Psi \text{ the original action is obtained.}}$

For $i, j = 1, 2$ and $Q_1 = Q_2$ it gives as SUSY vacua the $\mathbb{CP}^1$ variety.

Integrating out V the dual action is obtained.

One gets the equation of motion

$$0 = (D_+\bar{D}_-\bar{\Psi}_a + D_-\bar{D}_+\Psi_a + \{\chi, D_+\Psi_a\} + \{\bar{\chi}, \bar{D}_+\bar{\Psi}_a\} + (\bar{\Phi}e^{2QV_0}e^V T_a\Phi))\Delta V_a, \quad (7)$$

with $\chi = e^{-V}D_-e^V$ and where $\Delta V = e^{-V}\delta e^V$ is gauge invariant.

Note that in the generic case the dual fields are the semi-chiral superfields $D_+\Psi_a$ and $D_+\bar{\Psi}_a$.

When $\{\chi, D_+\Psi_a\} = \{\bar{\chi}, \bar{D}_+\bar{\Psi}_a\} = 0$ the dual superfields are twisted chiral $D_-\bar{D}_+\Psi_a$ and $D_+\bar{D}_-\bar{\Psi}_a$.

Let us work now for the SU(2) global symmetry case:
This requires $\boxed{\text{two chiral sf with equal charge}}$ under the U(1) gauge symmetry:

$$
\begin{aligned}
L_{model} &= \int d^4\theta \left(\bar{\Phi}_i (e^{2QV_0+V})_{ij}\Phi_j + \mathrm{tr}(\Psi\Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma})\right) \\
&- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0 - \frac{1}{2}\left(\int d^2\tilde{\theta} t\Sigma_0 + c.c.\right),
\end{aligned} \tag{8}
$$

with $i, j = 1, 2$, and the chiral and anti-chiral superfields constitute $SU(2)$ doublets.

GLSM with and SU(2) global symmetry

Integrating out V one gets:

$$e^{2QV_0}\bar{\Phi}_i e^V_{ij}\Phi_j = \sqrt{(X_1+\bar{X}_1)^2+(X_2+\bar{X}_2)^2+(X_3+\bar{X}_3)^2}. \qquad (9)$$

We take an Abelian direction in $SU(2)$ for the vsf $V = n_a\sigma_a$ with $n_a = const$, $\sum_a n_a^2 = 1$. The twisted-csf and the twisted gauge field strength have components

$$\begin{aligned} X_i &= x_i + \sqrt{2}\theta^+\bar{\chi}_+ + \sqrt{2}\bar{\theta}^-\chi_- + 2\theta^+\bar{\theta}^- G_i + ..., \qquad (10) \\ \Sigma_0 &= \sigma_0 + i\sqrt{2}\theta^+\bar{\lambda}_+ - i\sqrt{2}\bar{\theta}^-\lambda_- + 2\theta^+\bar{\theta}^-(D - iF_{01}) + ... \end{aligned}$$

In this case $\mathrm{tr}(\Psi\Sigma) = \frac{1}{2}X_a n_a|V|$. Integrating the vector superfield we have

$$|V| = 2QV_0 + \ln 2|\Phi_1|^2 + \ln(\mathcal{K}(X_i,\bar{X}_i,n_j)). \qquad (11)$$

Considering a gauging direction $V = |V| n_a \sigma_a$

The dual Lagrangian may be written as

$$
\begin{aligned}
L_{dual} &= \int d^4\theta \sqrt{(X_1+\bar{X}_1)^2 + (X_2+\bar{X}_2)^2 + (X_3+\bar{X}_3)^2} + \\
&+ \frac{1}{2}\int d^4\theta X_a n_a \ln(\mathcal{K}(X_i,\bar{X}_i,n_j)) + c.c. \\
&+ 2Q\int d\bar{\theta}^- d\theta^+ \left(X_a n_a - \frac{t}{4}\right)\Sigma_0 + 2Q\int d\bar{\theta}^+ d\theta^- \left(\bar{X}_a n_a - \frac{\bar{t}}{4}\right)\bar{\Sigma}_0 \\
&- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0.
\end{aligned}
\tag{12}
$$

We integrate the auxiliary fields to obtain the scalar potential.

Gauging direction $V = |V| n_a \sigma_a$

The scalar potential is given by

$$U = 16Q^2|\sigma_0|^2\mathcal{O}(x_i, \bar{x}_i, n_j) + 128Q^2e^2|x_a n_a - t/8|^2. \qquad (13)$$

$\mathcal{O}(x_i, \bar{x}_i, n_j)$ depends on the Kähler metric.

The supersymmetric vacua with $U = 0$ appear at $\sigma_0 = 0$ and $x_a n_a = t/8$.

The **dual variety** is given by:

$$X_a \in \mathbb{C}, \sum_a n_a^2 = 1. \qquad (14)$$

$$\sum_a X_a n_a = \frac{t}{2Q}, \; X_2 + \bar{X}_2 = X_3 + \bar{X}_3 = 0.$$

There is a quantum simmetry $X_a n_a \to X_a n_a + \frac{2\pi i k_a}{2Q}$, $k_a \in \mathbb{Z}$.

Gauging direction $V = |V| n_a \sigma_a$

In the dual model one can obtain an effective superpotential for the gauge field V_0.

The total twisted superpotential taking and Ansatz for the instanton contributions reads [Hori, Vafa,00]

$$\widetilde{W} = 2QX_a n_a \Sigma_0 - t\Sigma_0 + 2\mu e^{-X_a n_a}. \tag{15}$$

By integrating out X_a one gets the effective superpotential for the gauge field strength Σ_0:

$$W_{eff}(\Sigma_0) = -2Q\Sigma_0 \ln\left(\frac{Q\Sigma_0}{\mu}\right) + 2Q\Sigma_0 - t\Sigma_0. \tag{16}$$

This matches the result in the original model considering one-loop effects [Witten,93].

Gauging direction with $V = |V| n_a \sigma_a$, variable n_a

This is a non Abelian direction of the gauged group. In this case there is also the Lagrangian contribution term $\frac{2Q}{4}\int d\bar{\theta}^- d\theta^+ \left(X_a \bar{D}_+ n_a\right) D_- V_0 + c.c.$ (for non constant n_a).

In the dual model the SUSY vacua is given by the real dimension 2 space:

$$\begin{aligned} \sum_a x_a n_a &= \frac{t}{2Q} - \frac{B_1}{2An_1}, \\ x_2 + \bar{x}_2 &= x_3 + \bar{x}_3 = 0. \end{aligned} \tag{17}$$

Thus we observe that the dualization in a family of abelian dualities inside the non-Abelian group gives a dual model with the same geometry than the dual model.

This connects with the recent observation of [Gu,Sharpe,18] where their Ansatz to relate non-Abelian mirrors takes a dualization inside the Cartan subalgebra.

Dualization of a GLSM leading to a Grassmanian

Consider a GLSM with $U(k)$ gauge group, vector superfield V_0 and n chiral superfields Φ_i in the fundamental:

$$
\begin{aligned}
L_1 &= \int d^4\theta \left(\sum_{i=1}^{n} \bar{\Phi}_i^{\alpha} (e^{2V_0})_{\alpha\beta} \Phi_i^{\beta} \right) \\
&- \int d^4\theta \, \mathrm{Tr} \; \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 - \frac{1}{2} \left(\mathrm{Tr} \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right),
\end{aligned}
$$

There is a global $U(n)$ symmetry that can be dualized introducing the vsf V and Lagrange multipliers:

$$
\begin{aligned}
L_1 &= \int d^4\theta \left(\sum_{i=1}^{n} \bar{\Phi}_i^{\alpha} (e^{2V_0})_{\alpha\beta} (e^{2V})^{ij} \Phi_j^{\beta} \right) + \mathrm{Tr}(\Psi\Sigma) + \mathrm{Tr}(\bar{\Psi}\bar{\Sigma}) \\
&- \int d^4\theta \, \mathrm{Tr} \; \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 - \frac{1}{2} \left(\mathrm{Tr} \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right),
\end{aligned}
$$

Integrating Ψ one gets the original model.
Integrating V one gets the dual model.
We are currently exploring these dualities.

Conclusions

We have implemented non-Abelian T-duality for (2,2) 2D GLSMs, obtaining the general e.o.m after integrating the gauged field.

We analyzed an SU(2) example and a family of Abelian-duals embedded in the non-Abelian T-duality.

For the SU(2) case we extended the solutions to a general direction inside the group, studying the geometry of the dual models.

We took steps toward the inclusion of non-perturbative effects: An Ansatz reproduces correctly the effective potential for the U(1) gauge field V_0.

Future work

Study the connection between the non-Abelian T-duality in the GLSM to Mirror Symmetry.

Explore the T-duality for a GLSM with a non-Abelian gauge group.

Study explicit examples of dualities for CY varieties, possibly determinantal.

Formulate the non-Abelian duality for (0,2) 2d GLSMs [Hugo].

Relate the scalar components of fields from dual models in order to simplify the equations of motion.

¡Muchas gracias!