

Large-N properties of Inflation and the Inflaton Field Range

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 groningen

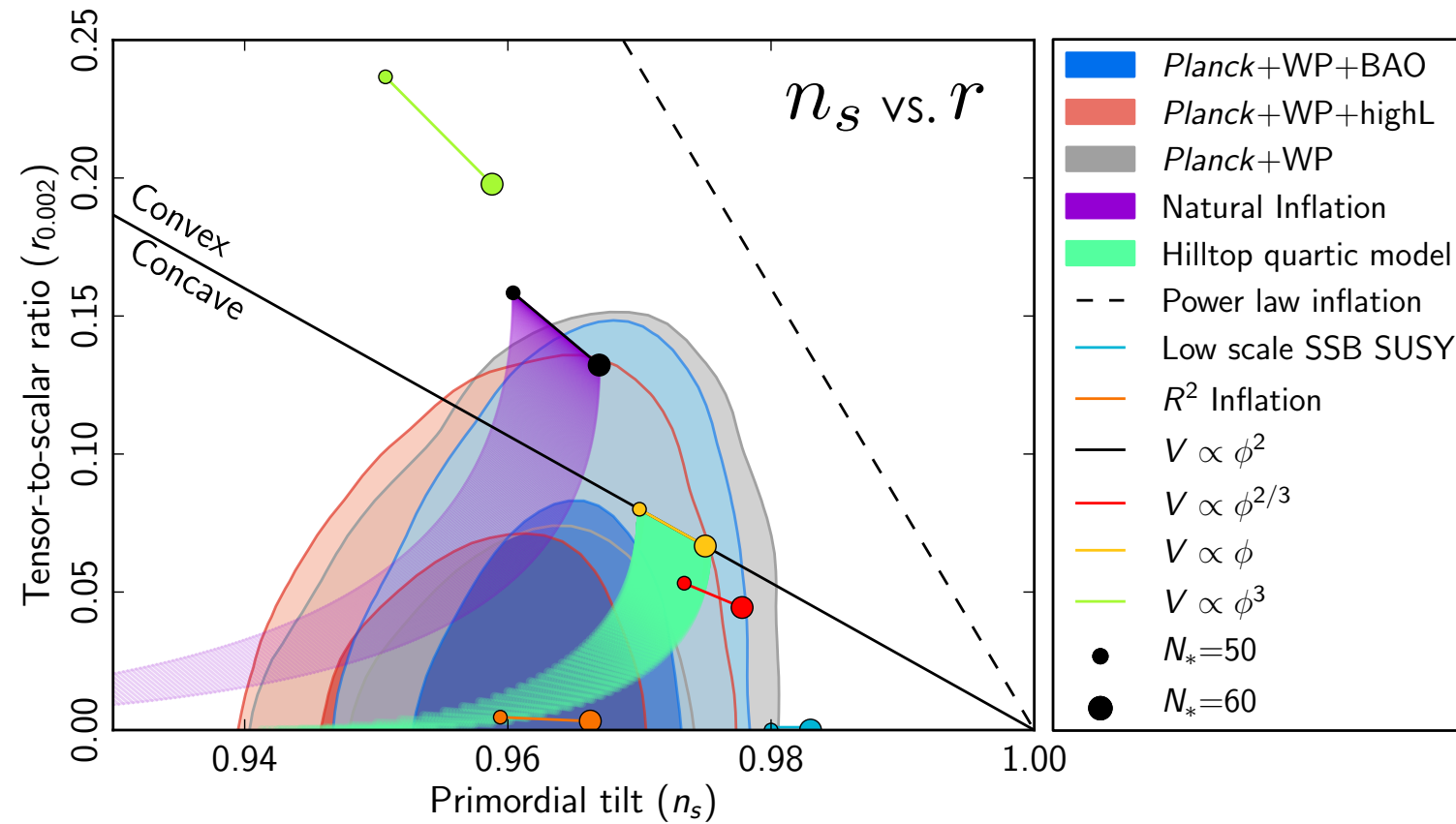
1405.7399 + in progress
w/García-Bellido, Scalisi, Roest

Contents:

- * CMB observations and Inflation
- * Large-N Inflation
 - Hubble flow parameters
 - Universality classes (in (n_s, r) plane)
 - (non-)Universality properties in field range $\Delta\phi$
 - Field range degeneracies
 - On the Lyth bound and universality
- * Summary

Planck 2013

$$n_s = 0.9603 \pm 0.0073, \quad r < 0.11$$

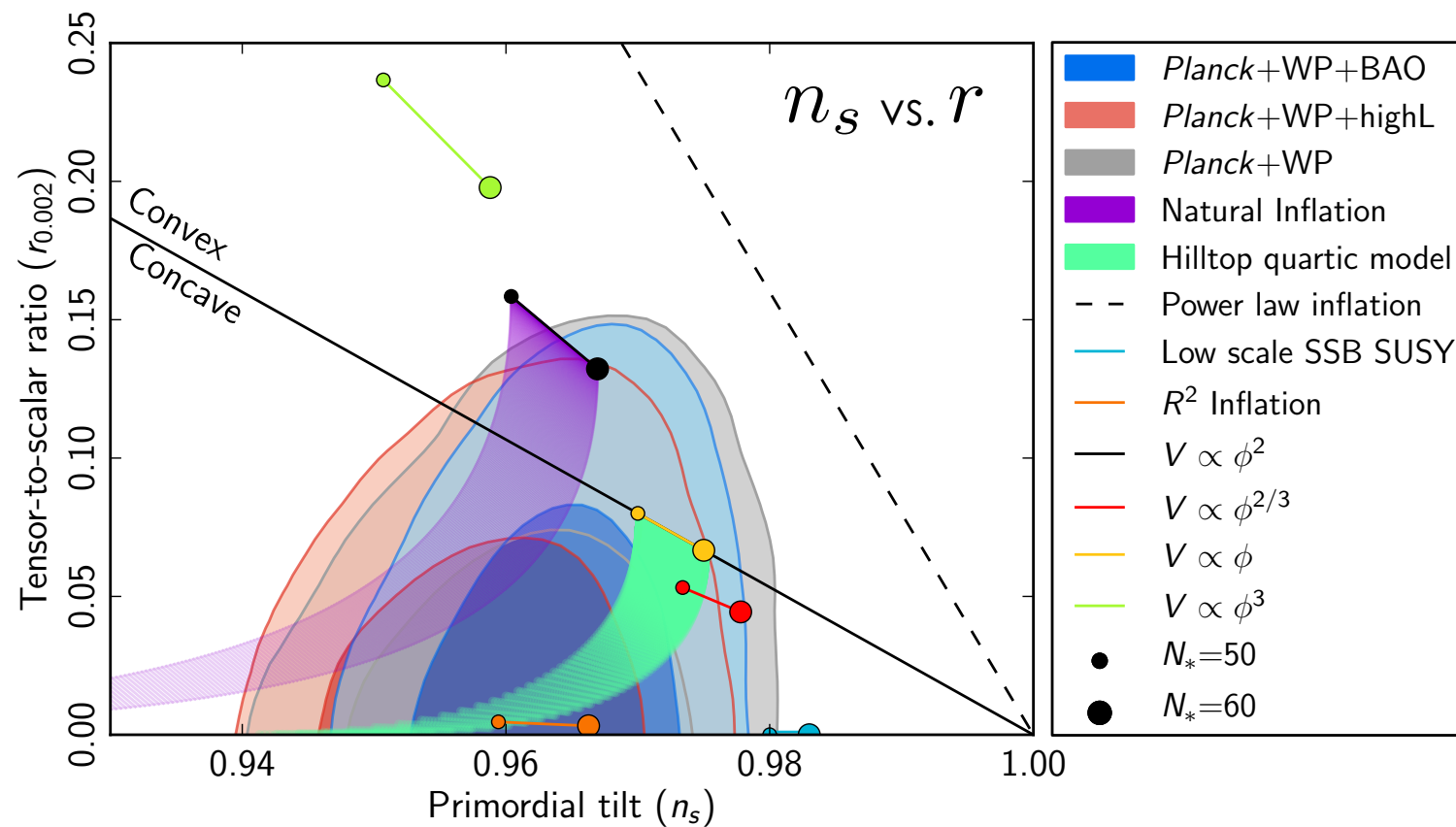


n_s = spectral index

r = tensor 2 scalar ratio

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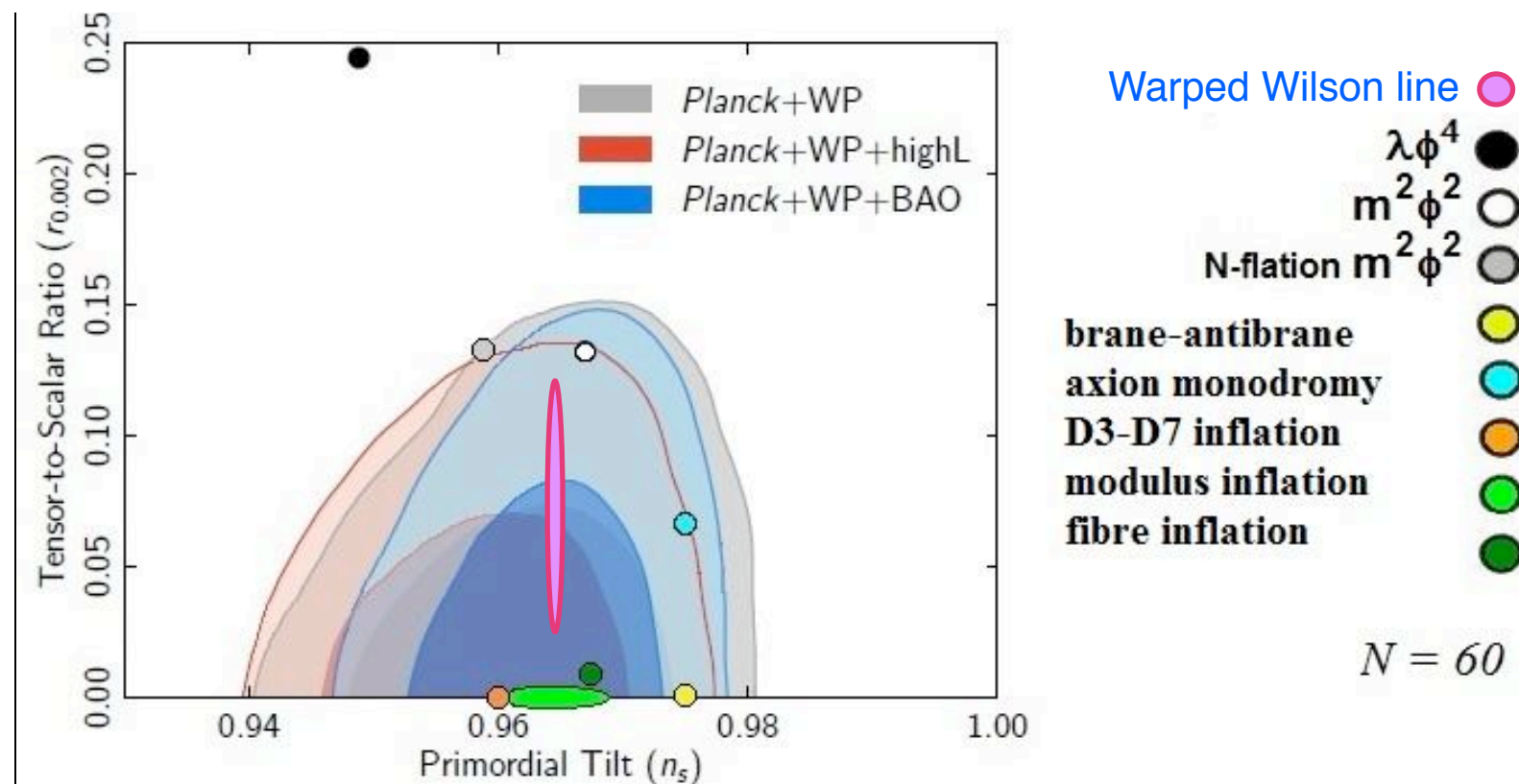
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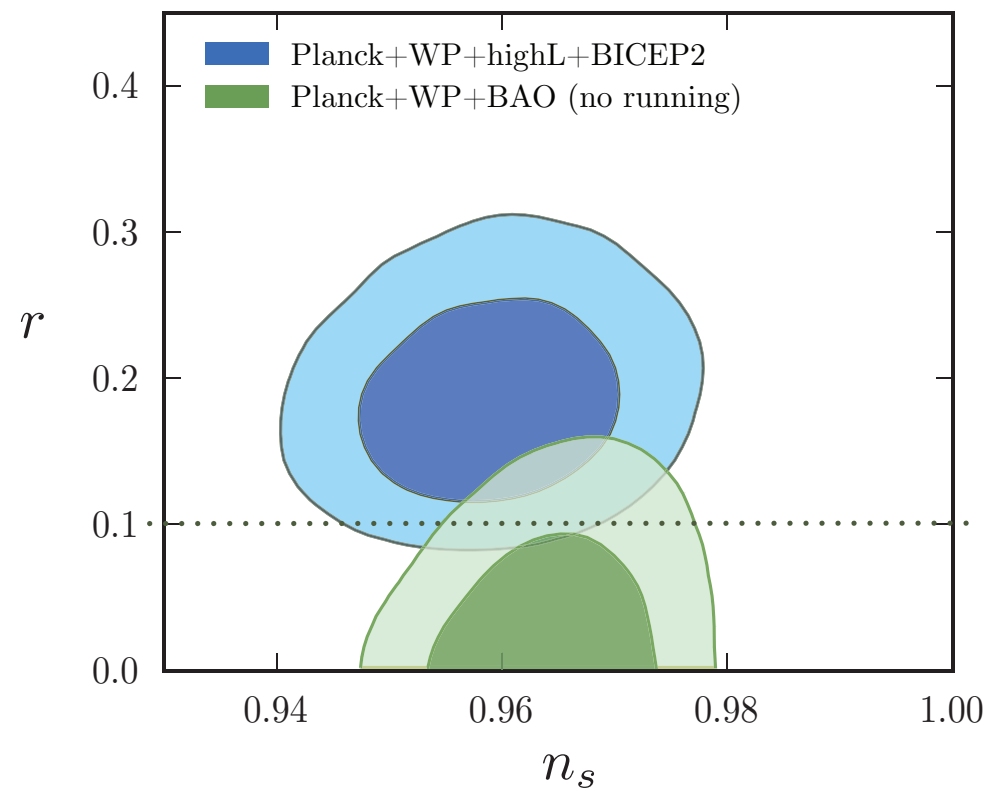
Field Theory Inflation models



String Theory Inflation Models

BICEP 2014

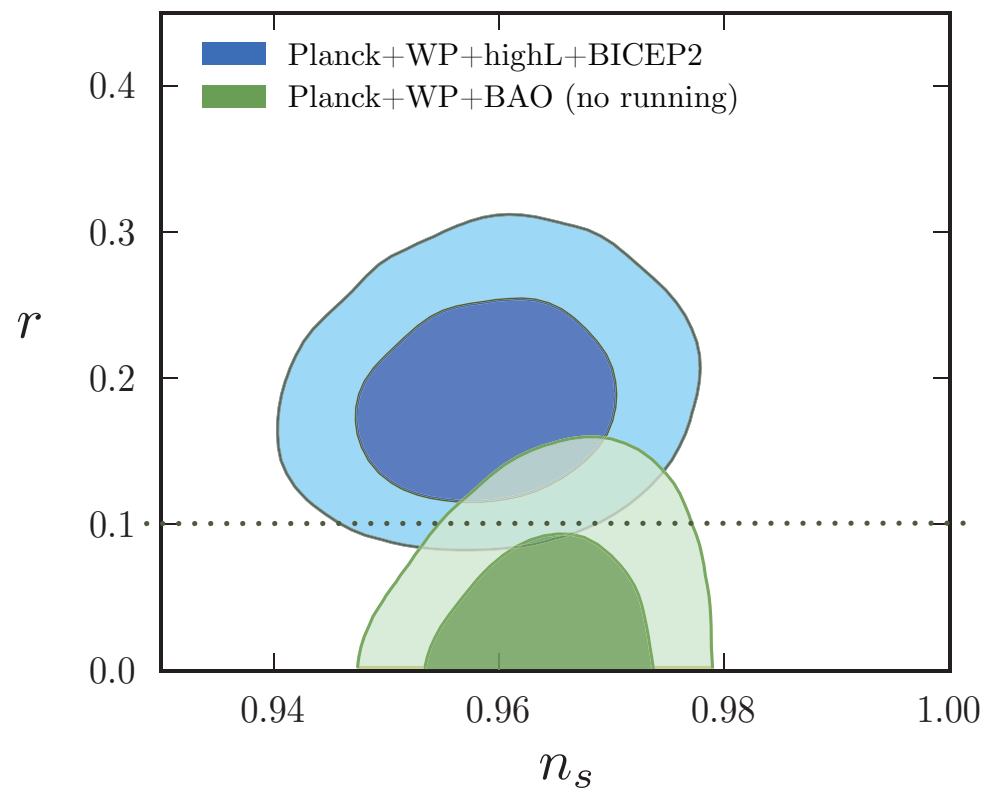
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BICEP 2014

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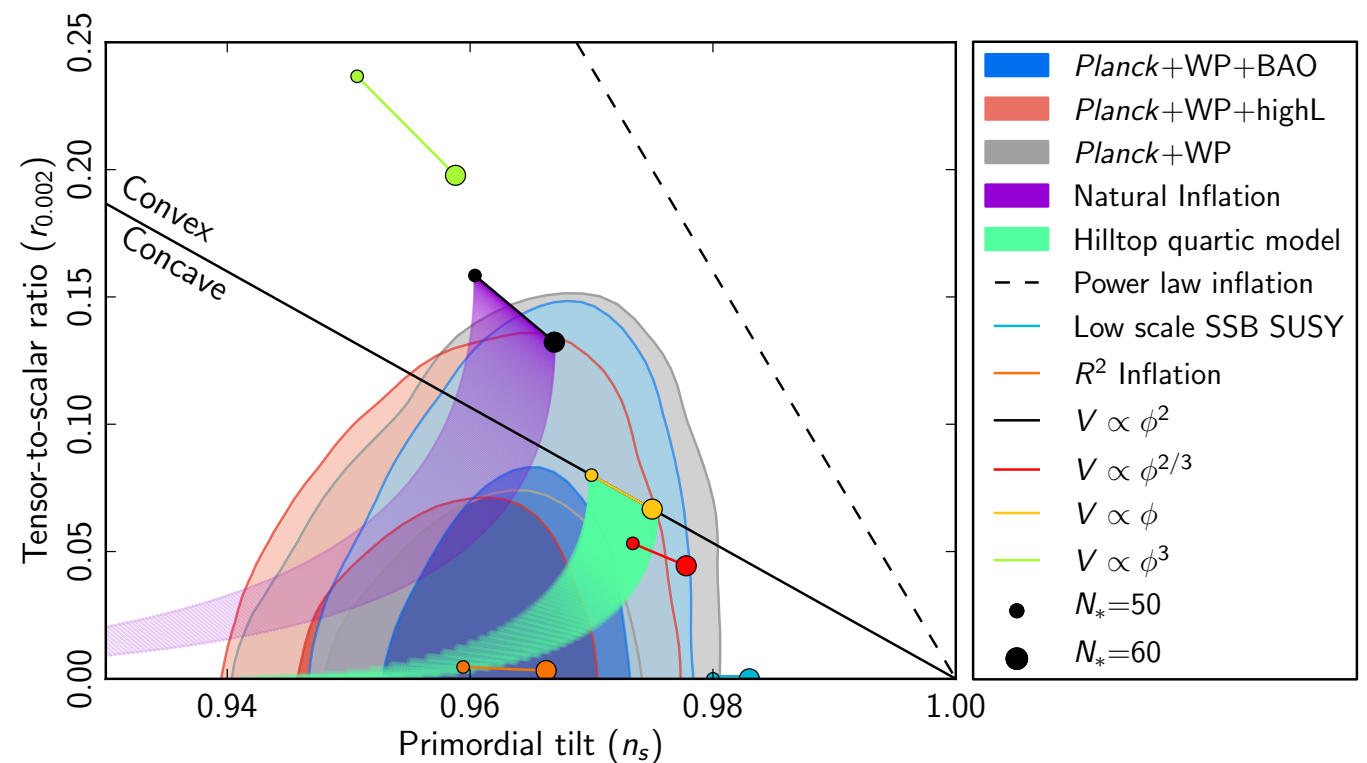
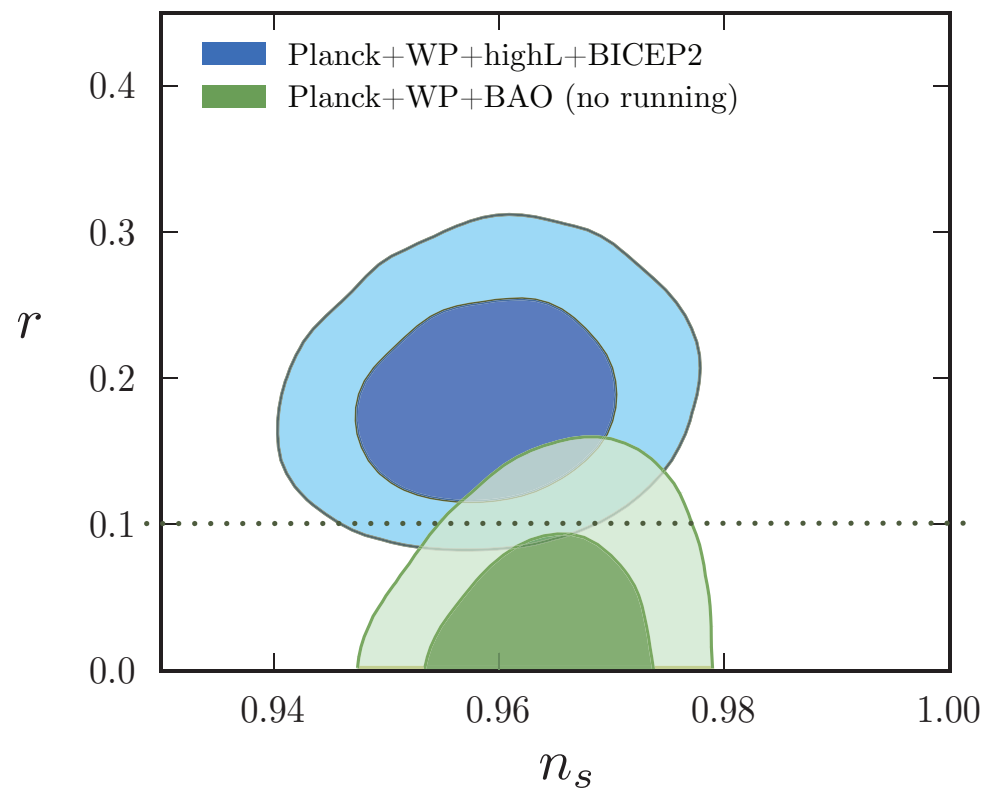


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- Inflation occurred
- *(super)-Planckian field excursion* $\Delta\phi \gtrsim M_{Pl}$
- Scale of inflation $\sim 10^{-2} M_{Pl}$
- Several string models would be in tension with data

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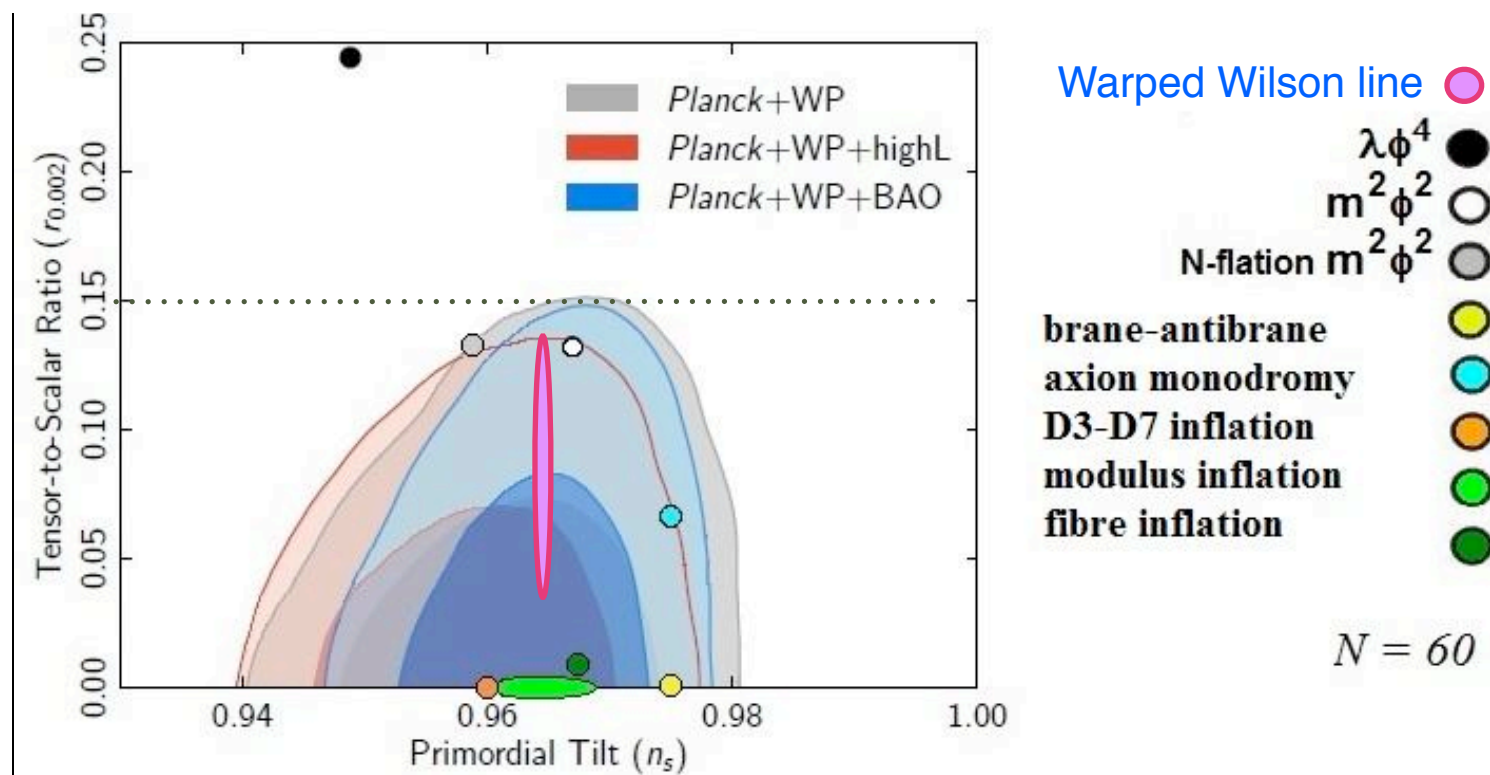
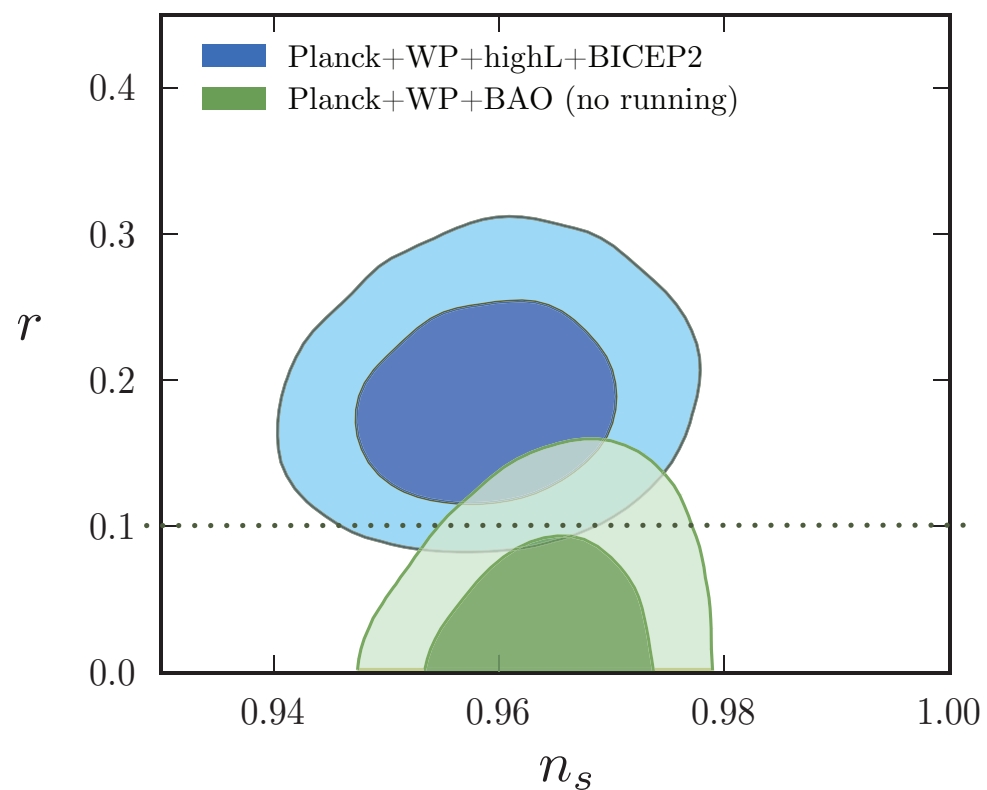


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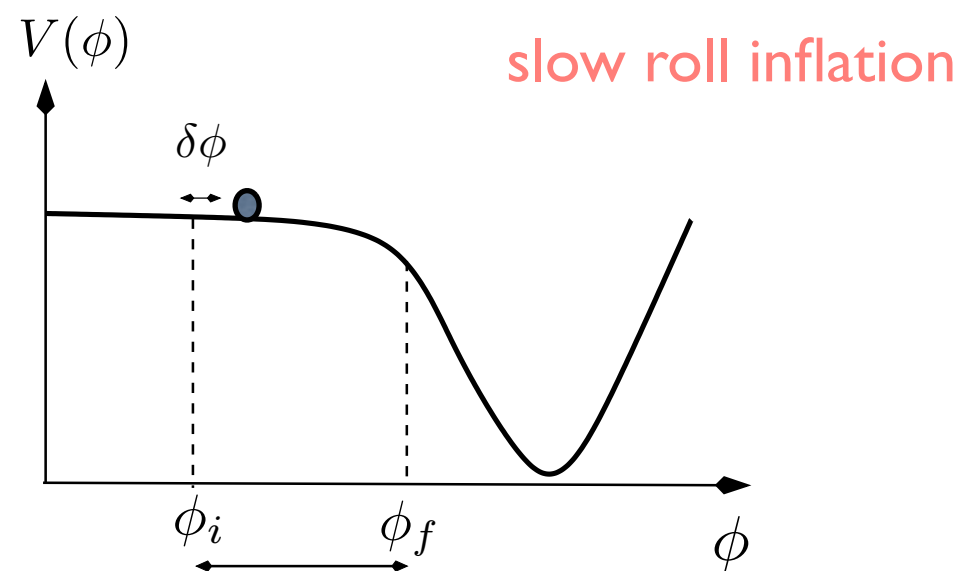


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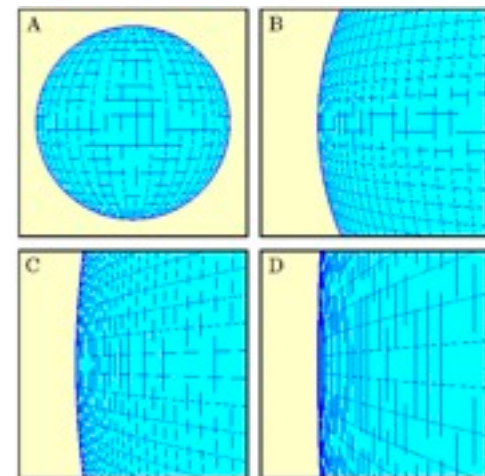
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Cosmological Inflation

An early period of quasi deSitter expansion, driven by a (single) scalar field, whose quantum fluctuations gave rise to the LSS we observe today: *inflation*

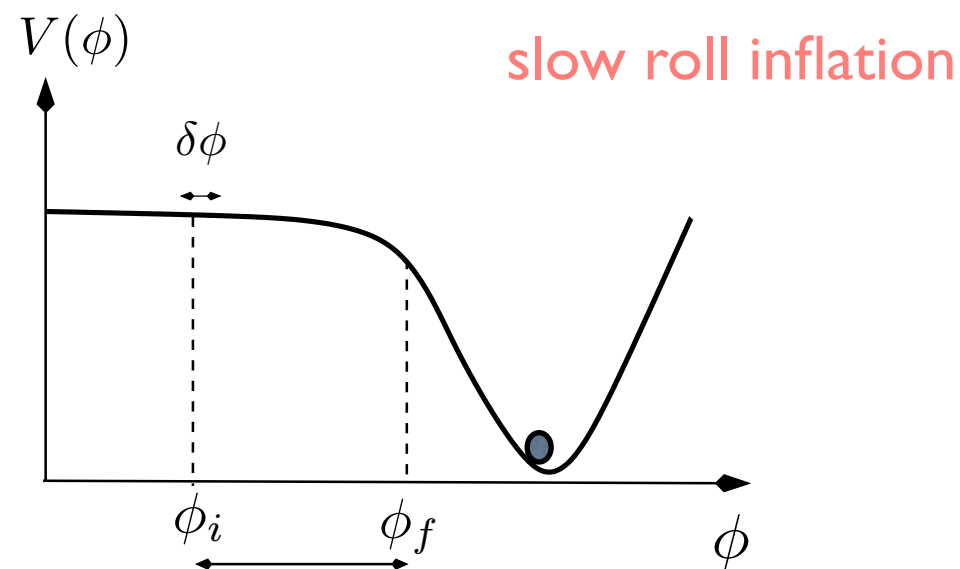


It also explains why the universe is approximately homogeneous and spatially flat (flatness, horizon problems). Explains absence of monopoles and other topological defects

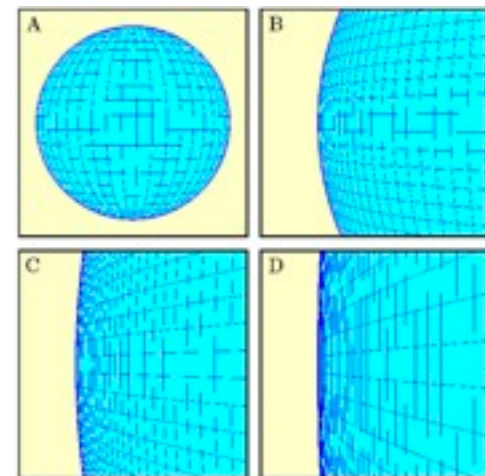


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Inflation in its simplest version (*single field slow roll*) gives generic predictions on the properties of the density perturbations:

- They are *approximately scale invariant*, $n_s \neq 1$
- Primordial *gravitational waves*, r
- They are *approximately Gaussian*
- Statistically *isotropic* and *homogeneous* to a high degree.

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These cosmological observables translate into properties of the inflationary model and thus we can use CMB data to classify models of inflation according to their generic predictions for these parameters

Hubble flow parameters

Cosmological evolution encoded in Hubble parameter H , which determines the inflationary observables (n_s , r)

Can be elegantly described by Hubble flow equations, with

$$\epsilon_0 = H \left(= \frac{\dot{a}}{a} \right) \quad \epsilon_{n+1} = \frac{d \log \epsilon_n}{dN}$$

[Schwarz, Terrero-Escalante, García '01]
[Schwarz, Terrero-Escalante '04]

and their evolution as a function of *number of e-folds*:

($N \sim 0$ @ end of inflation)

$$N = N_\star - N_e \equiv \ln \frac{a_e}{a_\star} = \int_{t_\star}^{t_e} H dt$$

$$\left(\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1 + \omega) \right)$$

In terms of these, the observable parameters become:

$$n_s - 1 = \epsilon_2 - 2\epsilon_1, \quad r = 16\epsilon_1$$

and are evaluated at *horizon crossing* $N_\star \sim 50 - 60$

Inflation at large- N

Solution of flatness/horizon problems as well as reheating set a lower bound on $N \gtrsim 50$

However there is no compelling reason for an upper bound on N .

It seems natural that inflation extends a long way further into the past than the portion that we can observe.

Together with current observational values on n_s and r , this suggest $1/N$ as a natural small parameter to expand the cosmological variables.

Inflation at large-N

A large-N expansion is a powerful tool to organize different inflationary models in terms of their cosmological predictions.

It is possible to identify a number of classes where physically different inflationary scenarios would predict the same values of n_s and r in the leading $1/N$ approximation

[Kallosh-Linde '13; Mukhanov '13]

[Roest '13; García-Bellido, Roest '14]

N-formalism of Inflation

This formulation does not depend on the mechanism that drives the inflationary period, nor on the details of the underlying microscopic model

However, strictly within the *slow-roll approximation*, the Hubble flow functions are equivalent to the *slow-roll parameters* in terms of the potential V for a scalar field ϕ :

$$\epsilon_0 = V^{-1/2}, \quad \epsilon_1 = \epsilon, \quad \epsilon_2 = -4\epsilon + 2\eta$$

where

$$\epsilon = \frac{1}{2} \left(\frac{V_\phi}{V} \right)^2, \quad \eta = \frac{V_{\phi\phi}}{V}$$

The link between these two equivalent formulations is provided by the relation

$$\frac{d\phi}{dN} = \sqrt{2\epsilon_1}$$

Universality classes

- Perturbative class: $\epsilon_1 = \frac{\beta}{N^p}, \quad \epsilon_2 = -\frac{p}{N} \quad \epsilon_0 = \exp \left[\beta \int N^{-p} dN \right]$
($p \geq 0$)

e.g. exponential inflation, $p < 1$,
monomial chaotic inflation, $p = 1$,
inverse hilltop inflation, $1 < p < 2$,
Starobinsky-like, $p = 2$,
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- Logarithmic class: $\epsilon_1 = \frac{\beta}{N^p \ln^q N}, \quad \epsilon_2 = -\frac{p}{N} - \frac{q}{N \ln N} \quad (q \geq 0)$

e.g. Kähler inflation ($p = 2, q = 1/2$)

$$\epsilon_0 = \exp \left[\beta \int N^{-p} \ln^{-q} N \right]$$

- Non-perturbative class: $\epsilon_1 = e^{-2cN}, \quad \epsilon_2 = -2c \quad \epsilon_0 = \exp \left[-\frac{e^{-2cN}}{2c} \right]$

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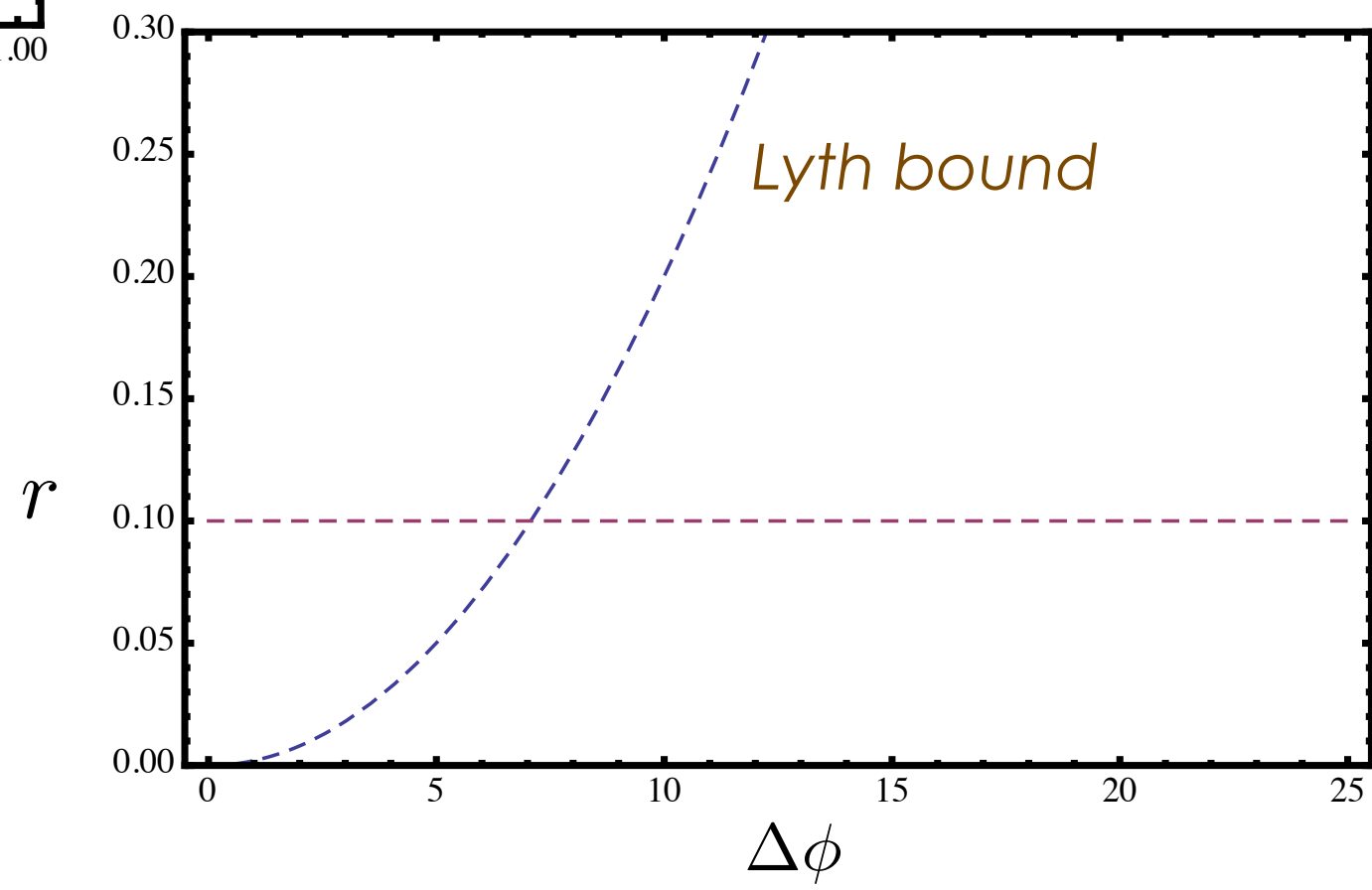
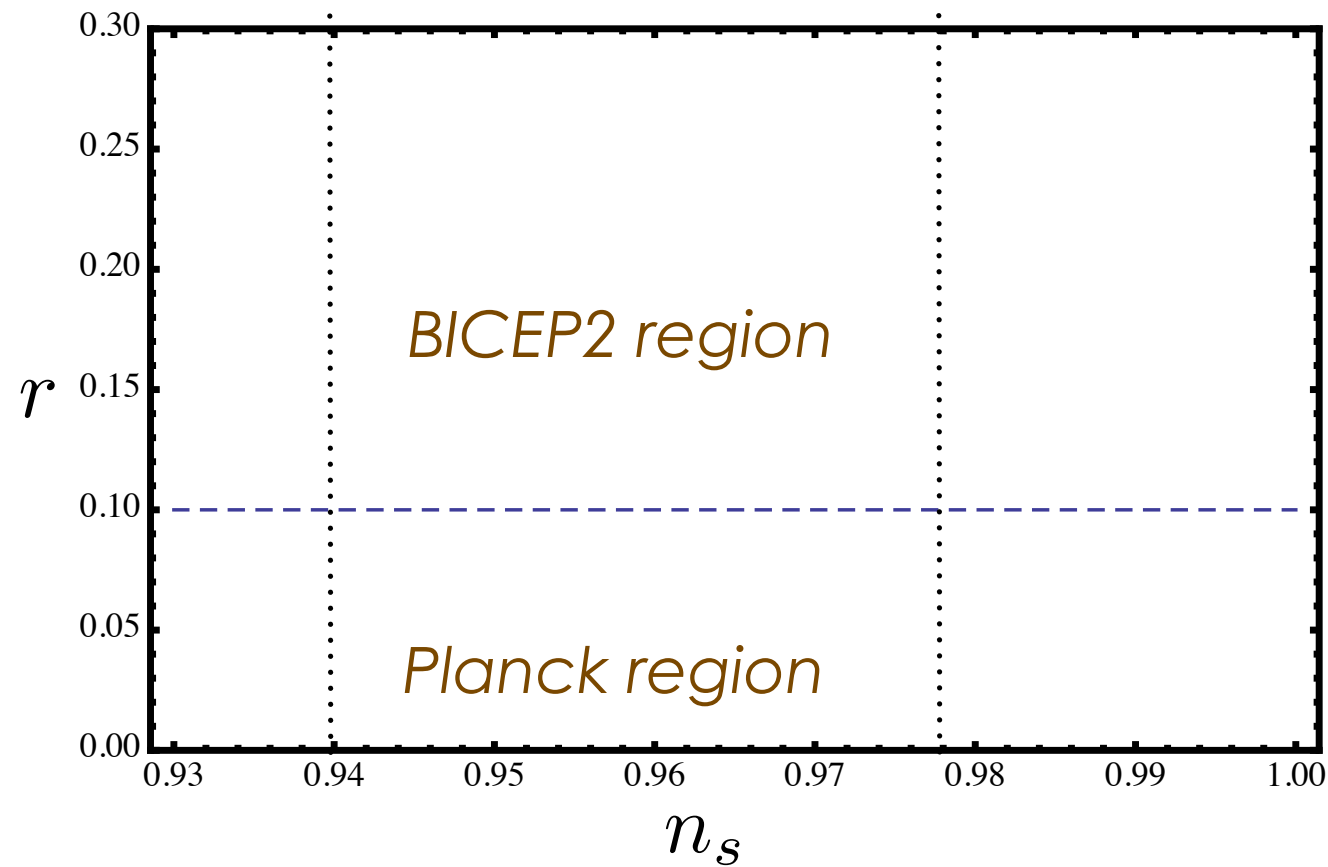
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e.g. natural inflation $V = V_0 [1 - \cos(\phi/\phi_0)]$

Universality properties & degeneracies of the Inflaton field range $\Delta\phi$

[García-Bellido, Scalisi, Roest, IZ '14]



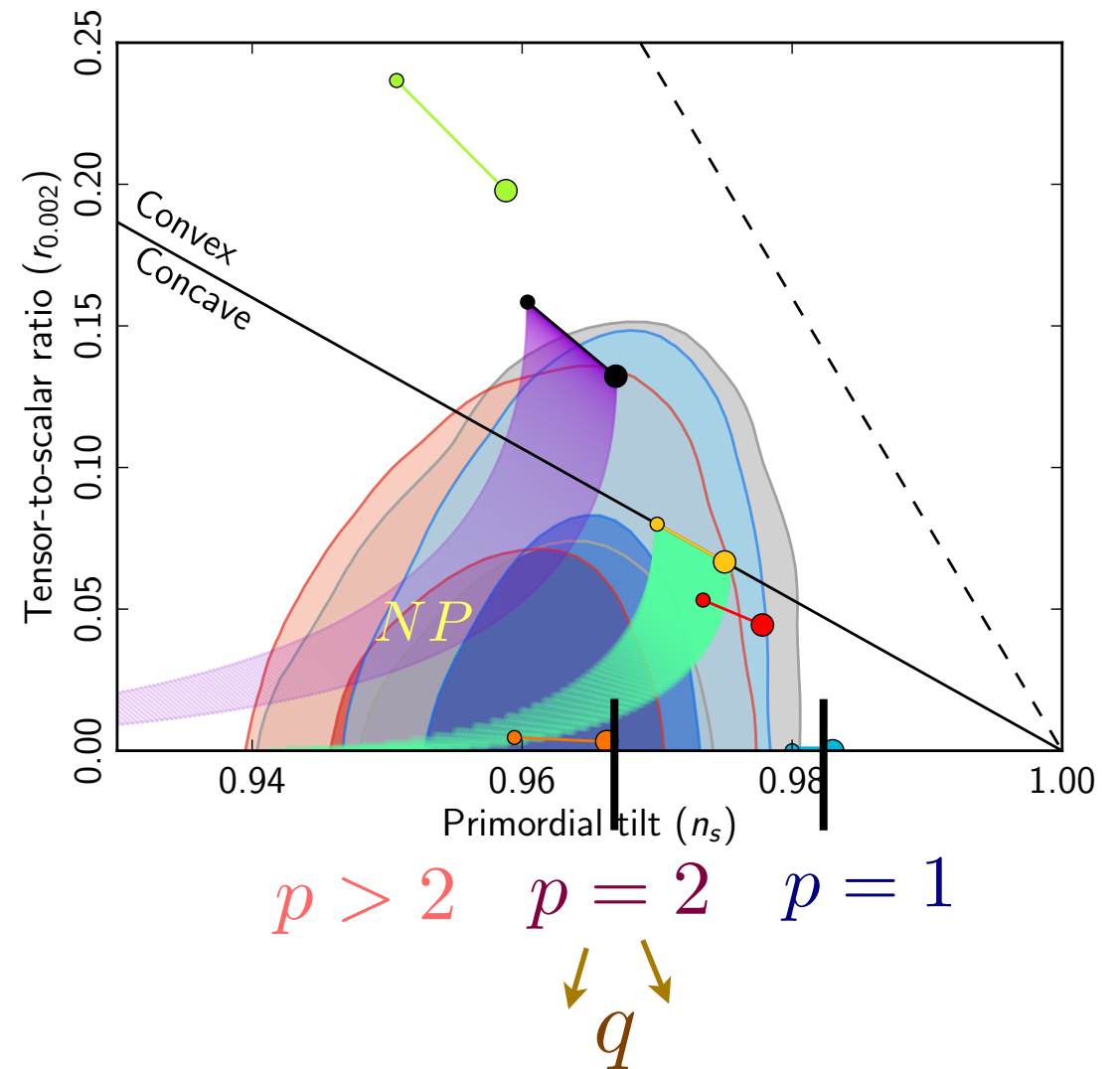
Universality properties in (n_s, r) plane

Universality classes of inflationary models can be identified in the (n_s, r) plane given their predictions. They divide the plane in various regions:

[Roest '13; García-Bellido, Roest '13]

[Roest '13; García-Bellido, Roest '14]

$$\epsilon_1 = \begin{cases} \frac{\beta}{N^p} \\ \frac{\beta}{N^p \ln^q N} \\ e^{-2cN} \end{cases}$$



Universality and the field range

The number of e-folds is related to the scalar field excursion via

$$\frac{d\phi}{dN} = \sqrt{2\epsilon_1}$$

Can integrate it along the *entire inflationary trajectory* to find the total field range $\Delta\phi$:

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- Perturbative class:

- ▶ $0 \leq p < 2$, $\Delta\phi = \frac{2\sqrt{2\beta_0}}{(2-p)} N^{1-p/2} - \phi_e$ universal
(large field)
- ▶ $p = 2$, $\Delta\phi = \sqrt{2\beta_0} \log N - \phi_e$ universal
(‘medium’ field)
- ▶ $p > 2$, $\Delta\phi = \phi_e + \frac{2\sqrt{2\beta_0}}{(2-p)} N^{-p/2+1}$ non-universal
(small field)

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- ▶ $0 \leq p < 2$, $\Delta\phi = \frac{2\sqrt{2\beta_0}}{(2-p)} (\ln N)^{-q/2} N^{1-p/2} - \phi_e$ universal (large field)

- ▶ $p = 2$, $\rightarrow q$

- ▶ $p > 2$, $\Delta\phi = \phi_e + \frac{2\sqrt{2\beta_0}}{(2-p)} (\ln N)^{-q/2} N^{-p/2+1}$ non-universal (small field)

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$$\blacktriangleright p = 2, \quad \left\{ \begin{array}{lll} q < 2, & \Delta\phi = \frac{2\sqrt{2\beta_0}}{2-q} (\ln N)^{1-q/2} - \phi_e & \begin{array}{l} \text{universal} \\ \text{Kähler inflation} \\ (q = 1/2) \end{array} \\ q = 2, & \Delta\phi = \frac{\sqrt{2\beta_0}}{2-q} \ln N (\ln N) - \phi_e & \text{universal} \\ q > 2, & \Delta\phi = \phi_e + \frac{2\sqrt{2\beta_0}}{2-q} (\ln N)^{-q/2+1} & \begin{array}{l} \text{non-} \\ \text{universal} \end{array} \end{array} \right.$$

Field range and tensor to scalar ratio

We can *unify* all cases by writing the field range in terms of r :

$$\frac{d\phi}{dN} = \sqrt{2\epsilon_1} = \frac{\sqrt{2\beta_0}}{N^{p/2}(\log N)^{q/2}}, \quad r = 16\epsilon_1$$

$$0 \leq p < 2, \quad \Delta\phi \sim \frac{2}{(2-p)} \left(\frac{r}{8}\right)^{1/2} N \quad \forall \quad q \quad \text{chaotic, hilltop}$$

$$p = 2 \quad \left\{ \begin{array}{ll} q < 2 : & \Delta\phi \sim \frac{2}{(2-q)} \left(\frac{r}{8}\right)^{1/2} N \ln N \\ q = 2 : & \Delta\phi \sim \left(\frac{r}{8}\right)^{1/2} N \ln N \ln(\ln N) \\ q > 2 : & \Delta\phi \sim \phi_e \end{array} \right. \quad \begin{array}{l} \text{Kähler inflation} \\ \text{Starobinsky-like} \end{array}$$

$$p > 2, \quad \Delta\phi \sim \phi_e \quad \forall \quad q \quad \text{hilltop}$$

Field range and tensor to scalar ratio

For $p > 2$ there is no universality in general. Example: *hilltop*

$$V = V_0 \left[1 - \left(\frac{\phi}{\phi_0} \right)^n \right] \quad (n > 2)$$

$$p = \frac{2(n-1)}{(n-2)} > 2 \quad (\phi_0 \ll 1)$$

$$\Delta\phi \sim \phi_e = \left(\sqrt{\frac{2}{8^{n-2}}} \right)^{1/(n-1)} \left(\frac{n-2}{n} \right)^{1/2} N r^{(n-2)/2(n-1)}$$

$$n = 3, \quad \Delta\phi \propto r^{1/4}$$

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Universality at Large-N and the Lyth bound

Original Lyth bound was derived assuming monotonically increasing ϵ_1 . Using

[Lyth '96]

$$d\phi = \sqrt{\frac{r}{8}} dN$$

$$(M_p^{-2} = 8\pi G = 1)$$

one can derive the bound:

$$\Delta\phi \geq \left(\frac{r}{0.002}\right)^{1/2} \left(\frac{N}{60}\right)$$

thus for r of percent level, the field range is *super-Planckian*

Universality at Large-N and the Lyth bound

We can refine the Lyth bound for the full field range using our large- N result, which can be written as

$$1 \leq p < 2, \quad \Delta\phi \sim \left(\frac{r}{0.0005}\right)^{1/2} \frac{1}{(2-p)} \left(\frac{N}{60}\right) \quad \forall \quad q \quad \text{chaotic, } i\text{-hilltop}$$

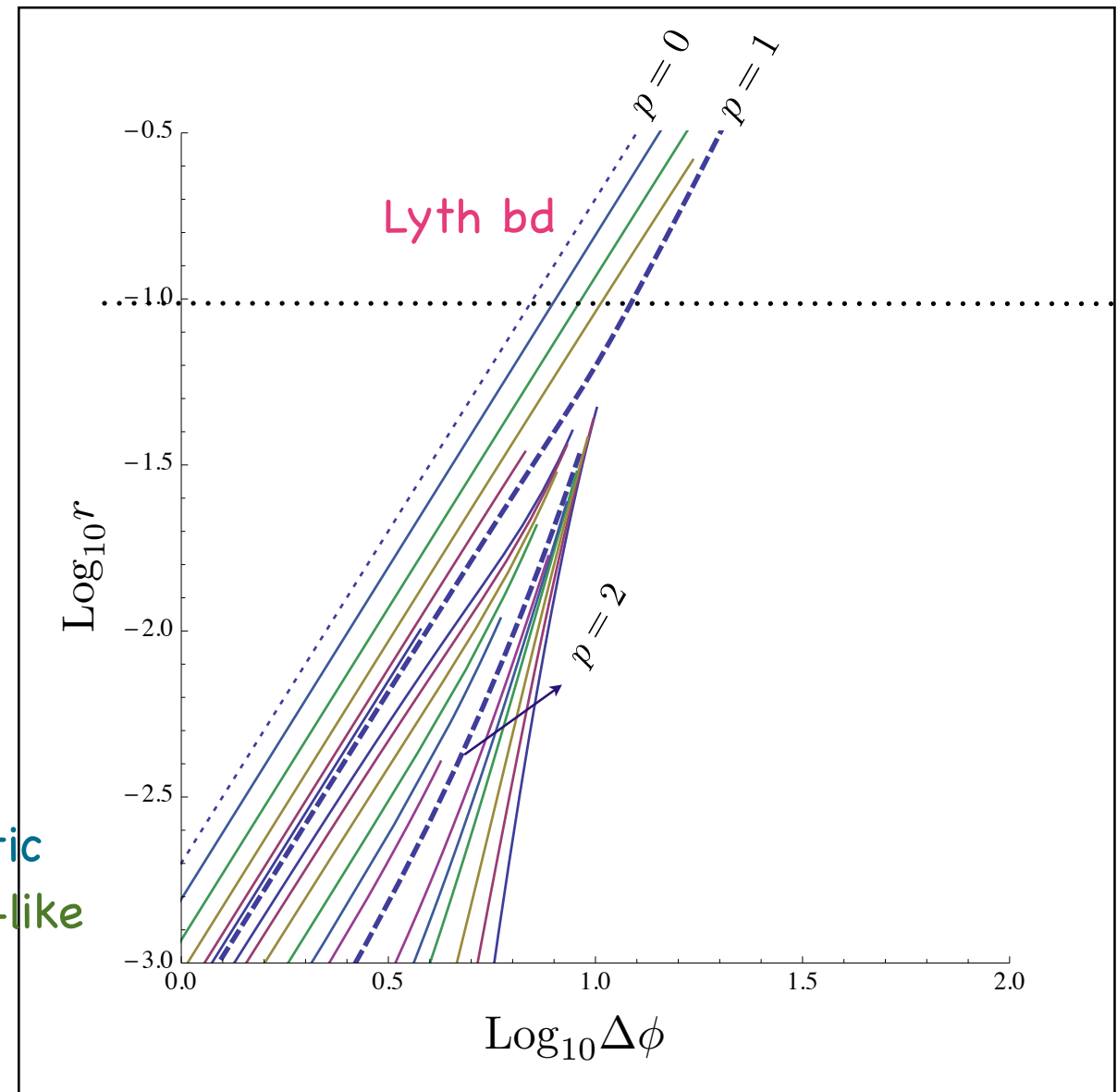
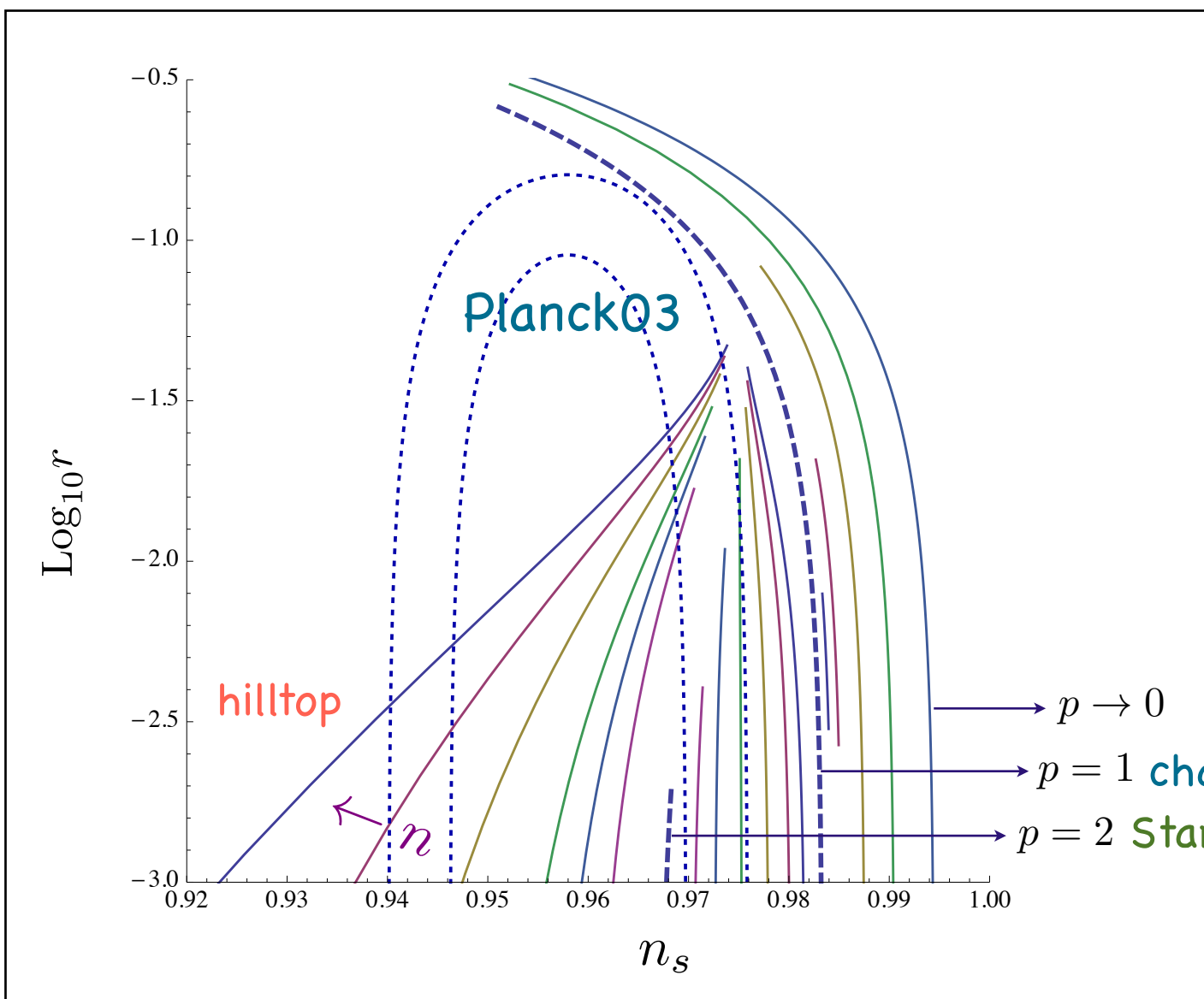
$$p = 2, \begin{cases} q < 2 : & \Delta\phi \sim \left(\frac{r}{3 \times 10^{-5}} \right)^{1/2} \frac{1}{(2-q)} \left(\frac{N \ln N}{60 \ln 60} \right) \\ q = 2 : & \Delta\phi \sim \left(\frac{r}{6 \times 10^{-5}} \right)^{1/2} \left(\frac{N \ln N \ln(\ln N)}{60 \ln 60 \ln(\ln 60)} \right) \end{cases} \begin{array}{l} \text{Kähler inflation} \\ \text{Starobinsky-like} \end{array}$$

$$p > 2, \quad \Delta\phi \sim \phi_e = \left(\sqrt{\frac{2}{8^{n-2}}} \right)^{1/(n-1)} \left(\frac{n-2}{n} \right)^{1/2} N r^{(n-2)/2(n-1)} \quad \text{hilltop}$$

$$n = 3, \quad \Delta\phi \sim \left(\frac{N}{60}\right) \left(\frac{r}{3 \times 10^{-6}}\right)^{1/4}$$

Universality classes and observations

Perturbative class full integration



$$V = V_0 \left[1 - \left(\frac{\phi}{\phi_0} \right)^n \right]$$

$$V = V_0 \left[1 - e^{-\phi/\phi_0} \right]$$

$$V = V_0 \left(\frac{\phi}{\phi_0} \right)^n$$

Further (non-)universality relations

- Using the expression for n_s we can relate further $\Delta\phi$ to n_s and r

$$n_s - 1 = -\frac{p}{N}$$

- $0 \leq p < 2$, $\Delta\phi = \frac{2p}{(2-p)} \frac{1}{(1-n_s)} \left(\frac{r}{8}\right)^{1/2}$

- $p = 2$ $\left\{ \begin{array}{ll} q < 2, & \Delta\phi = \frac{4}{(2-q)} \frac{1}{(1-n_s)} \ln\left(\frac{2}{1-n_s}\right) \left(\frac{r}{8}\right)^{1/2} \\ q = 2, & \Delta\phi = \frac{2}{(1-n_s)} \ln\left(\frac{2}{1-n_s}\right) \ln\left[\ln\left(\frac{2}{1-n_s}\right)\right] \left(\frac{r}{8}\right)^{1/2} \end{array} \right.$

- $p > 2$, $(p = 2, q > 2)$: $\Delta\phi = \phi_e$

$$\Delta\phi \sim \phi_e = \left(\frac{n-2}{n}\right)^{1/2} \frac{(n-1)}{(n-2)} \frac{2^{n/(n-1)}}{(1-n_s)} \left(\frac{r}{8}\right)^{(n-2)/2(n-1)}$$

Degeneracies of the field range

Given a point in the inflationary data (n_s, r) is the field range necessarily fixed?

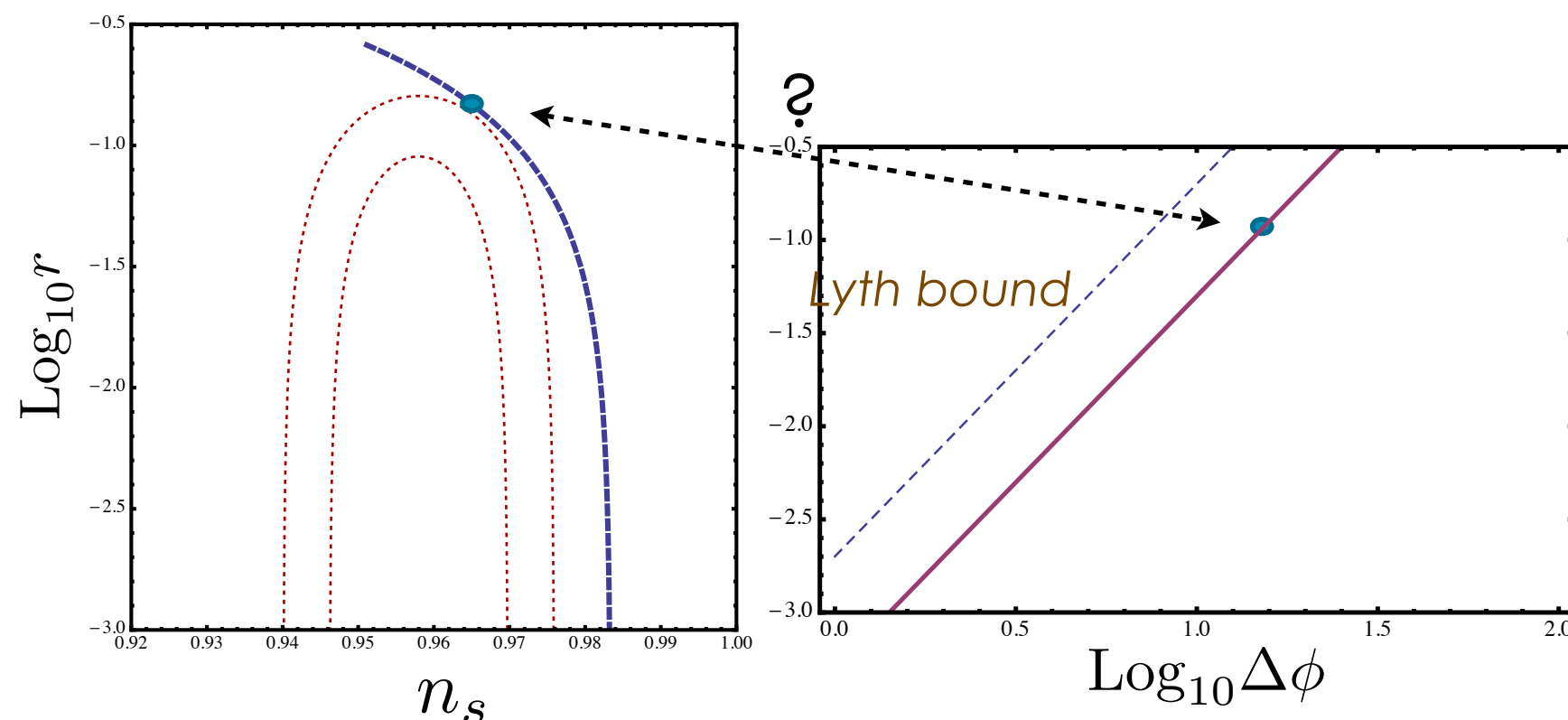
In particular, taking quadratic inflation as a benchmark model, is the field range unique?

Chaotic inflation has $p=1$,
$$\Delta\phi = \frac{2\sqrt{2\beta_0}}{(2-p)} N^{1-p/2} - \phi_e \quad \epsilon_1 = \frac{\beta}{N^p}$$

$$\Rightarrow \Delta\phi_c = 2\sqrt{2\beta} \left(N_\star^{1/2} - N_e^{1/2} \right) \quad \text{with} \quad \Delta N = N_\star - N_e = 60$$

N_e set by

$$\epsilon_1 = 1$$



$$\Delta\phi_c \sim 14.14$$

Degeneracies of the field range

- Perturbative class: $\epsilon_1 = \frac{\beta}{N^p}$, $\epsilon_2 = -\frac{p}{N}$

We want to reproduce a point in (n_s, r) plane with different $(p, \beta, \Delta N)$

Boils down to equating the Hubble parameters of each model:

$$\epsilon_i^c = \epsilon_i^p, \quad i = 1, 2$$

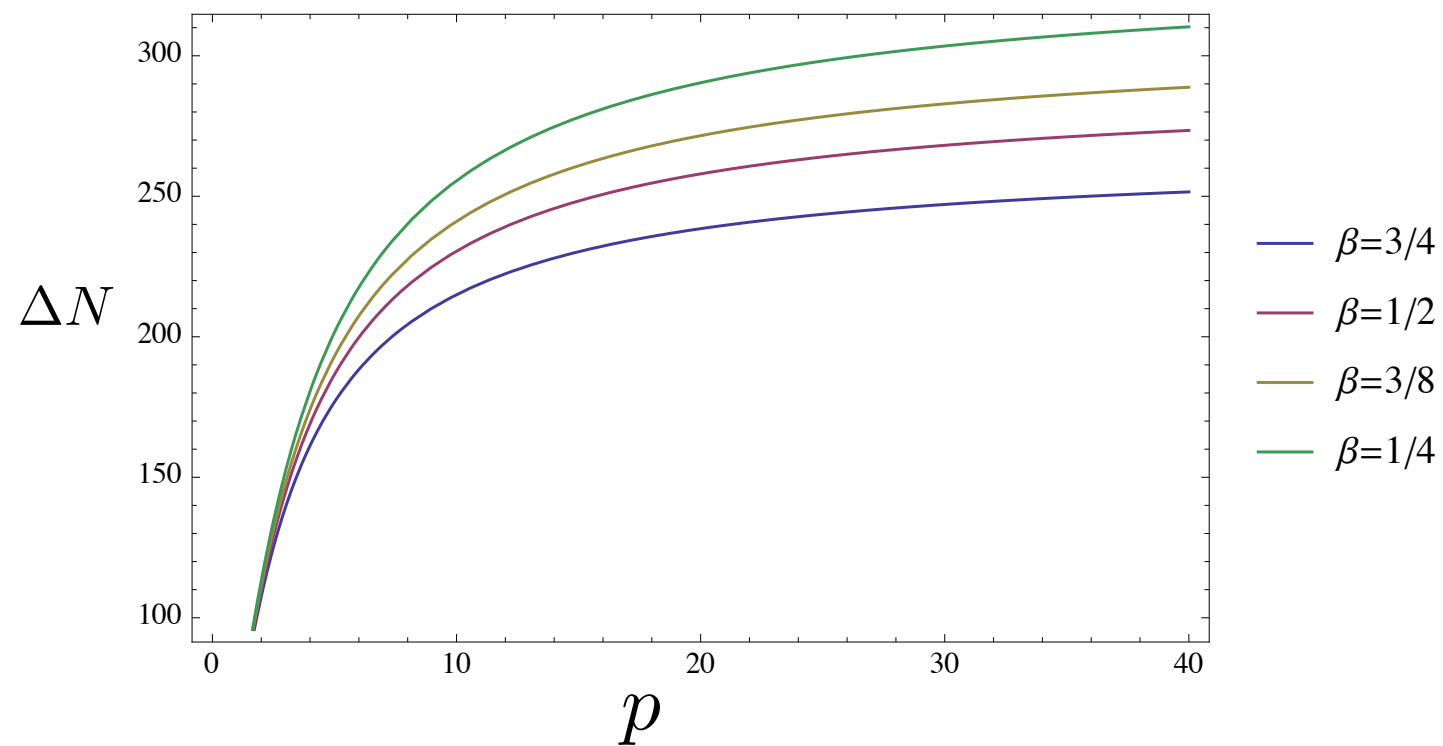
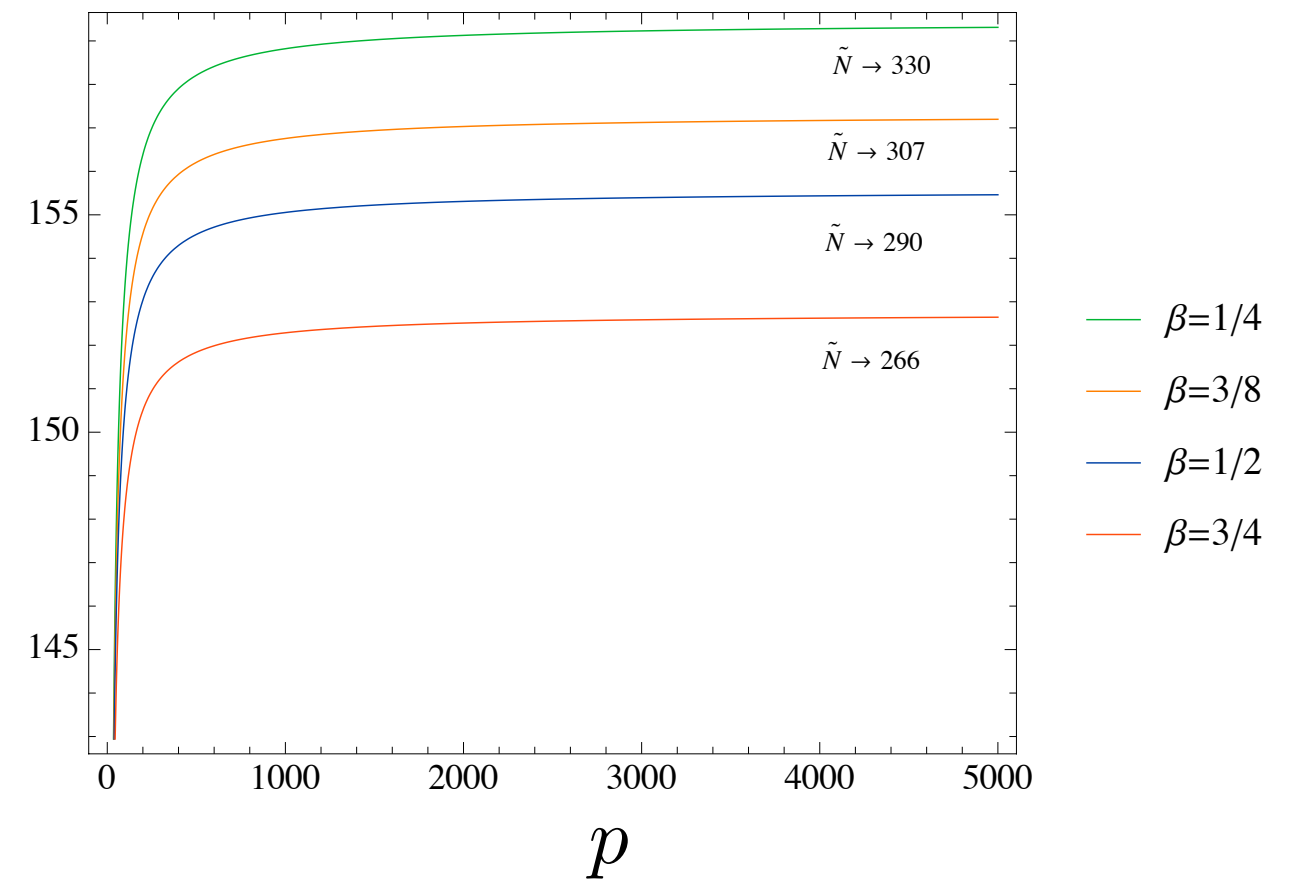
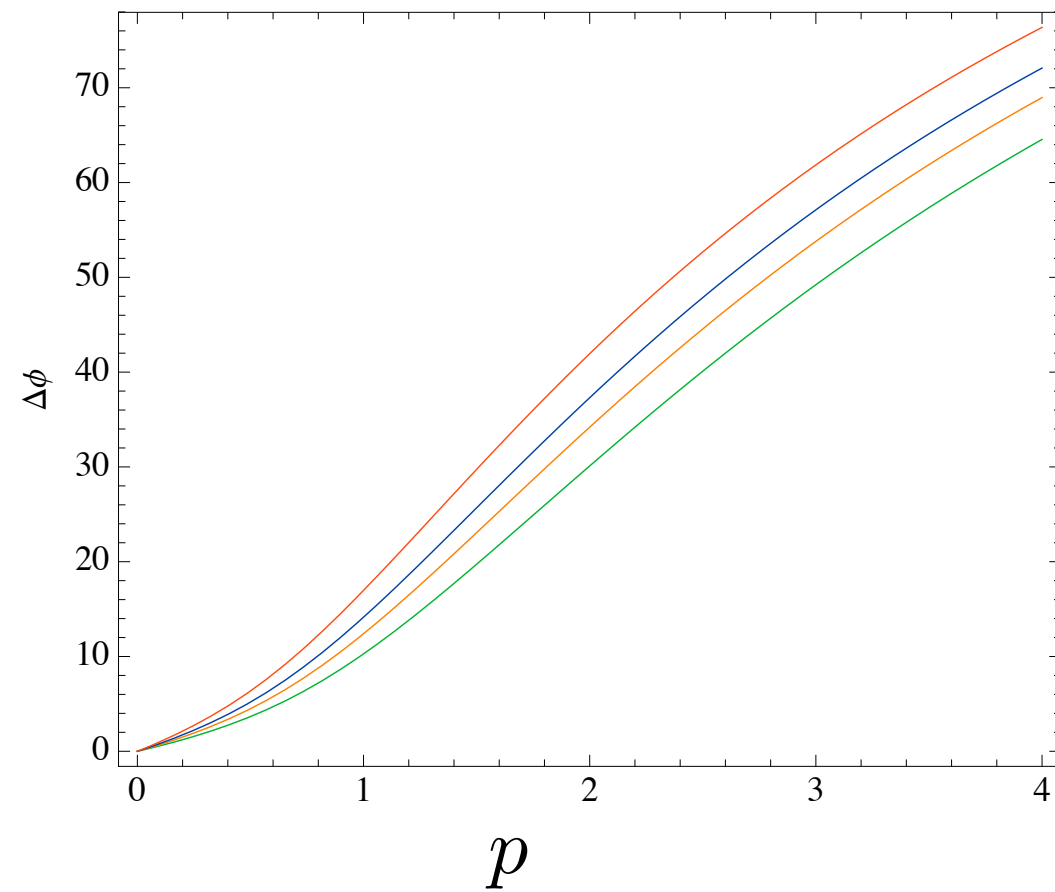
$$\Rightarrow \Delta N = p N_\star \left[1 - \left(\frac{\beta_c}{N_\star} \right)^{1/p} \right] \quad N_\star = \beta_c + 60,$$
$$\Delta\phi = 2\sqrt{2\beta_c} \frac{p}{2-p} \left[N_\star^{\frac{1}{2}} - \beta_c^{\frac{2-p}{2p}} N_\star^{\frac{p-1}{p}} \right] \quad \beta_c = 1/2$$

When $p \rightarrow \infty$

$$\Delta N \rightarrow N_\star \ln \frac{N_\star}{\beta_c}, \quad \Delta\phi \rightarrow 2\sqrt{2} N_\star^{1/2} \left[N_\star^{1/2} - \sqrt{\beta_c} \right]$$

$$\Delta N \rightarrow 290, \quad \Delta\phi \rightarrow 155.56$$

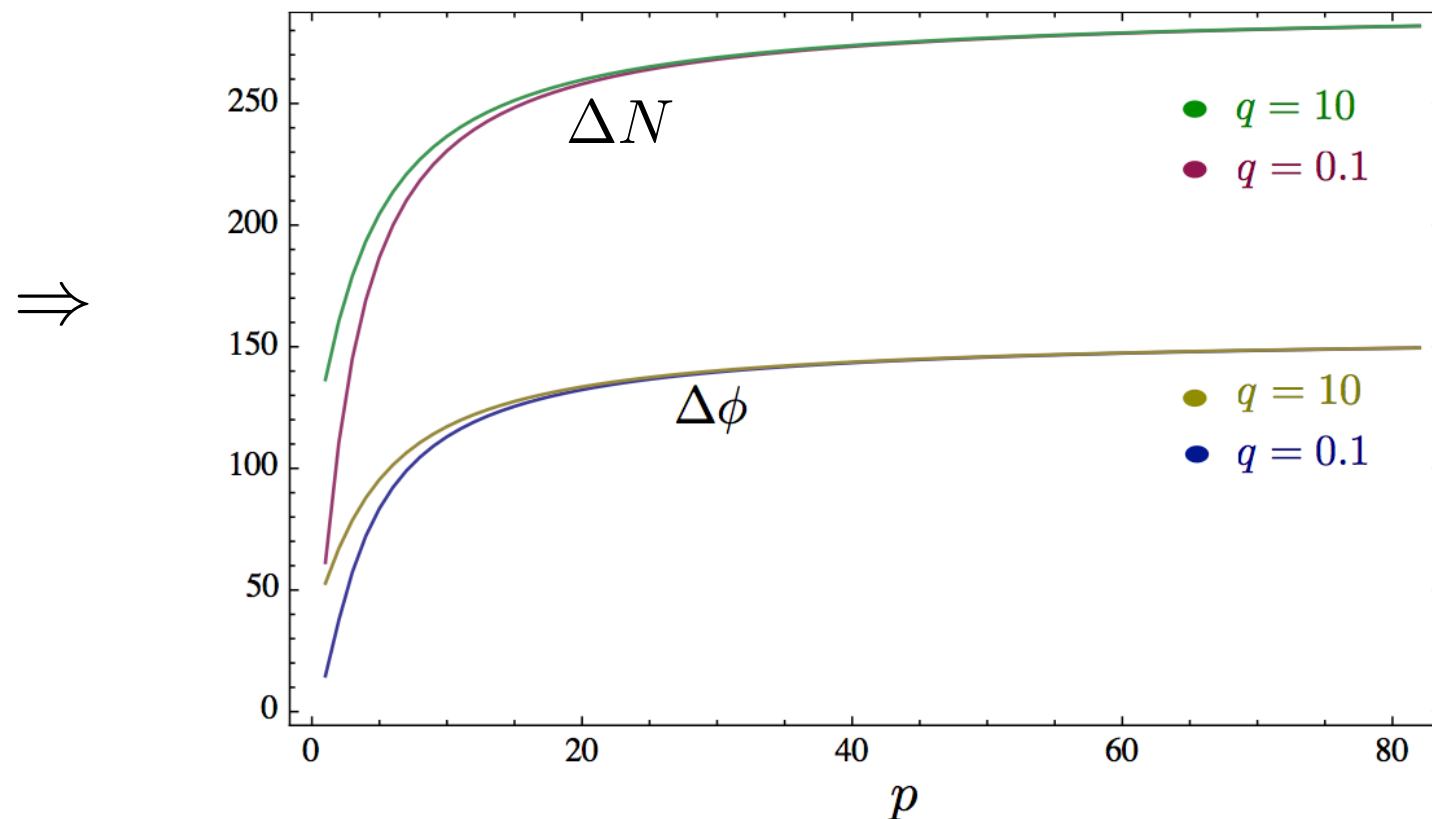
- Field range degeneracy in the perturbative class:



- Logarithmic class: $\epsilon_1 = \frac{\beta}{N^p \ln^q N}, \quad \epsilon_2 = -\frac{p}{N} - \frac{q}{N \ln N}$

As before, equating the Hubble flow parameters

$$\epsilon_i^c = \epsilon_i^p, \quad i = 1, 2$$



when $p \rightarrow \infty$

$$\Delta N \rightarrow 290, \quad \Delta \phi \rightarrow 155.56$$

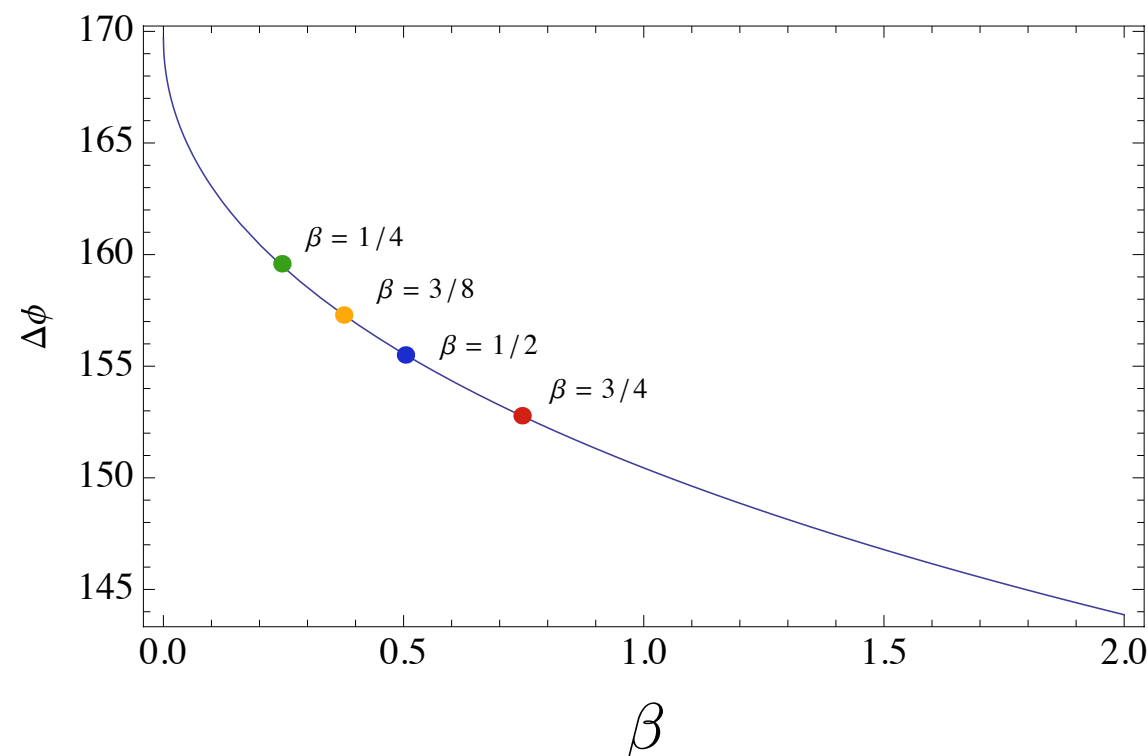
- Non-perturbative class: $\epsilon_1 = e^{-2cN}$, $\epsilon_2 = -2c$

Same procedure now fixes the field range and number of e-folds:

$$\epsilon_i^c = \epsilon_i^p, \quad i = 1, 2$$

$$\Delta N = N_\star \ln \frac{N_\star}{\beta} |_c \simeq 290$$

$$\Delta\phi = 2\sqrt{2} N_\star^{1/2} \left[N_\star^{1/2} - \sqrt{\beta_c} \right] |_c \simeq 155.56$$



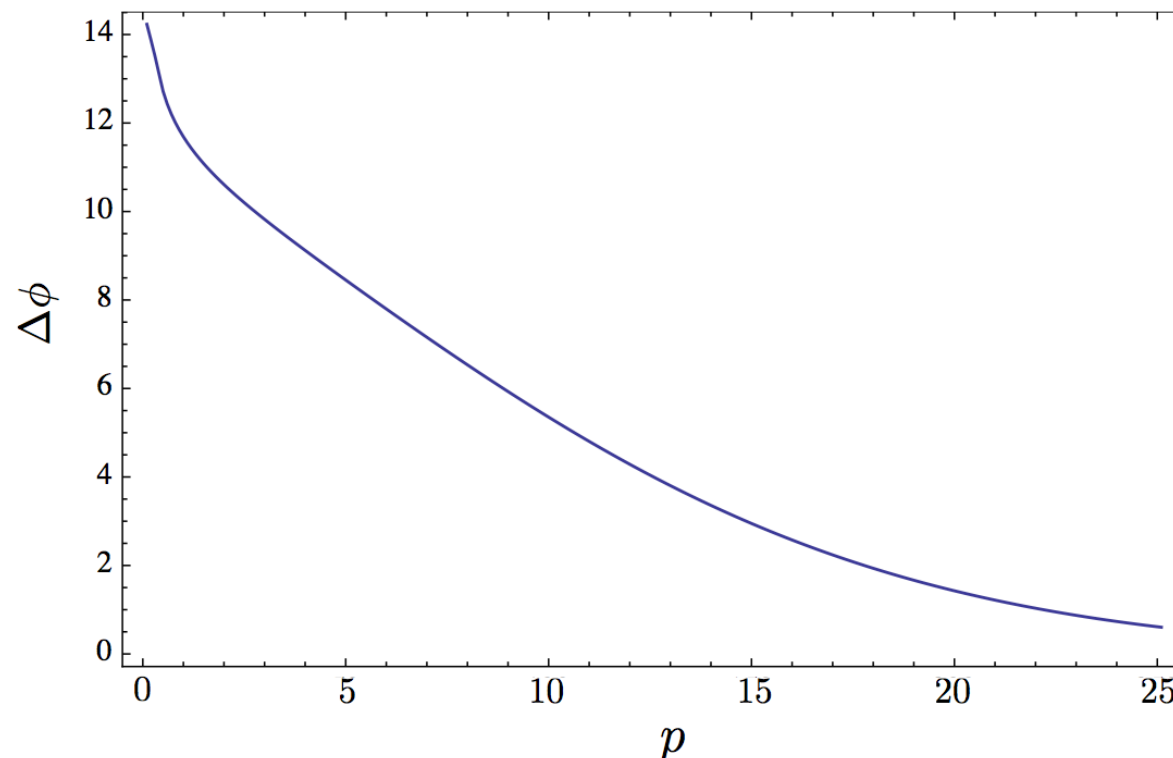
Sub-Planckian field range?

Fixing $\Delta N=60$, while letting p, q vary, it is possible to find sub-Planckian field ranges, by ending inflation when slow roll stops:

$$\epsilon_2 = 1$$

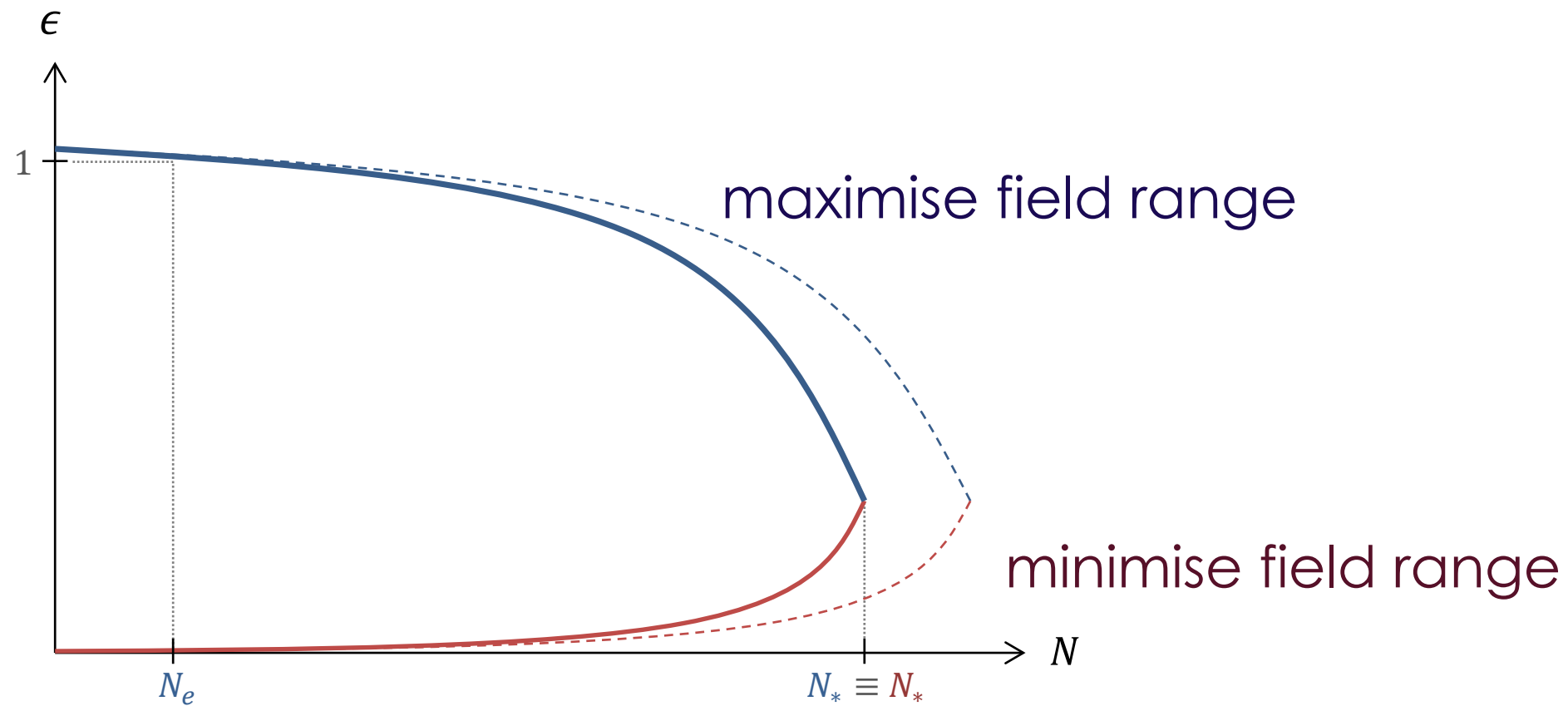
$$\text{but } \epsilon_1 = \epsilon, \quad \epsilon_2 = -4\epsilon + 2\eta$$

Accelerated expansion must end via some other mechanism:
e.g. by a field as in hybrid inflation



Summary

- Explored inflation in a (large)-N formalism. This allows for a useful classification of models in a $1/N$ expansion suggested by observation
- Analyzed universality properties of the field range at large-N for different classes of models
- Refined Lyth bound and found novel relations among field range, the tensor to scalar ratio and the spectral index.
- Studied the implications of CMB data for inflaton field range: to what extent one can infer $\Delta\phi$ from a measurement of (n_s, r)
- Found an upper bound for field range and number of e-folds.



$$\frac{d\epsilon}{dN} = 2\epsilon(\eta - 2\epsilon), \quad \frac{d\eta}{dN} = \epsilon(\eta - 3\epsilon)$$

Cannot have both right-hand sides vanishing at the same time when $\epsilon \neq 0$, thus cannot keep ϵ constant over a large range of e-foldings.

As a consequence, there is a limit on the number of e- foldings between horizon exit and the end of inflation, for a generic slow-roll model and translates into a limit on the field range during this period.