Chaos and spreading of entanglement in holography

Viktor Jahnke

ICN - UNAM

viktor.jahnke@correo.nucleares.unam.mx

August 2, 2018

Overview

1. Introduction
2. Quantum Chaos in a Nutshell
3. The Butterfly Effect in gauge/gravity duality
4. Bonds on chaos

Introduction

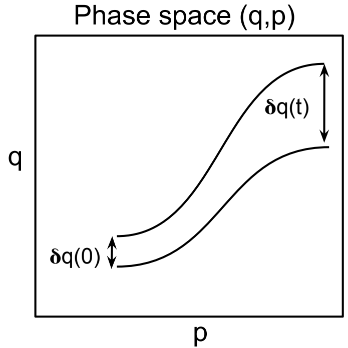
- chaos is a common property of thermal systems;
- in particular, strongly coupled thermal systems described by the boundary theory in AdS/CFT are chaotic;
- chaos-related properties of thermal systems provide us a deeper understanding of AdS/CFT.

Classical Chaos

The distance between nearby trajectories increases exponentially with time

$$|\delta q(t)| \approx e^{\lambda_L t} \, |\delta q(0)|$$

λ_L = Lyapunov exponent



From Classical to Quantum Chaos Kitaev 2015 Shenker, Stanford, Maldacena 2015

Sensitive dependence on initial conditions

$$\frac{\partial q(t)}{\partial q(0)} = \{q(t), p(0)\}_{\text{Poisson}} \to \frac{1}{i\hbar}[q(t), p(0))]$$

To avoid phase cancellations, one defines

$$C(t) = -\langle\, [q(t), p(0)]^2 \,\rangle$$

More generally, one replaces

$$q(t) \to W(t)$$
$$p(0) \to V(0)$$

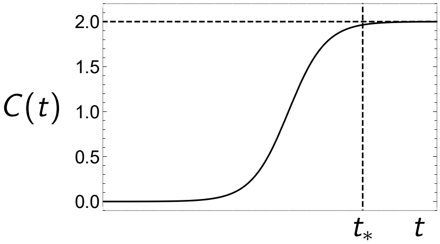
where W and V are hermitian operators, and consider

$$C(t) = -\langle\, [W(t), V(0)]^2 \,\rangle_\beta \to \text{diagnose chaos}$$

Quantum Chaos

$$C(t) = -\langle\, [W(t), V(0))]^2 \,\rangle_\beta = \langle\, W\ V\ V\ W \,\rangle_\beta + \langle\, V\ W\ W\ V \,\rangle_\beta$$
$$-\langle\, W\ V\ W\ V \,\rangle_\beta - \langle\, V\ W\ V\ W \,\rangle_\beta$$

For a chaotic system:

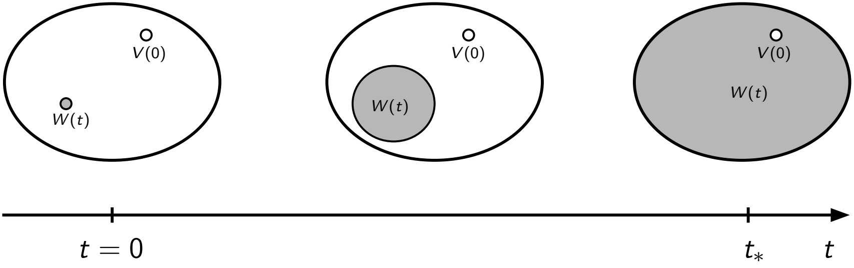


$\langle\, V\ W\ V\ W \,\rangle_\beta$ and $\langle\, W\ V\ W\ V \,\rangle_\beta$ vanish for $t > t_*$.

t_* = scrambling time. Generically $t_* \sim \log N_{\mathrm{d.o.f.}}$ and $C(t) \sim \frac{1}{N_{\mathrm{d.o.f.}}} e^{\lambda_L (t - t_*)}$.

Quantum Chaos - A cartoon of scrambling

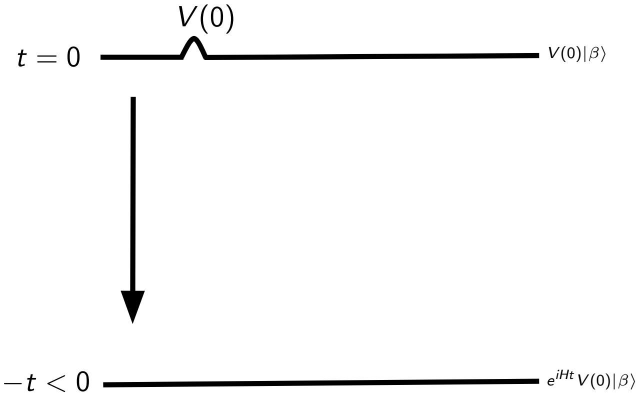
Space of degrees of freedom



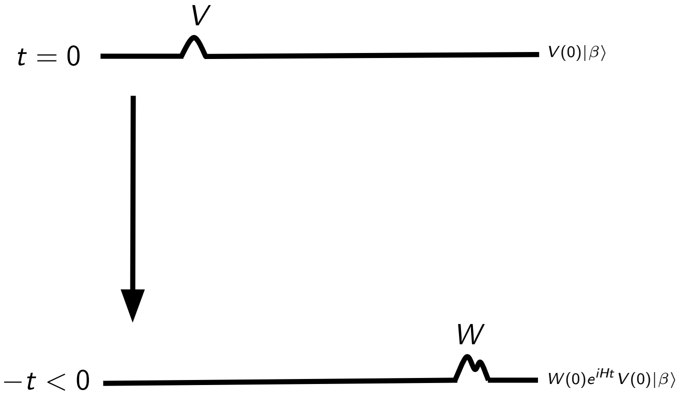
- $W(t)$ expands in the space of d.o.f. $\Longrightarrow C(t) = -\langle\, [W(t), V(0))]^2 \,\rangle_\beta$ grows.
- $C(t)$ saturates a constant value when $W(t)$ gets scramble among all d.o.f.
- Scrambling proceeds at a steady rate $\frac{\partial_t C}{C} = \lambda_L =$ Lyapunov exponent
- $\langle\, V\, W\, V\, W \,\rangle_\beta$ goes to zero at later times, $t > t_*$.

$$F(t) = \langle W(-t)\, V(0) \,|\, W(-t)\, V(0) \rangle_\beta$$

$$F(t) = \langle W(-t)\, V(0) \,|\, W(-t)\, V(0) \rangle_\beta$$

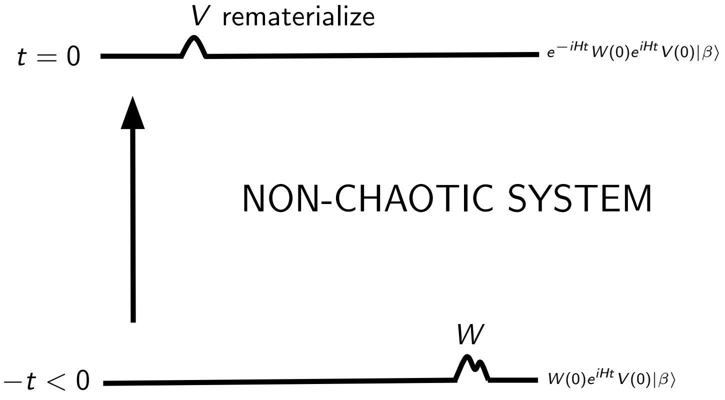


$$F(t) = \langle W(-t)\, V(0) \,|\, W(-t)\, V(0) \rangle_\beta$$



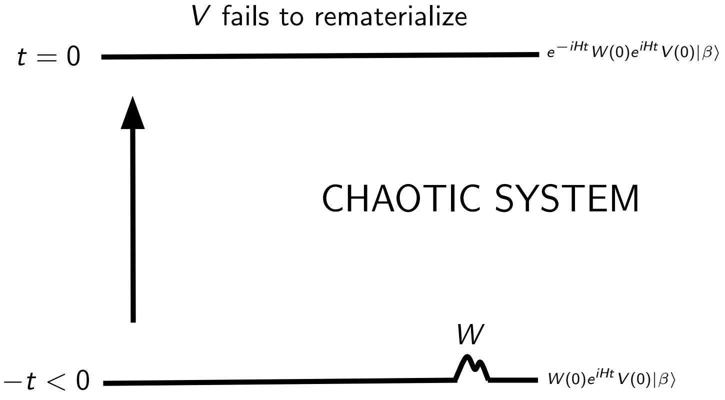
Quantum Chaos - OTOC for a *non-chaotic* system

$$F(t) = \langle W(-t)\,V(0)\,|\,W(-t)\,V(0)\rangle_\beta \approx 1$$



Quantum Chaos - OTOC for a *chaotic* system

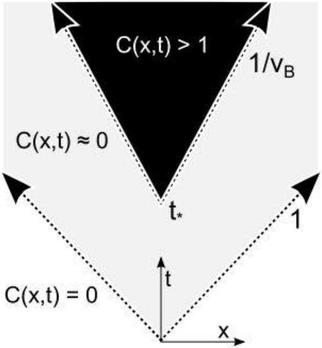
$$F(t) = \langle W(-t)\, V(0) \,|\, W(-t)\, V(0) \rangle_\beta = 0\,, \quad \text{for} \quad t \gtrsim t_*$$



If $V_0(0)$ and $W_x(t)$ are local operators, then

$$C(t,x) = -\langle\, [V_0(0), W_x(t)]^2\,\rangle_\beta \sim \frac{1}{N_{\mathrm{d.o.f.}}}\, e^{\lambda_L\left(t - t_* - \frac{|x|}{v_B}\right)}$$

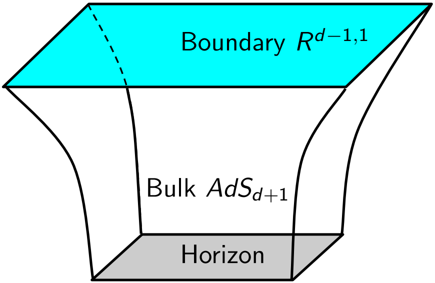
v_B = 'butterfly velocity' $\longrightarrow$ defines an effective light cone: $t = t_* + |x|/v_B$



Roberts, Stanford, Susskind 2014

Gauge/Gravity duality Maldacena 97

Also known as "AdS/CFT correspondence" or 'Holography'



BOUNDARY THEORY:
gauge theory, e.g. Conformal Field Theory

BULK THEORY:
gravity theory, e.g. Einstein gravity

WEAK/STRONG duality

Most known example:

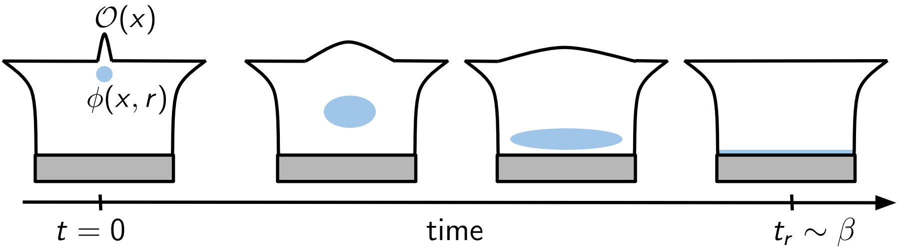
$$\mathcal{N} = 4 \text{ super Yang-Mills theory in } R^{3,1} \Longleftrightarrow \text{type IIB supergravity in } AdS_5 \times S^5$$

thermal system at the boundary $\Longleftrightarrow$ black hole in the bulk

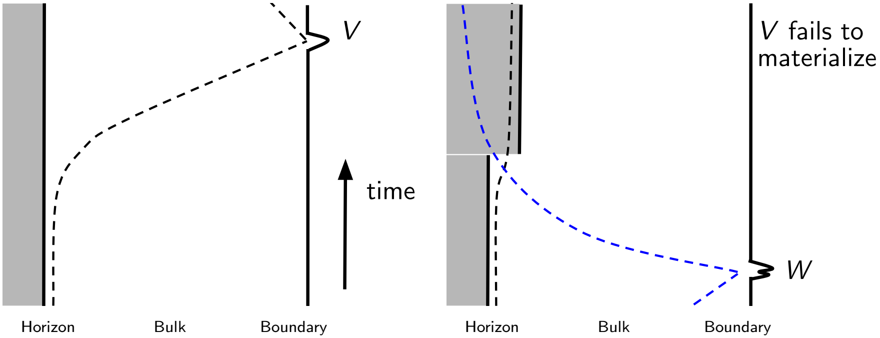
Operator $\mathcal{O}(x)$ at the boundary $\longleftrightarrow$ Field $\phi(x,r)$ in the bulk

$$Z_{\mathrm{bdry}}[J(x)] = Z_{\mathrm{bulk}}[\phi(x,r)]\Big|_{\phi(x,r_{\mathrm{bdry}})=J(x)}$$

A cartoon of thermalization



Chaos in Gauge/Gravity duality - description in the bulk

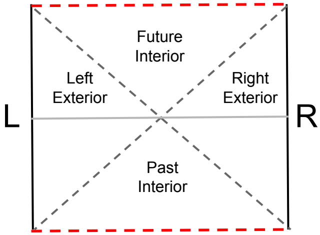


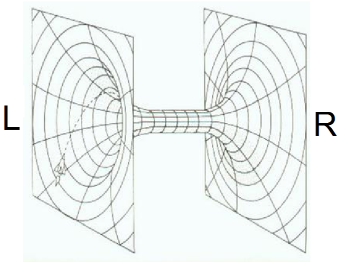
$E_W = E_0\, e^{\frac{2\pi}{\beta} t} \Longrightarrow$ the gravitational effect of W is important when $t \gtrsim t_*$

- W gets blue-shifted and increases the size of the horizon;
- V fails to materialize;
- the OTOC vanishes: $\langle W\, V | W\, V \rangle_\beta = 0 \Longrightarrow C(t,x) = 2$

Thermofield double (TFD) = Two-sided black hole (wormhole) Maldacena 2001

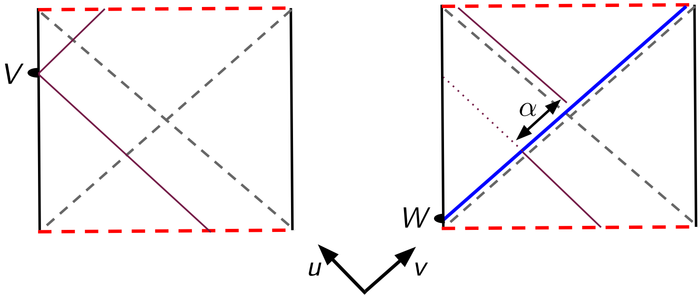
$$|TFD\rangle = \frac{1}{Z^{1/2}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \quad =$$





- highly atypical state;
- there is local entanglement between subsystems of L and R at $t = 0$;
- S_{BH} =total entanglement between L and R.

An early enough W produces a shock wave geometry



Unperturbed metric: $ds^2 = 2A(u,v)dudv + G_{ij}dx^i dx^j$

Shock wave metric: $v \to v + \theta(u)\alpha(t,x)$

$$C(t,x) = -\langle\, [V_0(0), W_x(t)]^2\, \rangle_\beta \longleftrightarrow \alpha(t,x)$$

Chaotic properties of the boundary theory

Shock wave profile:

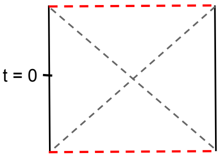
$$\left(\Box + M^2\right)\alpha(t,x) \sim e^{\frac{2\pi}{\beta}(t-t_*)} \Longrightarrow \alpha(t,x) \approx \frac{1}{N_{\mathrm{d.o.f.}}} e^{\frac{2\pi}{\beta}\left(t-t_*-\frac{|x|}{v_B}\right)}$$

For black holes:

- $\lambda_L = \frac{2\pi}{\beta}$
- $t_* = \frac{\beta}{2\pi}\log S_{\mathrm{BH}}$
- $v_{B,i}^2 = \frac{G'_{tt}(r_H)}{G_{ii}(r_H)\sum_k G^{kk}(r_H)G'_{kk}(r_H)}$

Homogeneous shocks: $\alpha(t) = \text{constant} \times e^{\frac{2\pi}{\beta}(t-t_*)}$

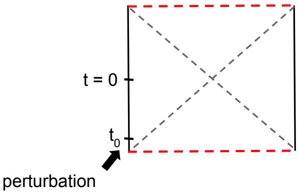
Unperturbed geometry



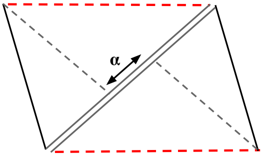
Atypical state

Localized pattern of entanglement

A small perturbation is added a long time in the past



Shock wave geometry



Typical state

Delocalized pattern of entanglement

- The two-sided black hole has a local pattern of entanglement at $t = 0$.
- A small perturbation added a long time in the past destroys this local pattern of entanglement.

Mutual Information *as a measurement of local entanglement*

Let A and B be subsystems (regions) of L and R asymptotic boundaries. The mutual information

$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

where $S(A)$ is entanglement entropy of the region A and so on.

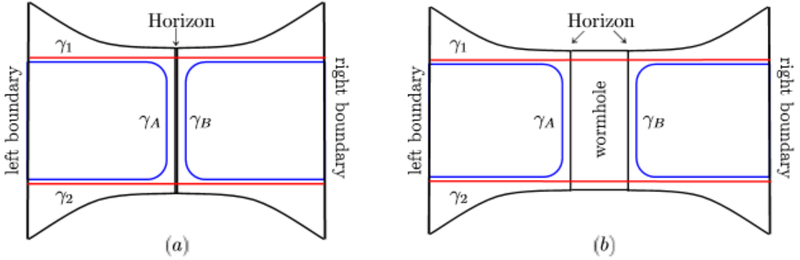
The mutual information provides an upper bound for correlations between A and B

$$\frac{\Big(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle\Big)^2}{2|\langle \mathcal{O}_A \rangle|^2 |\langle \mathcal{O}_B \rangle|^2} \leq I(A,B)$$

Wolf, Verstraete, Hastings 2008

$I(A,B) \rightarrow$ good measure of local entanglement

Disruption of the Mutual Information - Shenker, Stanford 2013



$$\boxed{I(A,B) = S(A) + S(B) - S(A \cup B)}$$

γ_A, γ_B = blue curves, $\quad \gamma_{A\cup B} = \gamma_1 \cup \gamma_2$ = red curves

$$S(A) = \frac{1}{4G_N}\ \text{Area}(\gamma_A), \quad S(B) = \frac{1}{4G_N}\ \text{Area}(\gamma_B)$$

$$S(A\cup B) = \frac{1}{4G_N}\ \min\left\{\text{Area}(\gamma_A) + \text{Area}(\gamma_B), \text{Area}(\gamma_{A\cup B})\right\}$$

$$I(A,B) = \frac{1}{4G_N}\left[\text{Area}(\gamma_A) + \text{Area}(\gamma_B) - \text{Area}(\gamma_{A\cup B})\right] \geq 0 \quad \text{or} \quad I(A,B) = 0$$

Chaos and Spreading of Entanglement

$$I(A,B;\alpha) = S_A + S_B - S_{A\cup B}(\alpha)\,, \qquad \alpha = \text{shock wave parameter}$$

Regularized entanglement entropy: $S^{\text{reg}}_{A\cup B}(\alpha) \equiv S_{A\cup B}(\alpha) - S_{A\cup B}(\alpha = 0)$

$$I(A,B;\alpha) = I(A,B;\alpha=0) - S^{\text{reg}}_{A\cup B}(\alpha)$$

$$\alpha \propto e^{\frac{2\pi}{\beta}t_0}\,, \qquad t_0 = \text{shock wave time}$$

After a scrambling time, $t_0 \gtrsim t_*$, $\boxed{\dfrac{d}{dt_0} S^{\text{reg}}_{A\cup B} = V_2\, s_{\text{th}}\, v_E}$

v_E = entanglement velocity

$\boxed{v_E \text{ controls the disruption of entanglement in shock wave geometries}}$

Bounds on chaos

- For system with large number of d.o.f.

$$\lambda_L \leq \frac{2\pi}{\beta} \quad \text{Maldacena, Shenker, Stanford 2015}$$

Black holes always saturate this bound.

- In holographic systems, null energy condition and isotropy imply:

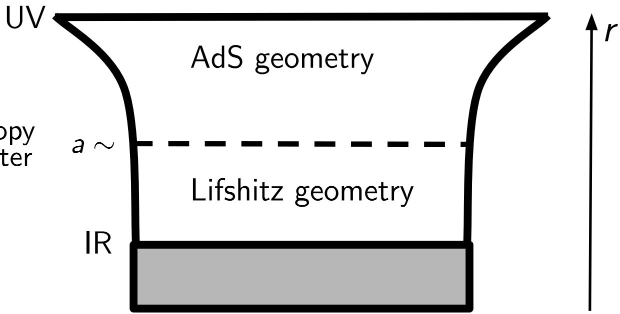
$$\begin{aligned} v_B &\leq v_B^{\mathrm{Sch}} = \sqrt{\frac{d}{2(d-1)}} \\ v_E &\leq v_E^{\mathrm{Sch}} = \frac{\sqrt{d}(d-2)^{\frac{1}{2}-\frac{1}{d}}}{[2(d-1)]^{1-\frac{1}{d}}} \quad \text{Mezei 2016} \end{aligned}$$

where d is the number of dimensions of the boundary theory

How general are these bounds?

Anisotropic black branes Mateos, Trancanelli 2011

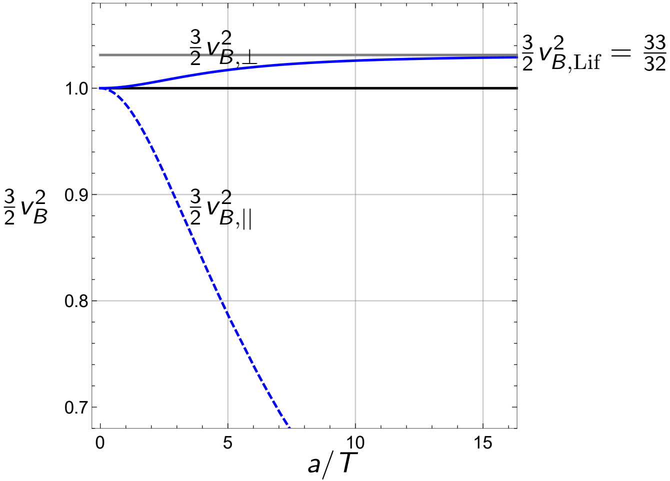
Mateos & Trancanelli model: SYM theory + theta-term



AdS-BH $\xrightarrow{a/T>>1}$ Lif-BH

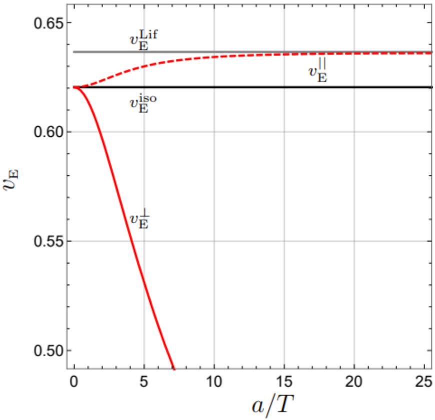
Designed to model anisotropy effects in a quark-gluon plasma

Chaos in anisotropic black branes Jahnke 2017



- $v_{B,\perp}^2$ violates the upper bound $v_B^2 \leq \frac{2}{3}$
- $v_{B,\perp}^2$ remains bounded by its value at the IR

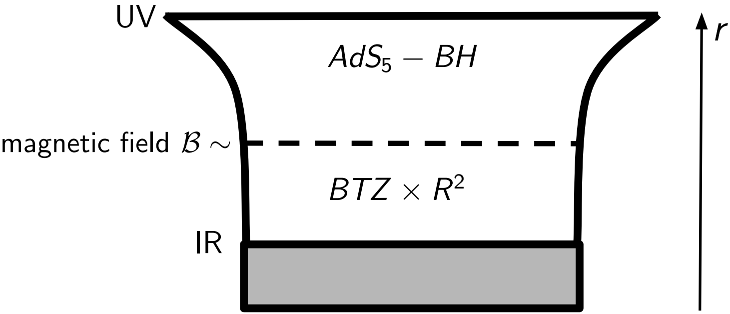
Chaos in anisotropic black branes Jahnke 2017



- $v_E^{||}$ violates the upper bound $v_E \leq \frac{\sqrt{2}}{3^{3/4}}$
- $v_E^{||}$ remains bounded by its value at the IR

Magnetic branes D'Hoker, Kraus 2009

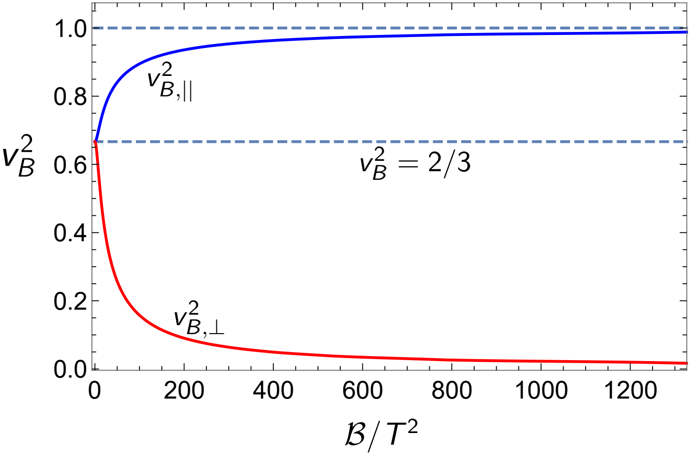
D'Hoker & Kraus model: SYM theory + magnetic field



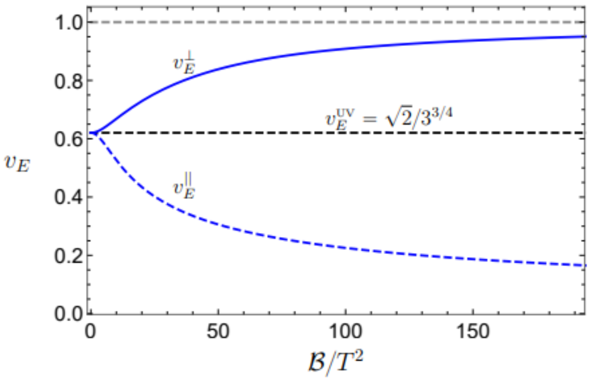
$$AdS_5 - BH \xrightarrow{\mathcal{B}/T^2>>1} BTZ \times R^2 \text{ or } CFT_4 \xrightarrow{\mathcal{B}/T^2>>1} CFT_2$$

Designed to model the effects of a magnetic field in the QGP

Chaos in magnetic black branes Ávila, Jahnke, Patiño 2018



- for a CFT_d, $v_B^2 = \frac{d}{2(d-1)}$
- $v_B^2 = 2/3$ for $d = 4$, and $v_B^2 = 1$ for $d = 2$.
- consistent with the RG flow $CFT_4 \xrightarrow{\mathcal{B}/T^2 >> 1} CFT_2$

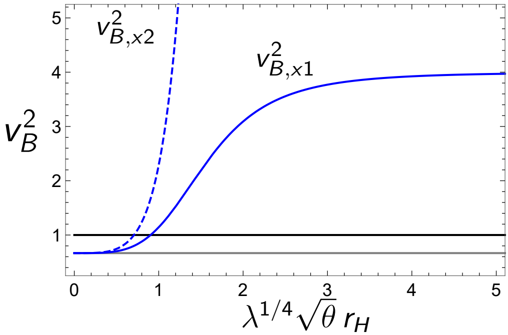


- for a CFT_d, $v_E = \frac{\sqrt{d}(d-2)^{\frac{1}{2}-\frac{1}{d}}}{[2(d-1)]^{1-\frac{1}{d}}}$
- $v_E = \frac{\sqrt{2}}{3^{3/4}}$ for $d = 4$, and $v_E = 1$ for $d = 2$.
- consistent with the RG flow $CFT_4 \xrightarrow{\mathcal{B}/T^2>>1} CFT_2$

Chaos in a non-commutative gauge theory (NC-SYM)

Fischler, Jahnke, Pedraza *in preparation*

Non-commutative geometry: $[x^2, x^3] = i\theta \Rightarrow$ non-asymp. AdS geometry

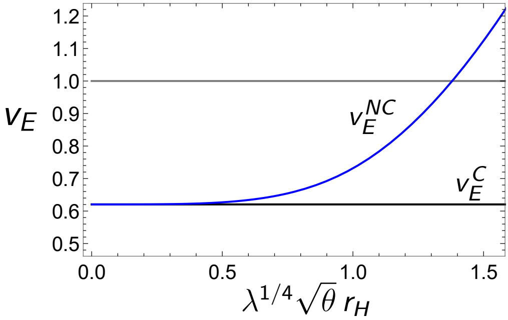


- v_B can become larger than the speed of light ($c = 1$)
- $v_B \leq 1$ for asymptotically AdS geometries Qi,Yang 2017
- non-Lorentz invariant boundary theory $\Longleftrightarrow$ non-asymp. AdS geometry

Chaos in a non-commutative gauge theory (NC-SYM)

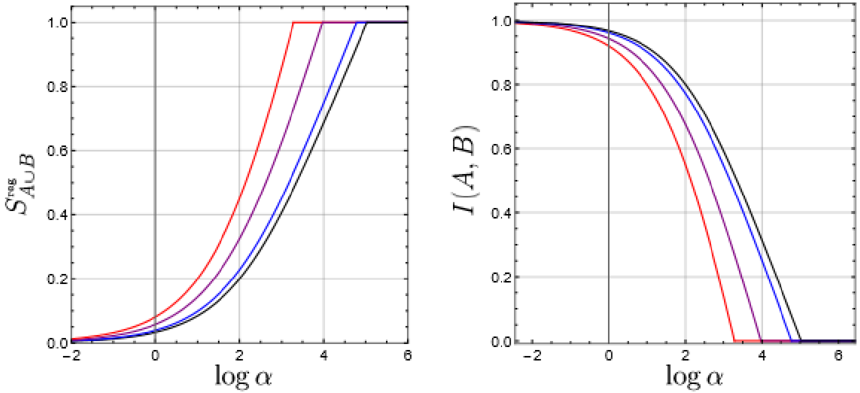
Fischler, Jahnke, Pedraza *in preparation*

Non-local effects in spreading of entanglement in the fuzzy sphere Sabella-Garnier 2017



- v_E can become larger than the speed of light ($c = 1$)
- $I(A, B) \geq 0$ imply $v_E \leq 1$ (Lorentz invariant theories)
- this is not necessarily true in non-Lorentz invariant theories

Disruption of the mutual information Fischler, Jahnke, Pedraza *in preparation*



- the non-commutative parameter increases from the right to the left
- $\log\alpha = \frac{2\pi}{\beta} t_0 + \ldots$, the quench starts after a scrambling time $t_0 \gtrsim t_*$
- similar to the behaviour observed in global quenches

Maldacena, Hartman 2013 Liu, Suh 2013

Conclusions and future directions

Findings:

- v_B and v_E violate previously suggested bounds, but remain bounded in asymptotically AdS geometries;
- v_B and v_E diagnose RG flows;
- homogeneous shock waves in eternal black hole geometries provide an additional example of a quench protocal.

Future directions:

- to study the chaotic properties of higher curvature gravity theories

  work in progress with Chernicoff, Avitua, Patiño
- to investigate connection between chaos and spreading of entanglement

THE END