Holographic complexity of anisotropic black branes.

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Outline

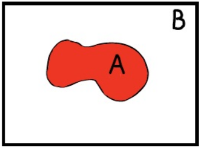
Quantum Field Theory in
$d + 1$ dimensions

Quantum Gravity in
$D > d + 1$ dimensions

Information Theory

- Entanglement

$$S_A = -\operatorname{Tr}_A \rho_A \log \rho_A$$

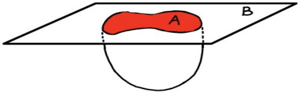


$$\Longleftrightarrow$$

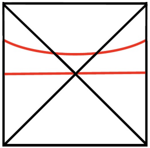
Gravity

- Einstein's equations, energy conditions and connectedness of spacetime.[Ryu, Takayanagi]

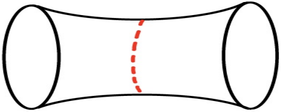
$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N^{d+2}}$$



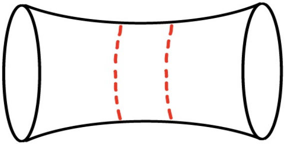
- In the context of eternal AdS-Schwarzchild BH the wormhole which connects the two sides grows linearly with time. [Hartman, Maldacena]



- Classically it grows forever.



- What is the CFT dual of this linear growth?
- Entanglement cannot be the answer since it stops growing at the scrambling time.

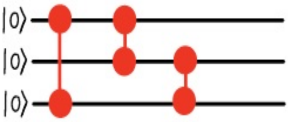


Complexity

Computational complexity is defined as the minimum number of gates required to produce $|\psi_T\rangle$ starting from $|\psi_R\rangle$.

$$|\psi_T\rangle = U\,|\psi_R\rangle$$

- It's a property of the reference state.
- Gates.- Unitaries that act only on two-qubits.
- Reference state is a unentangled state.
- Acting with a discrete set of gates would require some tolerance ϵ.

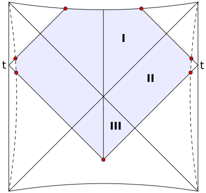


$$||\,|\psi_T\rangle - U\,|\psi_R\rangle\,||^2 \leq \epsilon$$

Complexity

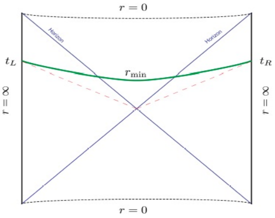
- Complexity grows linearly with time.
- It continues growing for late-times.

Complexity=Action [Brown et. al.]



$$C_A = \frac{I_{WdW}}{\pi}$$

Complexity=Volume [Susskind]



$$C_V = \max\left[\frac{\mathcal{V}}{G_N l}\right]$$

Lloyd's bound

Computers are physical systems: what they can and cannot do is dictated by the laws of physics. In particular, the speed with which a physical device can process information is limited by its energy.

"Seth Lloyd"

$$\frac{d\text{Complexity}}{dt} \leq \frac{2E}{\pi}$$

Complexity=Action

$$\lim_{t\to\infty} \frac{dC_A}{dt} = \frac{2M}{\pi}$$

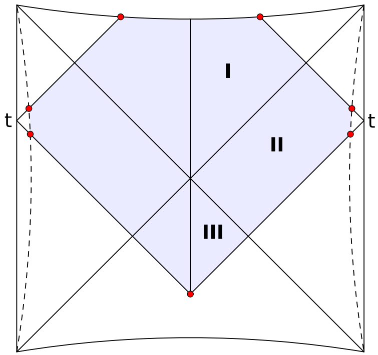
Complexity=Volume

$$\lim_{t\to\infty} \frac{dC_V}{dt} = \frac{8\pi M}{d-1}$$

Gravitational Action.

The Wheeler de Witt patch is the union of all spacial slices anchored between t_L and t_R.

- Null segments.
- Joints between null and spacelike segments.
- Joints between null and timelike segments.
- Joints between null and null segments.



Gravitational Action and boundary terms.

A well-defined variational principle for a finite domain of spacetime must also involve a contribution from the domain's boundary. [Lehner, Myers, Poisson, Sorkin]

$$I = I_{\text{bulk}} + \Sigma I_{\text{surface}} + \Sigma I_{\text{joint}} + \Sigma I_{\text{null-surface}} + \Sigma I_{\text{null-joint}} \tag{1}$$

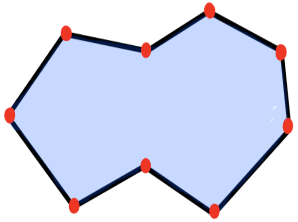
$$I_{\text{bulk}} = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^{d+1}x \sqrt{-g}\mathcal{L}$$

$$I_{\text{surface}} = \frac{1}{8\pi G_N} \int_{\mathcal{B}} d^d x \sqrt{|h|} K$$

$$I_{\text{joint}} = \frac{1}{8\pi G_N} \int_{\Sigma} d^{d-1}x \sqrt{\sigma}\eta$$

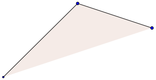
$$I_{\text{null-surface}} = \frac{1}{8\pi G_N} \int_{\mathcal{N}} d\lambda d\theta^{d-1} \sqrt{\gamma}\kappa$$

$$I_{\text{null-joint}} = \frac{1}{8\pi G_N} \int_{\Sigma'} d^{d-1}x \sqrt{\sigma} a$$

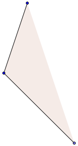


Additivity rules for null joints

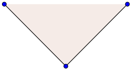
- Spacelike rule. $a = \log\left(-t \cdot k\right)$



- Timelike rule. $a = \log|s \cdot k|$



- Null rule. $a = \log\left(-\frac{1}{2} k \cdot \overline{k}\right)$



The Mateos and Trancanelli (MT) model is a solution of type IIB supergravity whose effective action in five dimensions can be written as

$$S = \frac{1}{16\pi G_N} \int_{\mathcal{M}} d^5x \sqrt{-g} \left[R + \frac{12}{L^2} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{2} e^{2\phi} (\partial\chi)^2 \right] + \ldots \tag{2}$$

In the Einstein frame the ten-dimensional geometry factorizes as $\mathcal{M} \times S^5$, where $\mathcal{M}$ satisfies the following properties:

- It is static and anisotropic (along the gauge theory directions).
- It possesses a horizon and it is regular on and outside the horizon.
- It approaches AdS_5 asymptotically.

Anisotropic Black Branes

- $\mathcal{N} = 4$ SYM + θ-parameter that depends linearly in the z-direction.
- Plasma created in a heavy ion collision is anisotropic.
- The D7-branes are wrapped on the S^5 factor, extent along the x and y directions and are homogeneously distributed along the z-direction with uniform density.

		t	x	y	z	u	S^5
N_c	D3	X	X	X	X		
n_{D7}	D7	X	X	X			X

Anisotropic Black Branes

The solution in Einstein frame takes the following form

$$ds^2 = L^2\, e^{-\phi(r)/2} \left[-r^2 \mathcal{F}(r)\, \mathcal{B}(r)\, dt^2 + \frac{dr^2}{r^2 \mathcal{F}(r)} + r^2 \left(dx^2 + dy^2 + \mathcal{H}(r)\, dz^2 \right) \right] \quad (3)$$

where

$$\chi = az, \qquad \phi = \phi(r), \qquad \mathcal{H} = e^{-\phi} \quad (4)$$

The above solution has a horizon at $r = r_H$ and the boundary is located at $r \to \infty$, where $\mathcal{F} = \mathcal{B} = \mathcal{H} = 1$ and $\phi = 0$. For small values of the anisotropy parameter a, the solution takes the following form

$$\begin{aligned}
\mathcal{F} &= 1 - \frac{r_H^4}{r^4} + \frac{a^2}{24 r^4 r_H^2} \left[8r^2 r_H^2 - 2r_H^2 (4 + 5\log 2) + (3r^4 + 7r_H^4) \log\left(1 + \frac{r_H^2}{r^2}\right) \right] \\
\mathcal{B} &= 1 - \frac{a^2}{24 r_H^2} \left[\frac{10 r_H^2}{r^2 + r_H^2} + \log\left(1 + \frac{r_H^2}{r^2}\right) \right] \\
\phi &= -\frac{a^2}{4 r_H^2} \log\left(1 + \frac{r_H^2}{r^2}\right)
\end{aligned}$$

Anisotropic Black Branes

- Temperature $T = \frac{r_H}{\pi} + \frac{(5\log 2 - 2)}{48\pi}\frac{a^2}{r_H^2}$

- Entropy $S = \frac{r_H^3}{4G_N}\left(1 + \frac{5a^2}{16r_H^2}\right)V_3$

- Mass $M = \left(\frac{3\pi^2 N^2 T^4}{8} + \frac{N^2T^2}{32}a^2\right)V_3$

- Pressure $P_x = P_y = \frac{\pi^2 N^2 T^4}{8} + \frac{N^2T^2}{32}a^2$ and $P_z = \frac{\pi^2 N^2 T^4}{8} - \frac{N^2T^2}{32}a^2$

Anisotropic Black Branes

With the aim to avoid unnecessary notation we adopt the following form for the metric.

$$ds^2 = -G_{tt}(r)dt^2 + G_{rr}(r)dr^2 + G_{ij}(r)dx^i dx^j \tag{5}$$

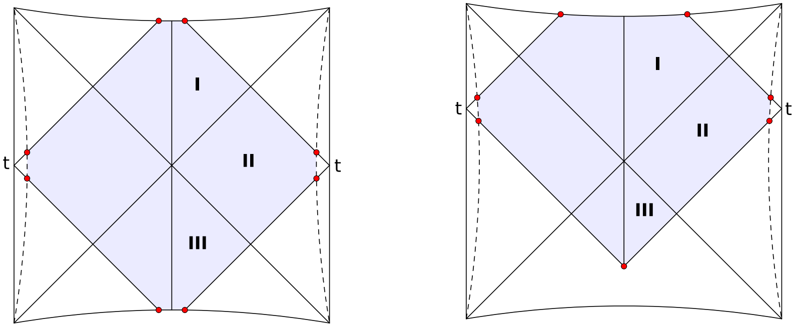
$$i, j = 1, 2, ..., d-1$$

where

$$G_{xx}(r) = G_{yy}(r) \quad \text{and} \quad G_{xx}(r) \neq G_{zz}(r)$$

Then, we can define the tortoise coordinate as

$$r^*(r) = \text{sgn}\left(G_{tt}(r)\right) \int dr \sqrt{\frac{G_{rr}}{G_{tt}}} \tag{6}$$



$0 \leq t \leq t_c$ $\qquad\qquad$ $t_c < t$

$$t_c = 2\left(r^*_\infty - r^*(0)\right) \qquad \text{where} \qquad r^*_\infty = \lim_{r\to\infty} r^*(r) \tag{7}$$

- Bulk contributions $I_{\text{bulk}}(t \leq t_c) = 2\left(I^{I}_{\text{bulk}} + I^{II}_{\text{bulk}} + I^{III}_{\text{bulk}}\right)$

$$I_{\text{bulk}}(t \leq t_c) = \frac{1}{4\pi G_N}\int_{\epsilon_0}^{r_{\max}} dr\sqrt{G_{tt}G_{rr}G}\mathcal{L}(r)\Big(r^*_\infty - r^*(r)\Big)\,. \tag{8}$$

- Surface contributions $I_{\text{surface}}(t \leq t_c) = I^{\text{future}}_{\text{surface}} + I^{\text{past}}_{\text{surface}} + I^{\text{bdry}}_{\text{surface}}$

$$\begin{aligned} I_{\text{surface}}(t \leq t_c) = {} & \frac{V_{d-1}}{4\pi G_N}\sqrt{\frac{G_{tt}G}{G_{rr}}}\left[\frac{\partial_r G_{tt}}{G_{tt}} + \frac{\partial_r G}{G}\right]\left[r^*_\infty - r^*(r)\right]\Big|_{r=r_\epsilon} + \\ & \frac{V_{d-1}}{8\pi G_N}\sqrt{\frac{G_{tt}G}{G_{rr}}}\left[\frac{\partial_r G_{tt}}{G_{tt}} + \frac{\partial_r G}{G}\right]\left[r^*_\infty - r^*(r)\right]\Big|_{r=r_{\max}} \end{aligned} \tag{9}$$

- Joint contributions $I_{\text{joint}} = I^{\text{sing}}_{\text{joint}} + I^{\text{bdry}}_{\text{joint}}$

Case $0 \leq t \leq t_c$ (divide and conquer)

- Summing all the contributions and then taking the derivative we obtain

$$\frac{dI_{\mathrm{WDW}}}{dt} = 0 \qquad \text{for} \qquad 0 \leq t \leq t_c\,. \tag{10}$$

Case $t_c < t$ (divide and conquer)

- Bulk contributions $I_{\rm bulk}(t_c < t) = 2\left(I^{I}_{\rm bulk} + I^{II}_{\rm bulk} + I^{III}_{\rm bulk}\right)$

$$I_{\rm bulk}(t_c < t) = I_{\rm bulk}(t \le t_c) + \frac{V_{d-1}}{8\pi G_N}\int_{\epsilon_0}^{r_m} dr\sqrt{G_{tt}G_{rr}G}\mathcal{L}(r)\left[\frac{t}{2} - r^*_\infty + r^*(r)\right] \quad (11)$$

- Surface contributions $I_{\rm surface}(t_c < t) = I^{\textsf{future}}_{\textsf{surface}} + I^{\textsf{past}}_{\textsf{surface}} + I^{\textsf{bdry}}_{\textsf{surface}}$

$$I_{\rm surface}(t_c < t) = I_{\rm surface}(t \le t_c) + \frac{V_{d-1}}{8\pi G_N}\sqrt{\frac{G_{tt}G}{G_{rr}}}\left[\frac{\partial_r G_{tt}}{G_{tt}} + \frac{\partial_r G}{G}\right]\left[\frac{t}{2} + r^*_\infty - r^*(r)\right]\Bigg|_{r=\epsilon_0} \quad (12)$$

- Joint contributions $I_{\textsf{joint}}(t_c < t) = I^{\textsf{null}}_{\textsf{joint}} + I^{\textsf{bdry}}_{\textsf{joint}}$

$$I_{\textsf{joint}}(t_c < t) = I_{\textsf{joint}}(t \le t_c) - \frac{V_{d-1}}{8\pi G_{\textsf{N}}}\sqrt{G(r_m)}\log\left|\frac{G_{tt}(r_m)}{\alpha^2}\right| \quad (13)$$

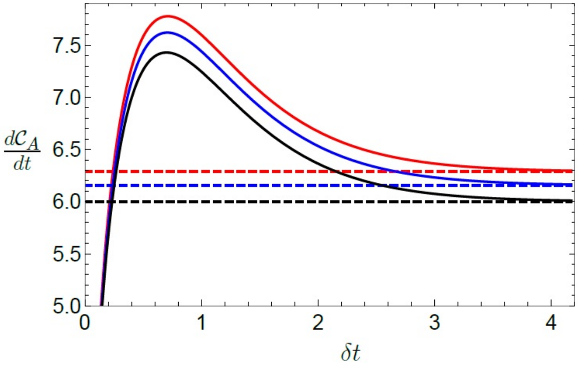
Case $t_c \leq t$ (divide and conquer)

- Summing all the contributions and then taking the derivative we obtain

$$\frac{dC_A}{dt} = \frac{V_{d-1}}{16\pi G_N}\left[\int_{\epsilon_0}^{r_m} dr\sqrt{G_{tt}G_{rr}G}\mathcal{L}(r) + \sqrt{\frac{G_{tt}G}{G_{rr}}}\left(\frac{\partial_r G_{tt}}{G_{tt}} + \frac{\partial_r G}{G}\right)\Big|_{r=\epsilon_0} \right. \\ \left. + \frac{1}{2}\sqrt{\frac{G_{tt}}{G_{rr}G}}\frac{dG}{dr_m}\log\left|\frac{G_{tt}(r_m)}{\alpha^2}\right| + \sqrt{\frac{G}{G_{rr}G_{tt}}}\frac{dG_{tt}}{dr_m}\right] \tag{14}$$

where r_m is the lower joint of the WdW patch.

Full time behavior



anisotropy parameter	r_h such that $T = 1/\pi$	black brane's mass
$a = 0.00$	$r_h = 1.000000$	$M = 3.00000$
$a = 0.45$	$r_h = 0.993778$	$M = 3.04931$
$a = 0.60$	$r_h = 0.988883$	$M = 3.08582$

- The late time behavior is

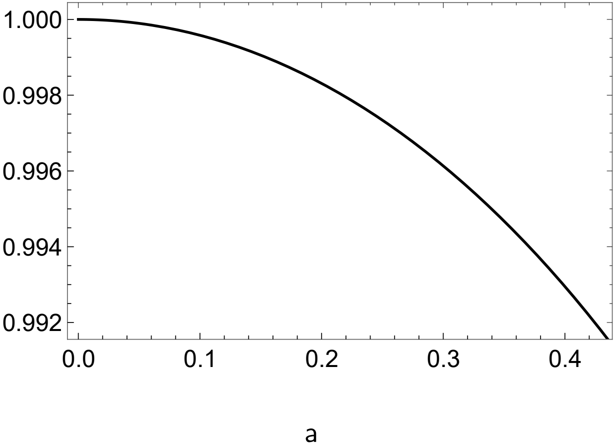
$$\frac{dI_{\mathrm{WDW}}}{dt} = \frac{V_3}{16\pi G_N}\left[6r_h^4 + \frac{r_h^2 a^2}{2}\left(5\log 2 - 1\right)\right] = 2M(a) \qquad (15)$$

Conclusions

- The late time rate of change of complexity matches the Lloyd's bound. This is a highly non-trivial match, because it means that the anisotropy increases the value of $2M$ and the late time value of $\frac{dI_{W\,dW}}{dt}$ precisely in the same amount.
- The full-time behavior shares a lot of similarities with the previous results obtained for isotropic systems.
- The effect of the anisotropy is to reduce t_c.
- Perhaps we need to test carefully the validity of the assumptions entering in the derivation of Lloyd's bound in a holographic setup.

Thanks for your attention!

Critical time plot $\frac{t_c(a)}{t_c(0)}$



$\frac{r_m}{r_H}$ plot