

A taste of flavor symmetries in realistic string compactifications

Saúl Ramos-Sánchez
UNAM

Mexicuerdas

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In collaboration with Y. Olguín-Trejo & R. Pérez-Martínez: [arxiv:1708.01595](#) & [arxiv:1808.0xxxx](#)

The need for discrete flavor symmetries

- Need to explain {
 - three flavors of SM particles
 - observed mass patterns
 - CKM, PMNS phases
 - neutrino physics
 - suppression of proton decay
 - baryogenesis
 - CP violation
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- Problem: Which one is THE right one ??

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A: permutation symmetries among fixed points $\rightarrow S_n$

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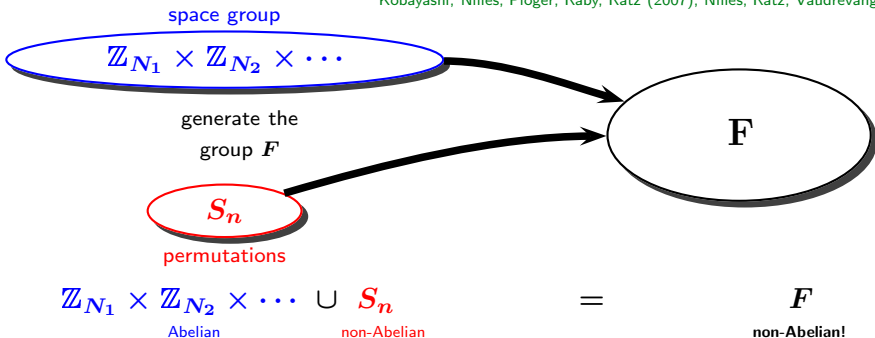
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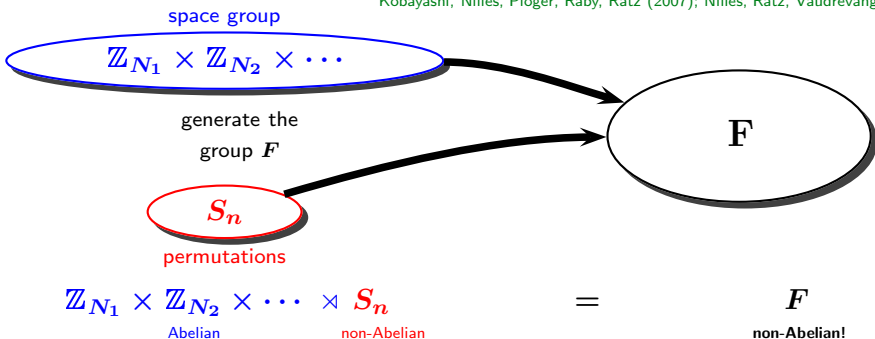
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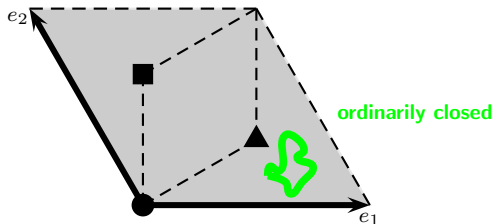
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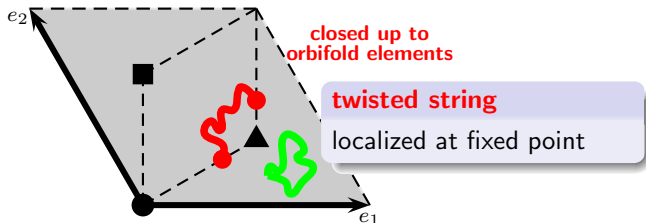
Untwisted and twisted closed strings in $\mathbb{T}^2/\mathbb{Z}_3$

Two types of massless *closed strings*:
ordinarily closed and *closed under the orbifold*



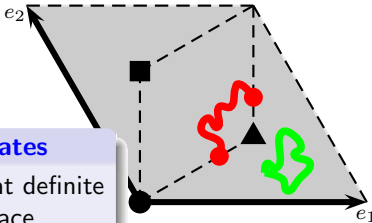
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localized twisted states

LEEF states appear at definite points in compact space

Twisted strings located at fixed points $\rightarrow$ LEEF states localized
Identified with **constructing elements** $g_\ell \in \mathcal{S}$ (boundary conditions for string closure)
$$g_\ell = (\vartheta^{k^{(\ell)}}, n_\alpha^{(\ell)} e_\alpha)$$

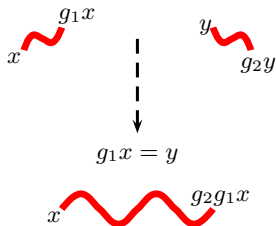
Invariance under S

- Interacting strings join for the time they interact:



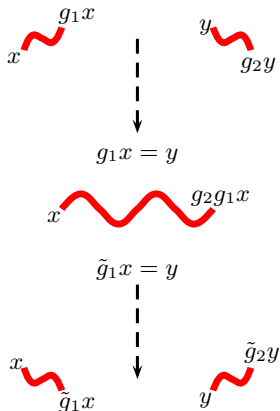
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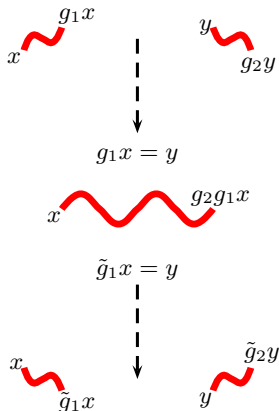
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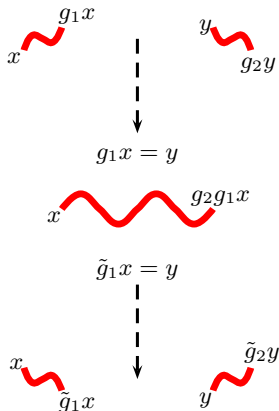
$$g_2 g_1 = \tilde{g}_2 \tilde{g}_1 = \mathbb{1}$$

for L interacting strings:

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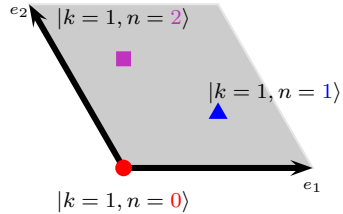
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space group selection rule:

$$\prod_{\ell=1}^L (\vartheta^{k^{(\ell)}}, n_{\alpha}^{(\ell)} e_{\alpha}) = (\mathbb{1}, \bigcup_{\ell} (\mathbb{1} - \vartheta^{k^{(\ell)}}) \Lambda)$$

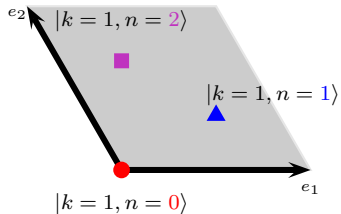
Two 2D examples: $\mathbb{T}^2/\mathbb{Z}_3$

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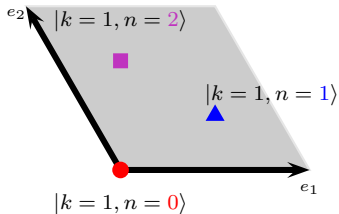
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$$\begin{aligned}\prod_{\ell=1}^L (\vartheta^{k^{(\ell)}} n^{(\ell)} e_1) &= (\vartheta^{\sum_{\ell} k^{(\ell)}}, (\sum_{\ell} n^{(\ell)}) e_1) \\ &\stackrel{!}{=} (\mathbb{1}, (0 \bmod 3) e_1)\end{aligned}$$

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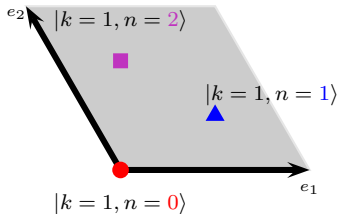
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$\Rightarrow \mathbb{Z}_3$'s acting on k, n charges:

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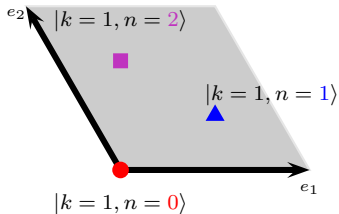
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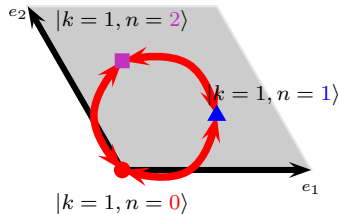
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permutations



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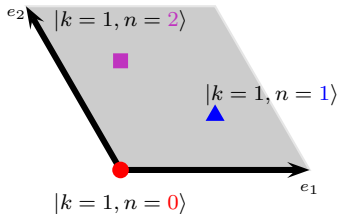
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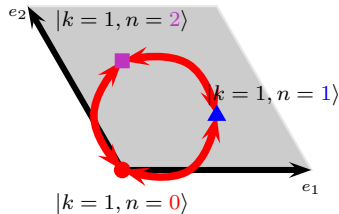
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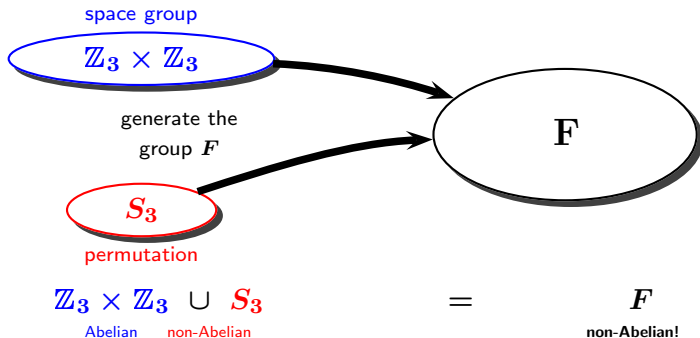


permutation symmetry:

$$S_3$$

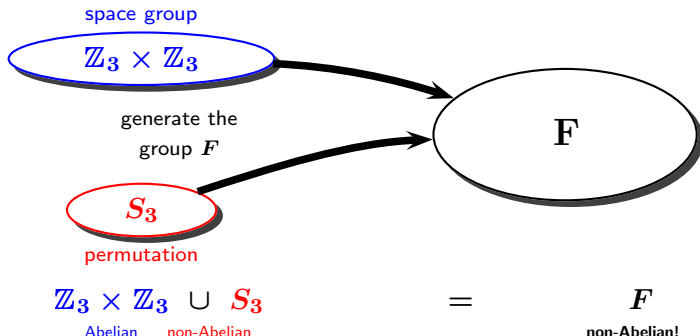
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- Flavor symmetry

$$F = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) = \Delta(54)$$

Two 2D examples: $\mathbb{T}^2/\mathbb{Z}_2 \times \mathbb{Z}_2$

An example with **roto-translations**

$$\mathbb{T}^2/\mathbb{Z}_2 \times \mathbb{Z}_2: \quad g_1 = (\vartheta, \tfrac{1}{2}e_1), \quad g_2 = (\omega, \tfrac{1}{2}e_2), \quad g_1, g_2 \in S$$

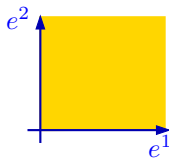
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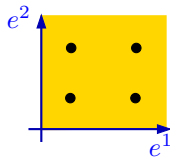
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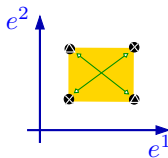
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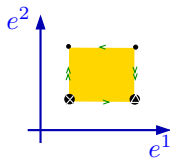
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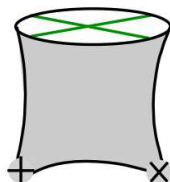
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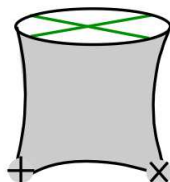
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$$(n_1, n_2) = (0, 1)$$

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Two fixed points: $g_f^{(\ell)} = (-\mathbb{1}, \tfrac{1}{2}(e_1 - e_2) + n_1^{(\ell)}e_1 + n_2^{(\ell)}e_2)$

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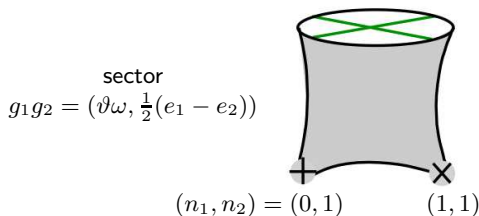
String interactions must be **invariant under S**

$$g_f^{(1)} g_f^{(2)} \cdots g_f^{(\ell)} \cdots g_f^{(L)} = \left(\mathbb{1}, \bigcup_{\ell} \tilde{\Lambda}_{\ell} \right), \quad \tilde{\Lambda}_{\ell} = (\mathbb{1} - \vartheta \omega) \Lambda = 2\Lambda,$$

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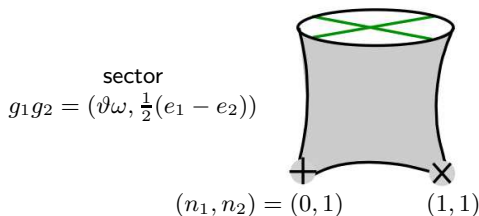


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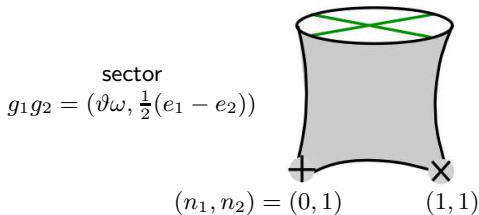
$$\vartheta^L \omega^L = (-\mathbb{1})^L = \mathbb{1} \quad \Rightarrow \quad L = 0 \bmod 2 \quad \Rightarrow \quad \mathbb{Z}_2$$

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Also, from permutations: $S_2 = \mathbb{Z}_2$

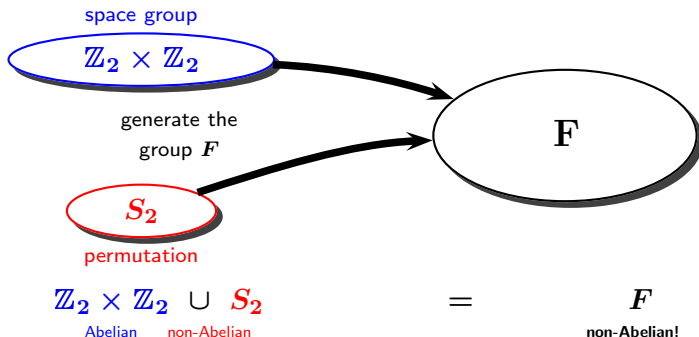
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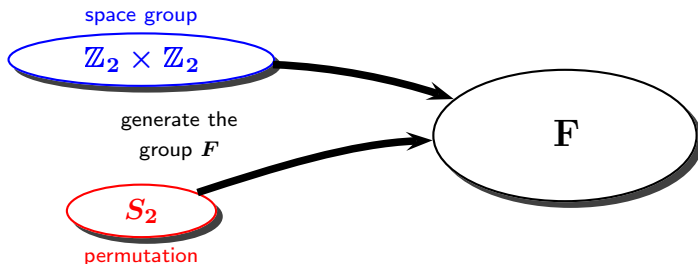
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From permutations: $S_2 = \mathbb{Z}_2$



$$\begin{array}{ccc} \mathbb{Z}_2 \times \mathbb{Z}_2 \cup S_2 & = & F \\ \text{Abelian} \quad \text{non-Abelian} & & \text{non-Abelian!} \\ \Rightarrow & F = S_2 \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2) = D_4 \end{array}$$

Examples in 1D, 2D

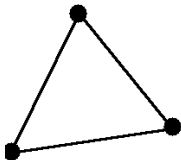


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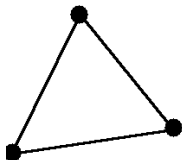


$$\rightarrow F = \Delta(54) = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3)$$

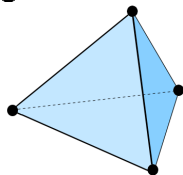
Examples in 1D, 2D



$$\rightarrow F = D_4 = S_2 \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$$



$$\rightarrow F = \Delta(54) = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3)$$

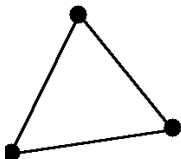


$$\rightarrow F = D_4 \times D_4 / \mathbb{Z}_2$$

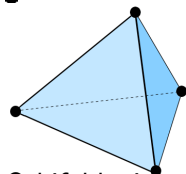
Examples in 1D, 2D



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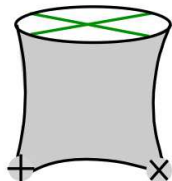


$$\rightarrow F = \Delta(54) = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3)$$



$$\rightarrow F = D_4 \times D_4 / \mathbb{Z}_2$$

Orbifold with **roto-translations**



$$\rightarrow F = D_4$$

Natural question 1:

Aiming at a **guiding principle** from an UV complete theory,...

Natural question 1:

Aiming at a **guiding principle** from an UV complete theory,...

what flavor symmetries are realized
in heterotic orbifolds?

Explore all 138 Abelian space groups **S** already known

Repeat previous steps for the all possible space groups

Orbifold label	Twist vector(s)	# of $\mathbb{Z}$ classes	# of affine classes
$\mathbb{Z}_3$	$\frac{1}{3}(1, 1, -2)$	1	1
$\mathbb{Z}_4$	$\frac{1}{4}(1, 1, -2)$	3	3
$\mathbb{Z}_6$ -I	$\frac{1}{6}(1, 1, -2)$	2	2
$\mathbb{Z}_6$ -II	$\frac{1}{6}(1, 2, -3)$	4	4
$\mathbb{Z}_7$	$\frac{1}{7}(1, 2, -3)$	1	1
$\mathbb{Z}_8$ -I	$\frac{1}{8}(1, 2, -3)$	3	3
$\mathbb{Z}_8$ -II	$\frac{1}{8}(1, 3, -4)$	2	2
$\mathbb{Z}_{12}$ -I	$\frac{1}{12}(1, 4, -5)$	2	2
$\mathbb{Z}_{12}$ -II	$\frac{1}{12}(1, 5, -6)$	1	1
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$\frac{1}{2}(0, 1, -1)$, $\frac{1}{2}(1, 0, -1)$	12	35
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$\frac{1}{2}(0, 1, -1)$, $\frac{1}{4}(1, 0, -1)$	10	41
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	$\frac{1}{2}(0, 1, -1)$, $\frac{1}{6}(1, 0, -1)$	2	4
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -II	$\frac{1}{2}(0, 1, -1)$, $\frac{1}{6}(1, 1, -2)$	4	4
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$\frac{1}{3}(0, 1, -1)$, $\frac{1}{3}(1, 0, -1)$	5	15
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$\frac{1}{3}(0, 1, -1)$, $\frac{1}{6}(1, 0, -1)$	2	4
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$\frac{1}{4}(0, 1, -1)$, $\frac{1}{4}(1, 0, -1)$	5	15
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$\frac{1}{6}(0, 1, -1)$, $\frac{1}{6}(1, 0, -1)$	1	1

Fischer, Ratz, Torrado, Vaudrevange (2013)

We find all possible flavor symmetries (without enhancement)

Orbifold	Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_2$ (1,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^4$
(1,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(1,3)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
(1,4)	—
(2,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^3$
(2,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(2,3)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
(2,4)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(2,5)	$(D_4 \times D_4)$
(2,6)	—
(3,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
(3,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(3,3)	$(D_4 \times D_4)/\mathbb{Z}_2$
(3,4)	—
(4,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
(4,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(5,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
(5,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(5,3)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(5,4)	$(D_4 \times D_4)$
(5,5)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(6,1)	$(D_4 \times D_4 \times D_4 \times D_4)/\mathbb{Z}_2^2$
(6,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(6,3)	D_4
(7,1)	$(D_4 \times D_4)$
(7,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(8,1)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(9,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
(9,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(9,3)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(10,1)	$D_4 \times \mathbb{Z}_2$

Orbifold	Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_2$ (10,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
(11,1)	$(D_4 \times D_4 \times D_4)/\mathbb{Z}_2$
(12,1)	$(D_4 \times D_4)$
(12,2)	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$\mathbb{Z}_2 \times \mathbb{Z}_4$ (1,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$
(1,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(1,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(1,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(1,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(1,6)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
(2,1)	$(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$
(2,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(2,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(2,4)	$D_4 \times \mathbb{Z}_4$
(2,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(2,6)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(3,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
(3,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(3,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(3,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(3,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(3,6)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(4,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
(4,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(4,3)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(4,4)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(4,5)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(5,1)	$(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$
(5,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$
(6,1)	$(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
(6,2)	$\mathbb{Z}_2 \times \mathbb{Z}_4$

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Orbifold	Flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(6,3)$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $(6,4)$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $(6,5)$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $(7,1)$ $D_4 \times \mathbb{Z}_4$ $(7,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$ $(7,3)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$ $(8,1)$ $(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^2$ $(8,2)$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $(8,3)$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $(9,1)$ $(D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$ $(9,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$ $(9,3)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$ $(10,1)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$ $(10,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_2$
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	$(1,1)$ $(D_4 \times D_4 \times \mathbb{Z}_6)/\mathbb{Z}_2$ $(1,2)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$ $(2,1)$ $(D_4 \times D_4 \times \mathbb{Z}_6)/\mathbb{Z}_2$ $(2,2)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -II	$(1,1)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$ $(2,1)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$ $(3,1)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$ $(4,1)$ $\mathbb{Z}_2 \times \mathbb{Z}_6$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1,1)$ $(\Delta(54) \times \Delta(54) \times \Delta(54))/\mathbb{Z}_3$ $(1,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(1,3)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(1,4)$ $\Delta(54) \times \Delta(54)$ $(2,1)$ $\Delta(54) \times \Delta(54)$ $(2,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(2,3)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(2,4)$ $\Delta(54) \times \Delta(54)$ $(3,1)$ $\Delta(54) \times \mathbb{Z}_3$ $(3,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(3,3)$ $\Delta(54) \times \Delta(54)$ $(4,1)$ $\Delta(54) \times \Delta(54)$ $(4,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$ $(4,3)$ $\Delta(54) \times \Delta(54)$ $(5,1)$ $\mathbb{Z}_3 \times \mathbb{Z}_3$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1,1)$ $\Delta(54) \times \mathbb{Z}_6$ $(1,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_6$

Orbifold	Flavor symmetry
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(2,1)$ $\Delta(54) \times \mathbb{Z}_6$ $(2,2)$ $\mathbb{Z}_3 \times \mathbb{Z}_6$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1,1)$ $(D_4 \times D_4 \times D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^3$ $(1,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(1,3)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(1,4)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(2,1)$ $(D_4 \times D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2^2$ $(2,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(2,3)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(2,4)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(3,1)$ $(D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2$ $(3,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(4,1)$ $(D_4 \times D_4 \times \mathbb{Z}_4^2)/\mathbb{Z}_2^2$ $(4,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(4,3)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(5,1)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$ $(5,2)$ $\mathbb{Z}_4 \times \mathbb{Z}_4$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1,1)$ $\mathbb{Z}_6 \times \mathbb{Z}_6$
Orbifold	Flavor symmetry
$\mathbb{Z}_3$	$(1,1)$ $(\Delta(54) \times \Delta(54) \times \Delta(54))/\mathbb{Z}_3^3$
$\mathbb{Z}_4$	$(1,1)$ $(D_4 \times D_4 \times D_4 \times D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2^4$ $(2,1)$ $(S_4 \times S_2 \times S_2) \ltimes (\mathbb{Z}_3^3 \times \mathbb{Z}_2^3)$ $(3,1)$ $(S_4 \times S_4) \ltimes (\mathbb{Z}_4^5 \times \mathbb{Z}_2^2)$
$\mathbb{Z}_6$ -I	$(1,1)$ $\Delta(54)$ $(2,1)$ $(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3$
$\mathbb{Z}_6$ -II	$(1,1)$ $\Delta(54) \times [D_4 \times D_4/\mathbb{Z}_2]$ $(2,1)$ $[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4 \times D_4]/\mathbb{Z}_2^2$ $(3,1)$ $[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4 \times D_4]/\mathbb{Z}_2^2$ $(4,1)$ $[\Delta(54) \times \mathbb{Z}_6]/\mathbb{Z}_3 \times [D_4/\mathbb{Z}_2]$
$\mathbb{Z}_7$	$(1,1)$ $S_7 \ltimes \mathbb{Z}_2^7$
$\mathbb{Z}_8$ -I	$(1,1)$ $(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$ $(2,1)$ $(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$ $(3,1)$ $S_4 \ltimes (\mathbb{Z}_8 \times \mathbb{Z}_4^2 \times \mathbb{Z}_2)$
$\mathbb{Z}_8$ -II	$(1,1)$ $(D_4 \times D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^3$ $(2,1)$ $(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2^2$
$\mathbb{Z}_{12}$ -I	$(1,1)$ $\Delta(54)$ $(2,1)$ $(\Delta(54) \times \mathbb{Z}_{12})/\mathbb{Z}_3$
$\mathbb{Z}_{12}$ -II	$(1,1)$ $(D_4 \times D_4)/\mathbb{Z}_2$

SRS (2017), Olguin-Irrejo, Pérez-Martínez, SRS (2018)

Natural question 2:

Wilson lines are needed to get MSSM-like models;

Ibáñez, Nilles, Quevedo (1987)

They break explicitly (the permutations in) flavor symmmetries...

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what flavor symmetries *survive*
in realistic compactifications?

Flavor in realistic orbifolds

Flavor symmetries in realistic models

- 1 Determine **all** (inequivalent) $E_8 \times E_8$ gauge embeddings in heterotic string

Flavor symmetries in realistic models

- ① Determine **all** (inequivalent) $E_8 \times E_8$ gauge embeddings in heterotic string
- ② Select **semi-realistic models**
 - 4D gauge group = $SU(3)_c \times SU(2)_L \times U(1)_Y \times G_{hidden}$
 - 3 generations of quarks and leptons + a pair H_u, H_d
 - $\sin^2 \theta_w(M_{GUT}) = 3/8$
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- 3 Determine the flavor breakdown due to Wilson lines in each case
- 4 Phenomenology?

Orbifolder needed as a tool (Nilles, SRS, Vaudrevange, Wingerter (2011))



Broadest known scan of MSSM-like models

Orbifold	Max # of independent WL	# of MSSM-like models with				Total
		0	1	2	≥ 3	
		vanishing WL				
$\mathbb{Z}_3$ (1,1)	3	0	0	0	0	0
$\mathbb{Z}_4$ (1,1)	4	0	0	0	0	0
(2,1)	3	149	0	0	0	149
(3,1)	2	27	0	0		27
$\mathbb{Z}_6$ -I (1,1)	1	30	0			30
(2,1)	1	30	0			30
$\mathbb{Z}_6$ -II (1,1)	3	26	337	0	0	363
(2,1)	3	14	335	0	0	349
(3,1)	3	18	335	0	0	353
(4,1)	2	44	312	0		356
$\mathbb{Z}_7$ (1,1)	1	1	0			1
$\mathbb{Z}_8$ -I (1,1)	2	230	38	0		268
(2,1)	2	205	41	0		246
(3,1)	1	389	0			389
$\mathbb{Z}_8$ -II (1,1)	3	1,604	398	21	0	2,023
(2,1)	2	274	231	0		505
$\mathbb{Z}_{12}$ -I (1,1)	1	556	0			556
(2,1)	1	555	0			555
$\mathbb{Z}_{12}$ -II (1,1)	2	279	84	0		363

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Orbifold	Max # of independent WL	# of MSSM-like models with vanishing WL					Total
		0	1	2	3	≥ 4	
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	6	1	152	52	0	205
	(2,1)	5	13	342	14	0	369
	(3,1)	5	4	400	40	0	444
	(5,1)	4	2	40	0	0	42
	(6,1)	4	344	57	0	0	401
	(7,1)	4	21	55	0	0	76
	(9,1)	3	25	2	0	0	27
	(10,1)	3	19	2	0	0	21
	(12,1)	2	3	0	0	0	3
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1,1)	4	454	8,637	1,463	26	10,580
	(1,6)	2	65	21	0	0	86
	(2,1)	4	260	4,683	1,131	81	6,158
	(2,4)	2	281	47	0	0	328
	(3,1)	3	13,117	3,637	103	0	16,857
	(4,1)	3	2,911	1,575	33	0	4,519
	(5,1)	3	1,311	742	63	0	2,116
	(6,1)	3	1,814	1,374	58	0	3,246
	(7,1)	3	1,481	1,122	64	0	2,667
	(8,1)	2	839	72	0	0	911
	(9,1)	2	1,620	522	0	0	2,142
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	(1,1)	2	467	116	0	0	583
	(2,1)	2	275	78	0	0	353
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	3	40	987	81	0	1,108
	(1,4)	1	8	0	0	0	8
	(2,1)	2	1,713	239	0	0	1,952
	(3,1)	2	6	0	0	0	6
	(4,1)	2	105	110	0	0	215
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1,1)	1	4,469	24	0	0	4,493
	(2,1)	1	495	45	0	0	540
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	3	599	12,091	2,258	5	14,953
	(2,1)	2	2,807	3,220	19	0	6,046
	(3,1)	2	2,039	875	6	0	2,920
	(4,1)	2	1,876	1,552	6	0	3,434
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1,1)	0	3,412	0	0	0	3,412

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Flavor symmetry with ℓ non-vanishing WLs				Total models
			$\ell = 1$	2	3	4	
$\mathbb{Z}_3$	(1,1)	3	$\Delta(54)^2$ 0	$\Delta(54) \times \mathbb{Z}_3^2$ 0	$\mathbb{Z}_3^3$ 0		0
$\mathbb{Z}_4$	(1,1)	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 0	$D_4^2 \times \mathbb{Z}_4$ 0	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 0	0
	(2,1)	3	$(S_2 \times S_2) \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ $(S_4 \times S_2) \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$S_2 \ltimes (\mathbb{Z}_4^2 \times \mathbb{Z}_2^2)$ 0 $S_4 \ltimes (\mathbb{Z}_4^3 \times \mathbb{Z}_2^3)$ 0	$\mathbb{Z}_4^4 \times \mathbb{Z}_2^2$ 149		149
	(3,1)	2	$S_4 \ltimes (\mathbb{Z}_4^4 \times \mathbb{Z}_2)$ 0	$\mathbb{Z}_4^4$ 27			27
$\mathbb{Z}_6$ -I	(1,1)	1	$\mathbb{Z}_3^2$ 30				30
	(2,1)	1	$\mathbb{Z}_6 \times \mathbb{Z}_3$ 30				30
$\mathbb{Z}_6$ -II	(1,1)	3	$[(D_4 \times D_4)/\mathbb{Z}_2] \times \mathbb{Z}_3^2$ 0 $\Delta(54) \times D_4 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_3^2$ 337 $\Delta(54) \times \mathbb{Z}_3^2$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 26		363
	(2,1)	3	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 335 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 14		349
	(3,1)	3	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times [(D_4 \times D_4)/\mathbb{Z}_2^2]$ 0 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times D_4$ 0	$D_4 \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 333 $[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2^2$ 2	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2^2$ 18		353
	(4,1)	2	$[(\Delta(54) \times \mathbb{Z}_6)/\mathbb{Z}_3] \times \mathbb{Z}_2$ 0 $[D_4/\mathbb{Z}_2] \times \mathbb{Z}_6 \times \mathbb{Z}_3$ 312	$\mathbb{Z}_6 \times \mathbb{Z}_3 \times \mathbb{Z}_2$ 44			356

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Flavor symmetry with ℓ non-vanishing WLs				Total models
			$\ell = 1$	2	3	4	
$\mathbb{Z}_7$	(1,1)	1	$\mathbb{Z}_7^2$ 1				1
$\mathbb{Z}_8$ -I	(1,1)	2	$D_4 \times \mathbb{Z}_8$ 38	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 230			268
	(2,1)	2	$D_4 \times \mathbb{Z}_8$ 41	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 204			246
	(3,1)	1	$\mathbb{Z}_8 \times \mathbb{Z}_4$ 389				389
$\mathbb{Z}_8$ -II	(1,1)	3	$(D_4 \times D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2$ 21	$D_4 \times \mathbb{Z}_8 \times \mathbb{Z}_2$ 398	$\mathbb{Z}_8 \times \mathbb{Z}_2^3$ 1,604		2,023
	(2,1)	2	$D_4 \times \mathbb{Z}_8$ 231	$\mathbb{Z}_8 \times \mathbb{Z}_2^2$ 274			505
$\mathbb{Z}_{12}$ -I	(1,1)	1	$\mathbb{Z}_3 \times \mathbb{Z}_3$ 556				556
	(2,1)	1	$\mathbb{Z}_{12} \times \mathbb{Z}_3$ 555				555
$\mathbb{Z}_{12}$ -II	(1,1)	2	$D_4 \times \mathbb{Z}_2$ 84	$\mathbb{Z}_2^3$ 279			363

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

6,563 $\mathbb{Z}_N$ heterotic orbifolds with MSSM-like properties

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLS	Max. # ℓ of WLS affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLS						Total
				$\ell = 1$	2	3	4	5	6	
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1,1)	6	6	$D_4^3/\mathbb{Z}_2^2$ 0	D_4^4 0	$D_4^4 \times \mathbb{Z}_2^2$ 0	$D_4^2 \times \mathbb{Z}_2^4$ 52	$D_4 \times \mathbb{Z}_2^6$ 152	$\mathbb{Z}_2^8$ 1	205
	(1,3)	4	2	$D_4^3 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0					0
	(2,1)	5	5	$D_4^4/\mathbb{Z}_2$ 0	$D_4^4 \times \mathbb{Z}_2$ 0	$D_4^4 \times \mathbb{Z}_2^2$ 14	$D_4 \times \mathbb{Z}_2^5$ 342	$\mathbb{Z}_2^7$ 13		369
	(2,3)	3	1	D_4^2 0						0
	(2,5)	3	1	$D_4 \times \mathbb{Z}_2^2$ 0						0
	(3,1)	5	3	$D_4^4 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 432	$\mathbb{Z}_2^5$ 12				444
	(3,3)	3	1	$D_4 \times \mathbb{Z}_2$ 0						0
	(4,1)	4	4	D_4^4 0	$D_4^4 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 0	$\mathbb{Z}_2^6$ 0			0
	(5,1)	4	4	D_4^4 0	$D_4^4 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 40	$\mathbb{Z}_2^6$ 2			42
	(5,4)	2	1	$D_4 \times \mathbb{Z}_2^2$ 0						0
	(6,1)	4	4	D_4^4 0	$D_4^4 \times \mathbb{Z}_2^2$ 0	$D_4 \times \mathbb{Z}_2^4$ 57	$\mathbb{Z}_2^6$ 344			401
	(6,3)	2	0	—						0
	(7,1)	4	2	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^2$ 76					76
	(9,1)	3	3	$D_4^4 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 2	$\mathbb{Z}_2^5$ 25				27
	(10,1)	3	1	$\mathbb{Z}_2^4$ 21						21
	(11,1)	3	3	$D_4^2 \times \mathbb{Z}_2$ 0	$D_4 \times \mathbb{Z}_2^3$ 0	$\mathbb{Z}_2^5$ 0				0

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

Flavor symmetries in MSSM-like models

Orbifold		Max. # of possible WLs	Max. # ℓ of WLs affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLs						Total
				$\ell = 1$	2	3	4	5	6	
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(12,1)	2	2	$D_4 \times \mathbb{Z}_2^2$ 0	$\mathbb{Z}_2^4$ 3					3
	(1,1)	4	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 26	$D_4^2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,463	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^3$ 8,637	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 454			10,580
	(1,6)	2	2	$D_4^2 \times \mathbb{Z}_4$ 21	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 65					86
	(2,1)	4	4	$(D_4^3 \times \mathbb{Z}_4)/\mathbb{Z}_2$ 81	$D_4^2 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,131	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^3$ 4,686	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 260			6,158
	(2,4)	2	1	$\mathbb{Z}_4 \times \mathbb{Z}_2$ 321						328
	(3,1)	3	3	$(D_4^2 \times \mathbb{Z}_4)$ 103	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 3,637	$\mathbb{Z}_4 \times \mathbb{Z}_2^4$ 13,117				16,857
	(4,1)	3	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 1,308	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 3,187					4,519
	(5,1)	3	3	$D_4^2 \times \mathbb{Z}_4$ 63	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2^2$ 742	$\mathbb{Z}_4 \times \mathbb{Z}_2^4$ 1,311				2,116
	(6,1)	3	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 884	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 2,325					3,246
	(7,1)	3	1	$\mathbb{Z}_4 \times \mathbb{Z}_2^2$ 2,229						2,667
	(8,1)	2	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 72	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 839					911
	(9,1)	2	2	$D_4 \times \mathbb{Z}_4 \times \mathbb{Z}_2$ 522	$\mathbb{Z}_4 \times \mathbb{Z}_2^5$ 1,620					2,142
$\mathbb{Z}_2 \times \mathbb{Z}_6$ -I	(1,1)	2	2	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_6$ 467	$\mathbb{Z}_2^2 \times \mathbb{Z}_6$ 116					583
	(2,1)	2	2	$D_4 \times \mathbb{Z}_2 \times \mathbb{Z}_6$ 275	$\mathbb{Z}_2^2 \times \mathbb{Z}_6$ 78					353
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1,1)	3	3	$\Delta(54)^2 \times \mathbb{Z}_3$ 81	$\Delta(54) \times \mathbb{Z}_3^3$ 987	$\mathbb{Z}_3^5$ 40				1,108
	(1,4)	1	1	$\mathbb{Z}_3^3$ 8						8
	(2,1)	2	2	$\Delta(54) \times \mathbb{Z}_3^2$ 239	$\mathbb{Z}_3^4$ 1,713					1,952
	(3,1)	2	1	$\mathbb{Z}_3^3$ 6						6

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Orbifold		Max. # of possible WLs	Max. # ℓ of WLs affecting the flavor symmetry	Flavor symmetry with ℓ of non-vanishing WLs						Total
				$\ell = 1$	2	3	4	5	6	
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(4,1)	2	1	$\mathbb{Z}_3^4$ 127						215
	(1,1)	1	1	$\mathbb{Z}_3^2 \times \mathbb{Z}_6$ 4,469						4,493
	(2,1)	1	1	$\mathbb{Z}_3^2 \times \mathbb{Z}_6$ 495						540
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1,1)	3	3	$(D_4^2 \times \mathbb{Z}_4^2)/\mathbb{Z}_2$ 2,258	$D_4 \times \mathbb{Z}_4^2 \times \mathbb{Z}_2$ 12,091	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 599				14,953
	(2,1)	2	2	$D_4 \times \mathbb{Z}_4^2$ 3,220	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 2,807					6,046
	(3,1)	2	1	$\mathbb{Z}_4^2 \times \mathbb{Z}_2$ 2,914						2,920
	(4,1)	2	1	$D_4 \times \mathbb{Z}_4^2$ 1,552	$\mathbb{Z}_4^2 \times \mathbb{Z}_2^2$ 1,876					3,434

SRS (2017), Olguín-Trejo, Pérez-Martínez, SRS (2018)

87,809 $\mathbb{Z}_N \times \mathbb{Z}_M$ heterotic orbifolds with MSSM-like properties

Pheno with stringy flavor

Lessons from models with D_4

Lebedev, Nilles, SRS, Ratz, Vaudrevange, Wingerter (2008)

- Most of the models ($\sim 46,000$) endowed with D_4
Oguín-Trejo, Pérez-Martínez, SRS
- Stringy constraint: $\mathbf{2} + \mathbf{1}_0$ family structure, 3^{rd} generation in the bulk
 $\rightarrow$ top Yukawa is the largest
- All exotics can be decoupled in SUSY vacua with $\mathbb{Z}_2^{\text{matter}}$ (and/or $\mathbb{Z}_4^R$)
- In those vacua, D_4 is spontaneously broken $\rightarrow$ pheno?
- Nice Yukawa matrices (with $\tilde{s} \sim 0.1$) $\rightarrow$ pheno?

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, Y_d = \begin{pmatrix} 0 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & 0 & \tilde{s}^7 \\ 0 & \tilde{s}^6 & \tilde{s}^7 \end{pmatrix}, Y_e = \begin{pmatrix} \tilde{s}^7 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & \tilde{s}^7 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}$$

- See-saw with 100's of r.h. neutrinos $\rightarrow$ nicer $M_{\text{see-saw}} \sim 10^{-2} M_{\text{GUT}}$
Buchmüller, Hamaguchi, Lebedev, SRS, Ratz

$\Delta(54)$ flavor symmetry

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- Simplest options: $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$
- No promising models in $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$ with $\Delta(54)$

Investigate $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifolds!

Flavor symmetries in $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- If all fixed points are degenerate, flavor symmetry is large:

$$F = S_3 \times S_3 \times S_3 \ltimes \left[\mathbb{Z}_3^\vartheta \times \mathbb{Z}_3^\omega \times (\mathbb{Z}_3^m)^3 \right]$$

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- In $\mathbb{Z}_3 \times \mathbb{Z}_3$ heterotic orbifolds, up to 3 **Wilson lines** (WLs) allowed
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- Models with **2 WLs** lead to flavor

$$F = S_3 \ltimes \left[\mathbb{Z}_3^\vartheta \times \mathbb{Z}_3^\omega \times (\mathbb{Z}_3^m)^3 \right] \supset \Delta(54) \text{ ☺}$$

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 - ③ 81 with $F = \Delta(54)^2 \times \mathbb{Z}_3$

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explanation for three generations!! 😊

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* Numbers statistically compatible with Nilles, Vaudrevange (2014), but we find many more models 😊

Example

$\Delta(54)$ string spectrum

#	irrep	$\Delta(54)$	label	#	anti-irrep	$\Delta(54)$	label
3	$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i				
3	$(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\bar{u}_i$				
3	$(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\bar{d}_i$				
3	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i				
3	$(\mathbf{1}, \mathbf{1})_1$	$\mathbf{3}_{11}$	$\bar{e}_i$				
3	$(\mathbf{1}, \mathbf{1})_0$	$\mathbf{3}_{12}$	$\bar{\nu}_i$				
1	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\mathbf{1}_0$	H_d	1	$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$\mathbf{1}_0$	H_u

Flavons

3	$(\mathbf{1}, \mathbf{1})_0$	$\mathbf{3}_{11}$	ϕ_i^u	
3	$(\mathbf{1}, \mathbf{1})_0$	$\mathbf{3}_{11}$	$\phi_i^{d,e}$	
3	$(\mathbf{1}, \mathbf{1})_0$	$\mathbf{3}_{12}$	ϕ_i^ν	
2	$(\mathbf{1}, \mathbf{1})_0$	$2 \cdot \mathbf{1}_0$	$s^{(d,e)}, s^u$	
128	$(\mathbf{1}, \mathbf{1})_0$	$77 \cdot \mathbf{1}_0 + 16 \cdot \mathbf{3}_{12} + \mathbf{3}_{11}$	N_i	

Exotic states

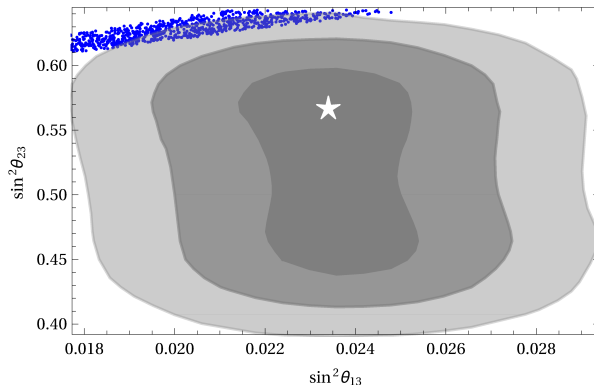
16	$(\mathbf{1}, \mathbf{2})_{\frac{1}{6}}$	$10 \cdot \mathbf{1}_0 + 2 \cdot \mathbf{3}_{12}$	v_i	16	$(\mathbf{1}, \mathbf{2})_{-\frac{1}{6}}$	$4 \cdot \mathbf{1}_0 + 4 \cdot \mathbf{3}_{12}$	$\bar{v}_i$
3	$(\mathbf{3}, \mathbf{1})_0$	$\mathbf{3}_{12}$	y_i	3	$(\overline{\mathbf{3}}, \mathbf{1})_0$	$3 \times \mathbf{1}_0$	$\bar{y}_i$
1	$(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{1}{3}}$	$\mathbf{1}_0$	z_i	1	$(\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$	$\mathbf{1}_0$	$\bar{z}_i$
7	$(\mathbf{1}, \mathbf{1})_{-\frac{2}{3}}$	$4 \cdot \mathbf{1}_0 + \mathbf{3}_{12}$	x_i	7	$(\mathbf{1}, \mathbf{1})_{\frac{2}{3}}$	$4 \cdot \mathbf{1}_0 + \mathbf{3}_{11}$	$\bar{x}_i$
51	$(\mathbf{1}, \mathbf{1})_{-\frac{1}{3}}$	$30 \cdot \mathbf{1}_0 + 7 \cdot \mathbf{3}_{12}$	w_i	51	$(\mathbf{1}, \mathbf{1})_{\frac{1}{3}}$	$24 \cdot \mathbf{1}_0 + 9 \cdot \mathbf{3}_{12}$	$\bar{w}_i$

$$\begin{aligned}
 W_Y &= y_{ijk}^u Q_i H_u \bar{u}_j \phi_k^u s_u + y_{ijk}^d Q_i H_d \bar{d}_j \phi_k^{(d,e)} s^{(d,e)} + y_{ijk}^e L_i H_d \bar{e}_j \phi_k^{(d,e)} s^{(d,e)} \\
 &+ y_{ijkl}^\nu L_i H_u \bar{\nu}_j + \lambda_{ijk} \bar{\nu}_i \bar{\nu}_j \bar{\phi}_k^\nu, \quad i, j, k = 1, 2, 3,
 \end{aligned}$$

$\Delta(54)$ stringy phenomenology: neutrinos with see-saw

Correlation atmospheric–reactor mixing angles compatible with **best fit**

Forero, Tortola, Valle (2014)

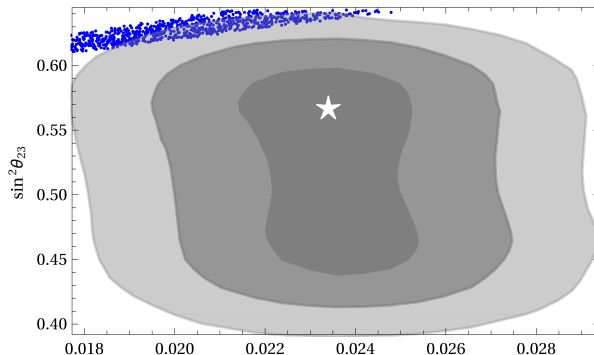


- atmospheric mixing angle: $51.3^\circ \lesssim \theta_{23} \lesssim 53.1^\circ$ (second octant) ☺
- reactor mixing angle: $7.8^\circ \lesssim \theta_{13} \lesssim 8.9^\circ$ ☺
- $6\text{meV} \lesssim m_{\nu_1} \lesssim 6.8\text{meV}$, $65\text{meV} \lesssim \sum m_\nu \lesssim 70\text{meV}$ ☺

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Correlation atmospheric–reactor mixing angles compatible with **best fit**

Forero, Tortola, Valle (2014)



Advantages of model:

- atmospheric θ_{23} in second octant) ☺
- reactor mixing only **normal hierarchy** admissible
- $6\text{meV} \lesssim m_{\nu_1}$ **Falsifiable** soon ☺

$\Delta(54)$ stringy phenomenology: down sector

Down-quark and charged-lepton sectors are symmetric

$$\Rightarrow \tan \theta_C^e \approx \frac{(M_e^D)_{12}}{(M_e^D)_{22}} \approx \sqrt{\frac{m_e}{m_\mu}} \quad \text{leptonic Gatto-Sartori-Tonin} \quad \text{😊}$$

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BUT all other quark and charged-lepton mixing angles are smaller than observed 😞

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Additional novel consequence

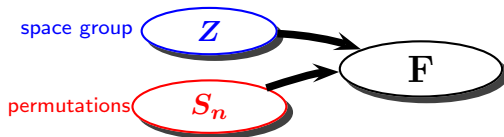
$$\frac{m_s - m_d}{m_b} \stackrel{!}{=} \frac{m_\mu - m_e}{m_\tau}.$$

more stringent than $b - \tau$ unification 😞

Punch line

- Flavor symmetries from compactification geometry:

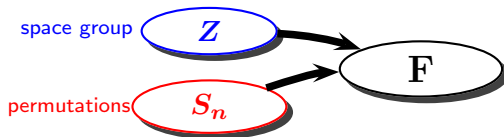
Kobayashi, Nilles, Plöger, Raby, Ratz (2007)



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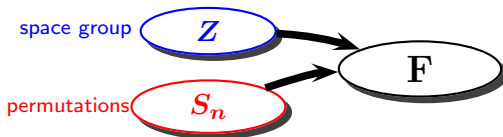


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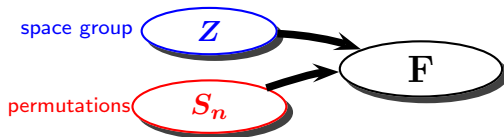


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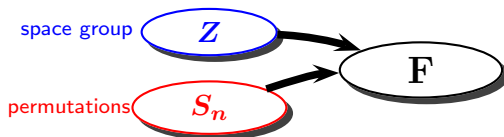


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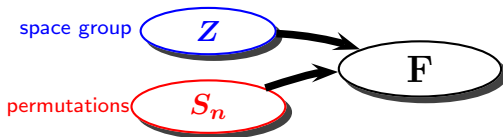


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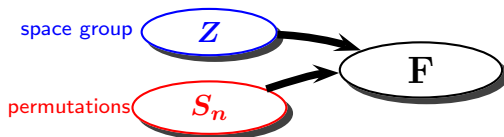


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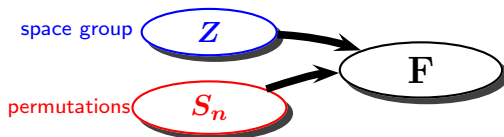


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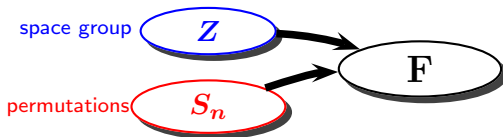
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Possibilities for explanation of CP-violation [Nilles et al, in progress]

Punch line

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Kobayashi, Nilles, Plöger, Raby, Ratz (2007)



- Full classification of flavor symmetries in Abelian orbifolds
- Full “classification” of Abelian heterotic orbifolds with $\sim 94,000$ nice models

- Full “classification” To do

- 50% of them exhibit
 - compare with other classifying tools
- 4% of them enjoy Δ
 - identify common properties
- 45% only Abelian flavor
 - study phenomenology
- Nice pheno, although
 - symmetry enhancement?
 - are there predictions?

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