Probing Conformal Field Theories

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Overview of work done in collaboration with
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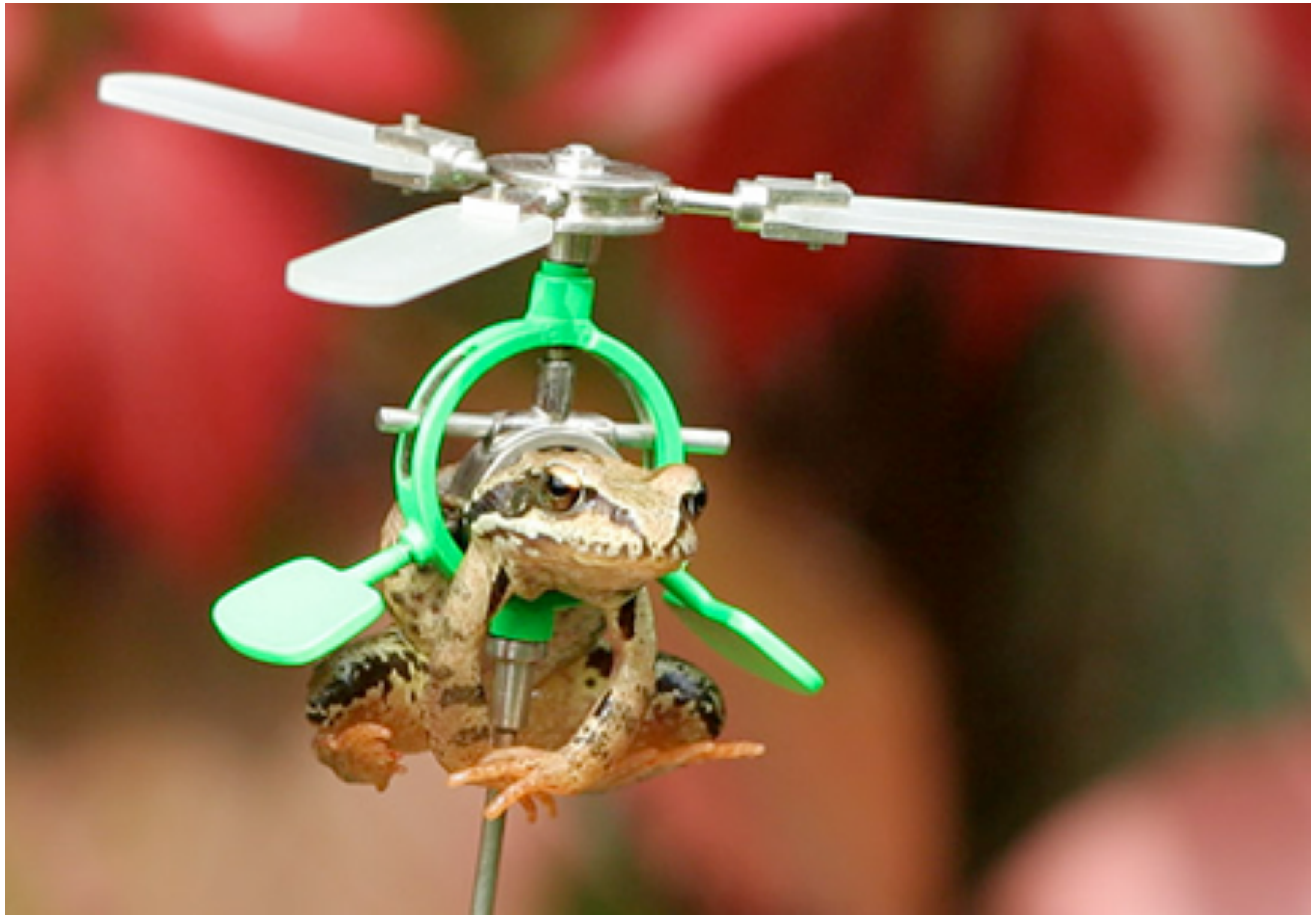
Birds and Frogs

Freeman Dyson

Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a time. I happen to be a frog, but many of my best friends are birds. The main theme of my talk tonight is this. Mathematics needs both birds and frogs. Mathematics is rich and beautiful because birds give it broad visions and frogs give it intricate details. Mathematics is both great art and important science, because it combines generality of concepts with depth of structures. It is stupid to claim that birds are better than frogs because skill as a mathematician. In his later years he hired younger colleagues with the title of assistants to do mathematical calculations for him. His way of thinking was physical rather than mathematical. He was supreme among physicists as a bird who saw further than others. I will not talk about Einstein since I have nothing new to say.

Francis Bacon and René Descartes

At the beginning of the seventeenth century, two great philosophers, Francis Bacon in England and René Descartes in France, proclaimed the birth of modern science. Descartes was a bird, and Bacon was a frog. Each of them described his vision of the future. Their visions were very different. Bacon said, "All depends on keeping the eye steadily fixed on the facts of nature." Descartes said, "I think, therefore I am." According to Bacon, scientists



Plan of the Bird Talk

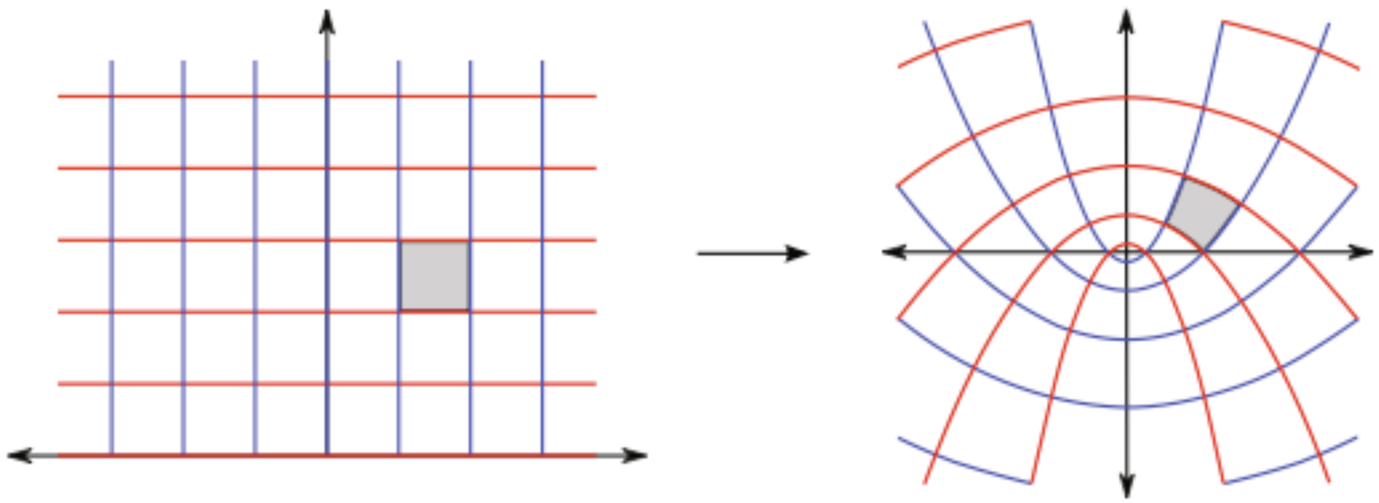
- CFTs: the first three seconds.
- What are CFTs good for?
- Taming the Conformal Zoo
- Back to the Bootstrap
- Scale vs. Conformal Invariance

- Exact results

Conformal Group

Conformal Transformation

$$g_{\mu\nu}(x) \to \Lambda(x) g_{\mu\nu}(x)$$



d>2 **SO(2,d): Poincaré + dilatations + special conformal transformations.**

What are CFTs good for?

- CFTs control 2nd order thermal phase transitions and quantum phase transitions.

- CFTs control the UV and IR of RG flows.

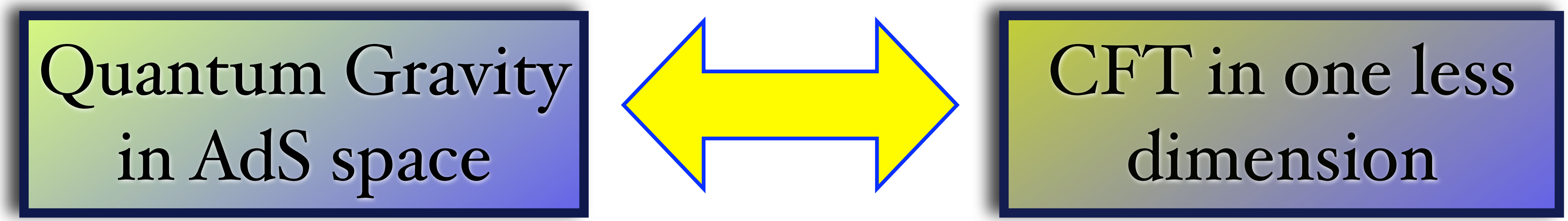
- CFTs define non-perturbative quantum gravity in asymptotic Anti de Sitter backgrounds.

What else might CFTs be good for?

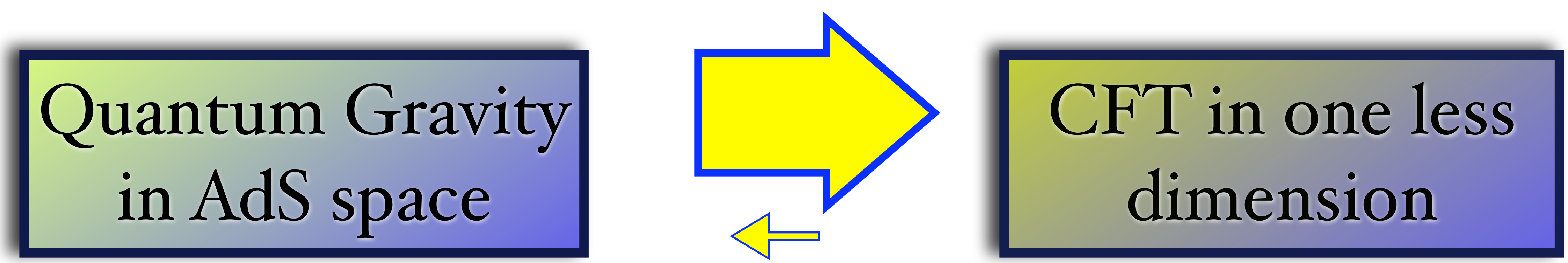
- Physics beyond the Standard Model: unparticles, conformal technicolor,...
- Holographic description of quantum gravity in deSitter space, inflation,...

Two words about AdS/CFT

What we say....



What we do....



Can we use CFTs to learn about specific issues of Quantum Gravity ?

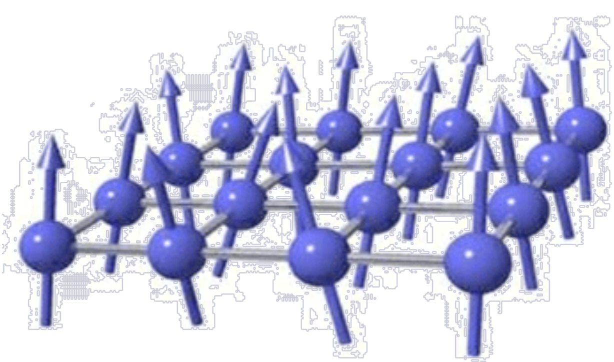
Taming the Conformal Zoo

(or CFTs, from 2 to 6)

2d CFTs

- In 2d, the conformal algebra is infinite-dimensional.

- 2d CFTs play an important role in statistical mechanics, in perturbative string theory, black hole entropy,...



- ...but today I won't talk about them.

$d=6$, a CFT frontier?

- There are free CFTs in all dimensions: free massless scalar boson, free massless fermion.

- In $d \neq 4$, Maxwell theory is scale invariant but not conformal.

  El-Showk, Nakayama, Rychkov 11

  Jackiw, Pi 11

- There are free CFT p-form generalizations of Maxwell theory in all even dimensions. **Witten 07**

d=6, a CFT frontier?

- There are known interacting 6d CFTs. They are poorly understood.
- I am not aware of any interacting CFT in d>6.....
- There are no supersymmetric CFTs in d>6.

Nahm 78

Are there unitary UV complete, interacting QFTs in d>6 ?

4d CFTs

- All the interacting 4d CFTs that I know (at finite N) are supersymmetric. (Nevertheless, I tend to believe that there exist finite N non-susy 4d CFTs).
- Furthermore, if they admit a Lagrangian, they are supersymmetric gauge theories.
- Some SCFTs are conformal for any value of the coupling, while others appear at the ends of RG flows.

4d CFTs: central charges

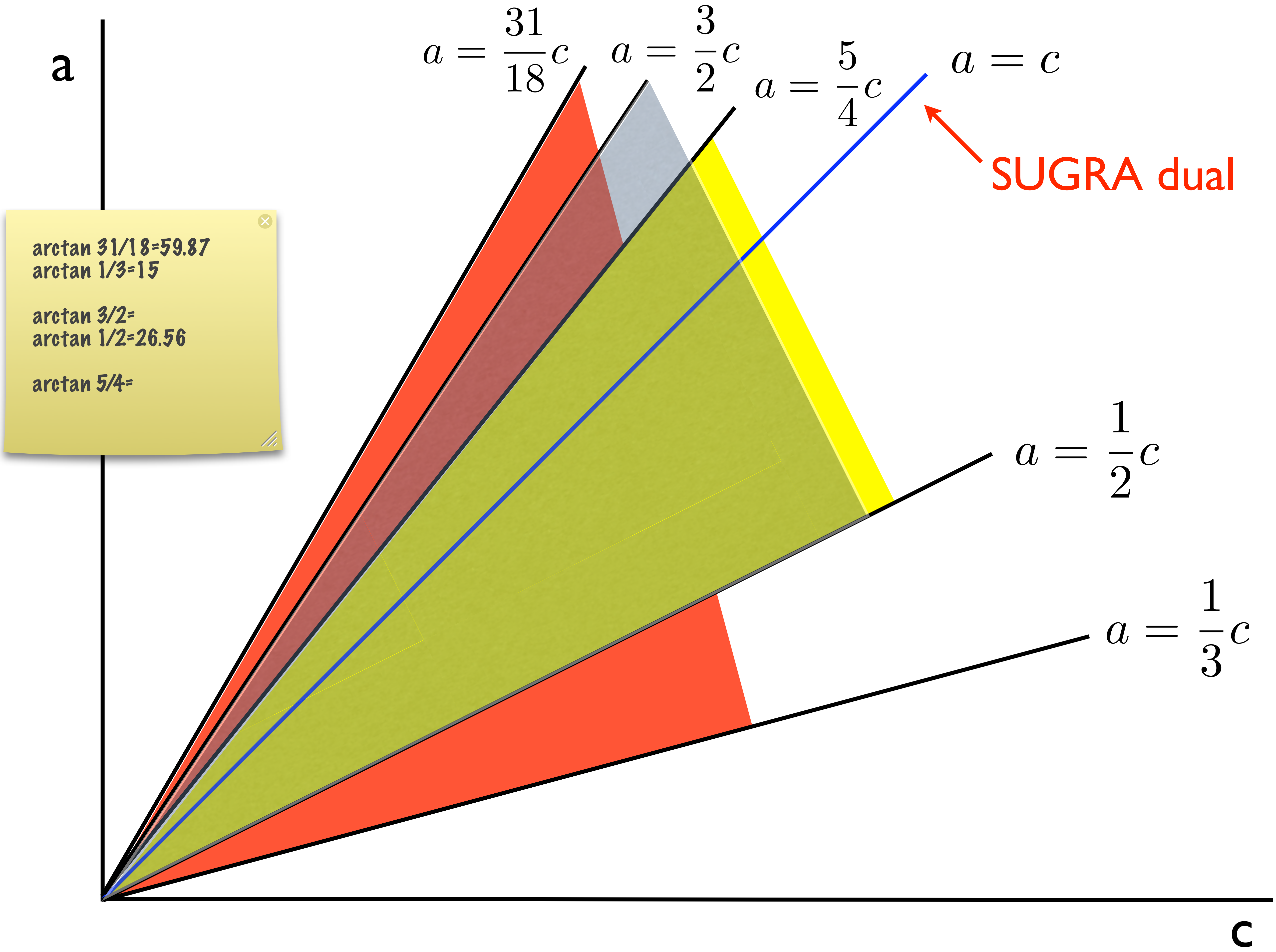
$$T^{\mu}_{\mu} = \frac{c}{16\pi^2} W^2 - \frac{a}{16\pi^2} E$$

- The ratio a/c is bounded from above and below. (positivity of the energy flux).

Hofman Maldacena 08
Shapere Tachikawa 08

- Having a (two-derivative) supergravity dual requires a=c (at large N).

Henningson Skenderis 98



a
c
$a = \frac{31}{18}c$
$a = \frac{3}{2}c$
$a = \frac{5}{4}c$
$a = c$
SUGRA dual
$a = \frac{1}{2}c$
$a = \frac{1}{3}c$
arctan 31/18=59.87
arctan 1/3=15
arctan 3/2=
arctan 1/2=26.56
arctan 5/4=

Are CFTs saturating the a/c bounds free?

Zhiboedov 13

Develop holography for 4d CFTs with $c \neq a$

Buchel, Myers, Sinha 08

Gadde, Pomoni, Rastelli 09

....

Back to the Bootstrap

(or everything old is new again)

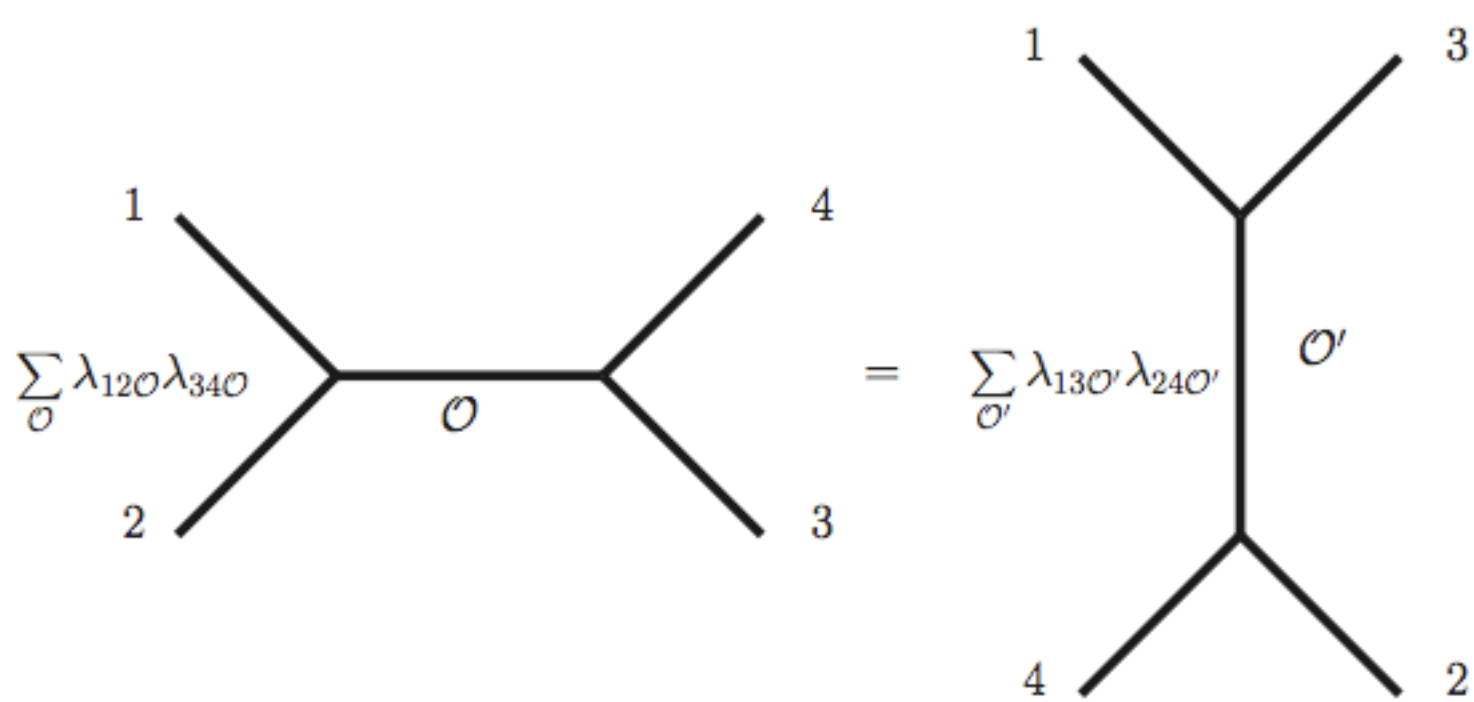
Consider a 4-point function

$$< \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4) >$$

OPE: $$\phi_1\phi_2 \sim \sum_{\mathcal{O}} \lambda_{12\mathcal{O}}\mathcal{O}$$

Conformal Bootstrap:

Ferrara, Grillo, Gato 73
Polyakov 74

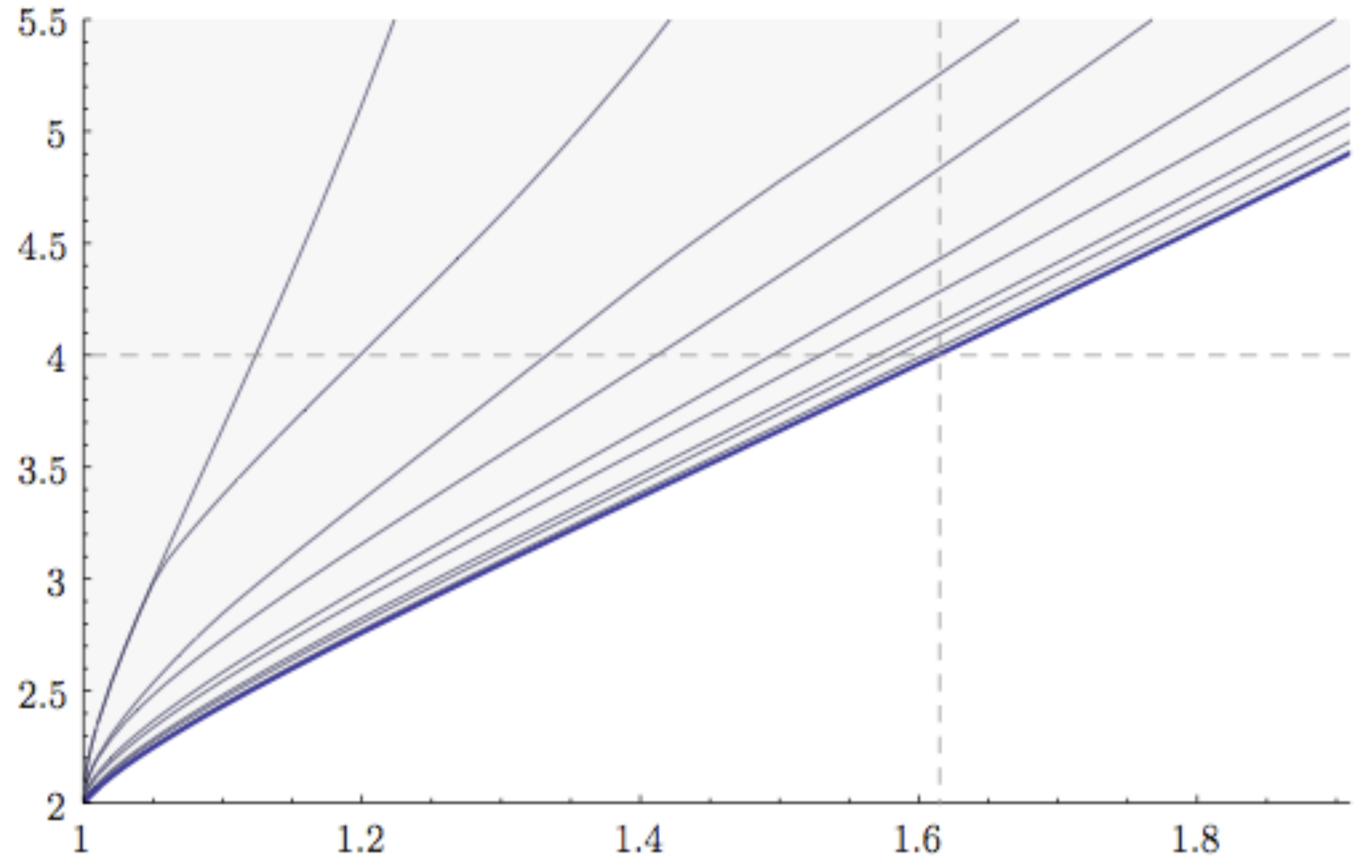


- In 2d, the conformal bootstrap is quite powerful.

Belavin, Polyakov Zamolodchikov 84

- In d >2, it has been dormant...until recently

e.g. : upper bound on $\Delta(\phi^2)$ as a function of $\Delta(\phi)$

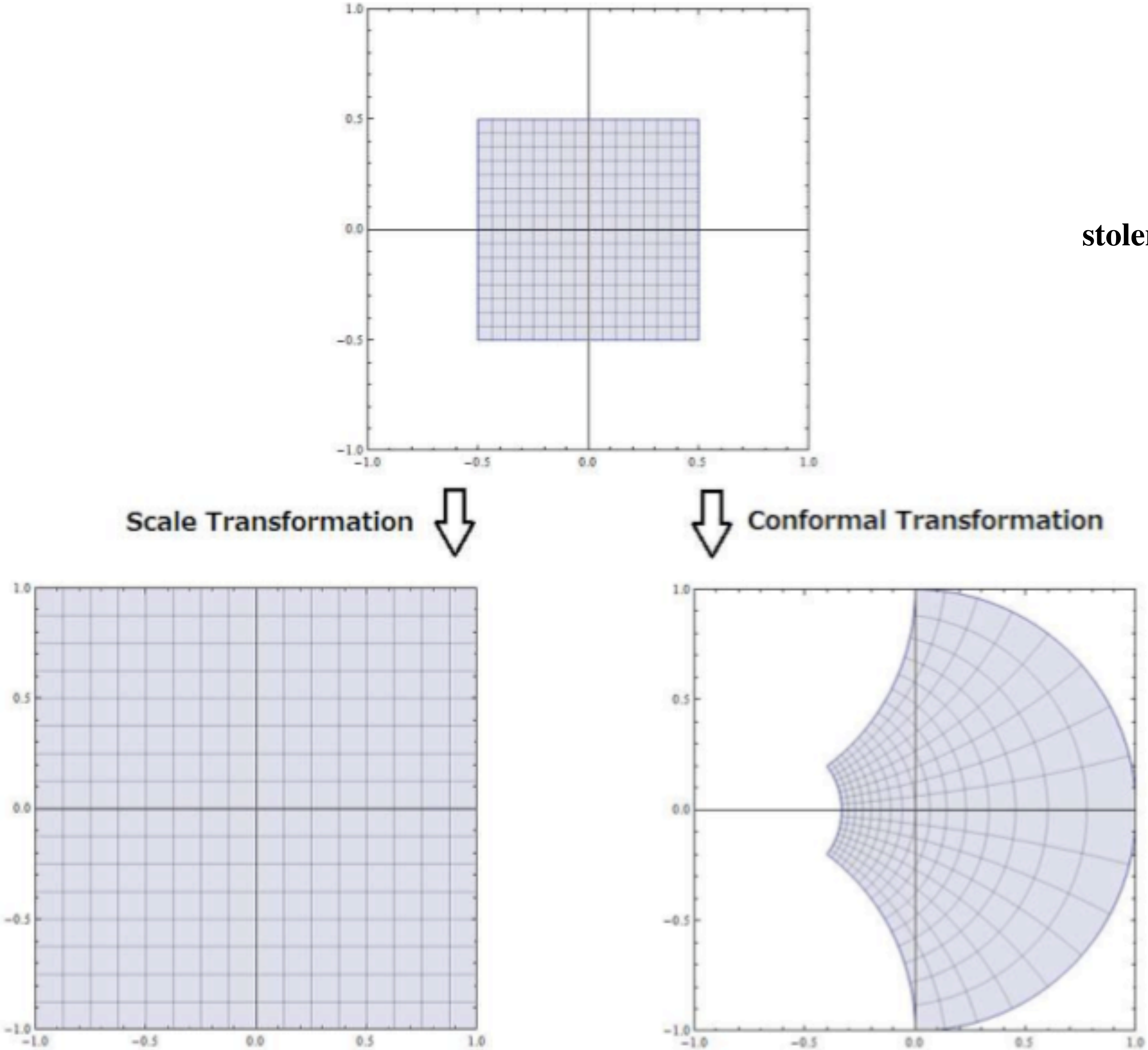


Ratazzi, Rychkov, Tonni, Vichi 08

....

Turn all this numerical insight into analytical arguments

Scale vs. Conformal Invariance



stolen from Nakayama 13

Scale vs. Conformal Invariance

- Mathematically,
  scale invariance $\underset{\Longleftarrow}{\not\Longrightarrow}$ conformal invariance.

- In 2d, under some conditions (unitarity, Poincaré invariance,...) it can be proven that scale invariance implies conformal invariance.

  Zamolodchikov 86

  Polchinski 88

- in 4d the folklore is that a similar theorem is true.

  Luty, Polchinski, Ratazzi 12

  Nakayama 13

Exact Results

(Frog time !!!)

Probing a CFT

- **Consider a heavy probe coupled to a CFT, in some rep. of the gauge group.**
- **It may be coupled to additional fields.**
- **Its world-line is prescribed. It defines a line operator (Wilson line,....).**
- **What are the fields it creates? Energy radiated? Momentum fluctuations?.....**

An example: Maxwell theory

Static particle

$$< \mathcal{L}(\vec{x}) > = q^2 \frac{1}{|\vec{x}|^4}$$

Coulomb

$$\mathcal{L} \sim F^2 \sim E^2 - \cancel{B^2}$$

Accelerated Particle

$$P = \frac{2}{3} q^2 \; a^\mu a_\mu$$

Larmor

In today's talk....

Probes will be infinitely heavy.

We will only consider probes in the vacuum state of the CFT.

Plan of the Frog Talk

- **External Probes in CFTs.**
- **Computing Bremsstrahlung functions.**
    - **AdS/CFT**
    - **Localization**
- **An application.**
- **Outlook.**

CFTs and local operators

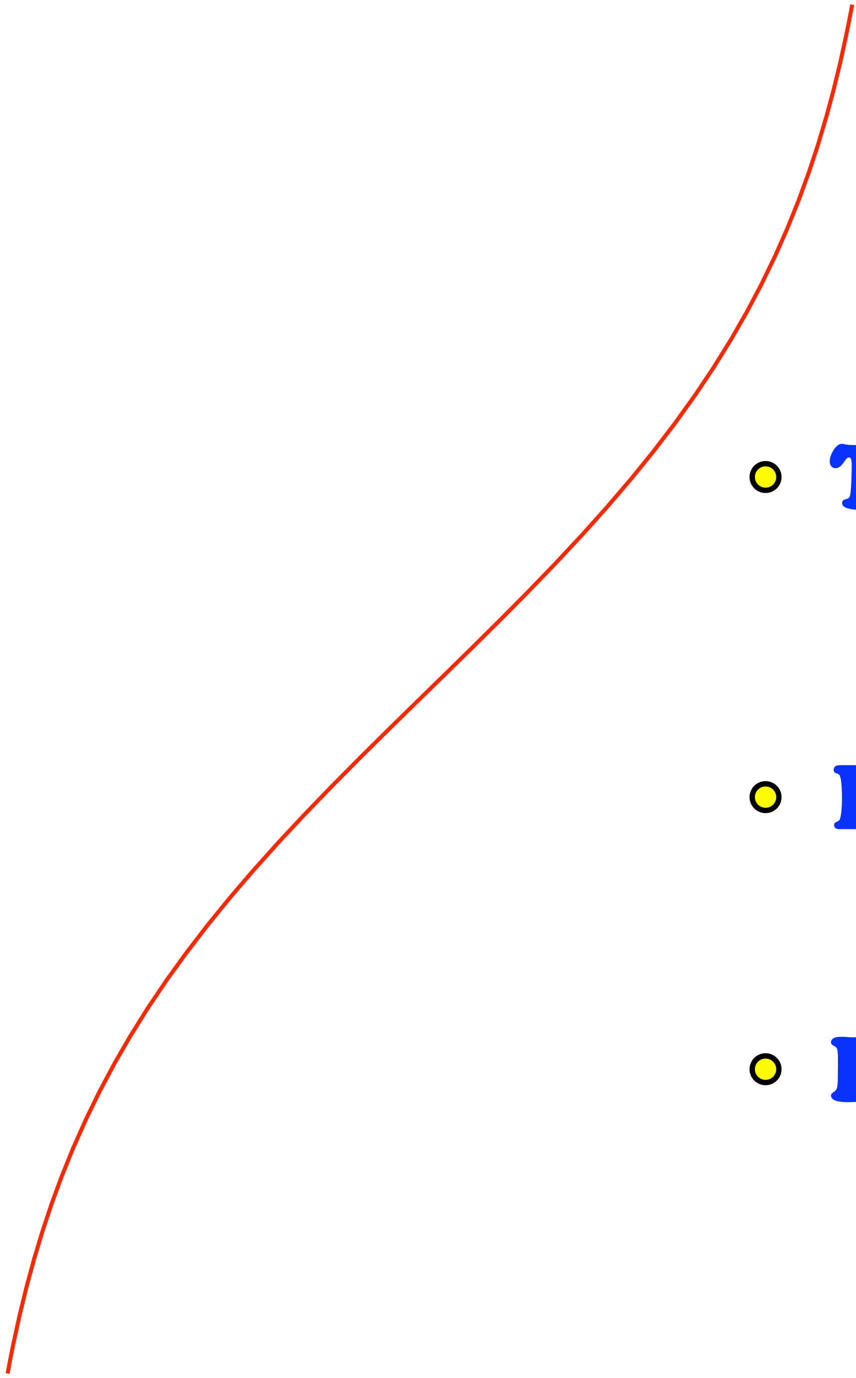
- **Conformal symmetry gives constraints on correlation functions, but it doesn't fix all coefficients.**

$$< \mathcal{O}_i(x)\mathcal{O}_j(0) >= \frac{\delta_{ij}}{(x)^{2\Delta_i(\lambda,N)}}$$

$$< \mathcal{O}_i(x_i)\mathcal{O}_j(x_j)\mathcal{O}_k(x_k) >= \frac{c_{ijk}(\lambda,N)}{(x_{ij})^{2\alpha_{ijk}}(x_{ik})^{2\alpha_{ikj}}(x_{jk})^{2\alpha_{jki}}}$$

- **We need additional tools to compute coefficients.**

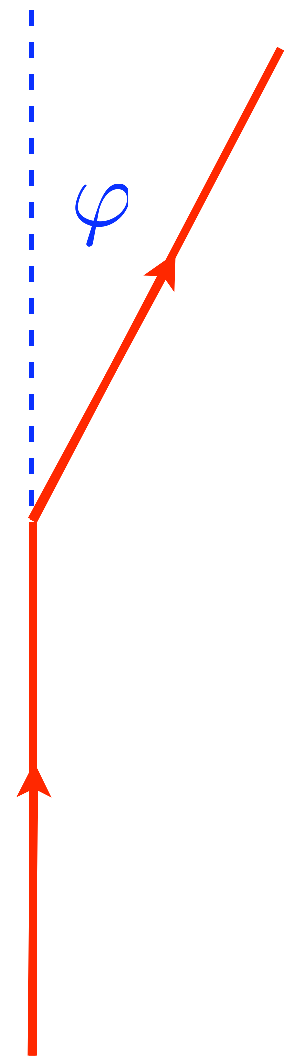
Let’s add external probes… (aka line operators)



- Timelike curve C
- Electrically/magnetically charged
- Representation R of G

→ Wilson operator, 't Hooft operator,....

Cusped Wilson loop: Radiation

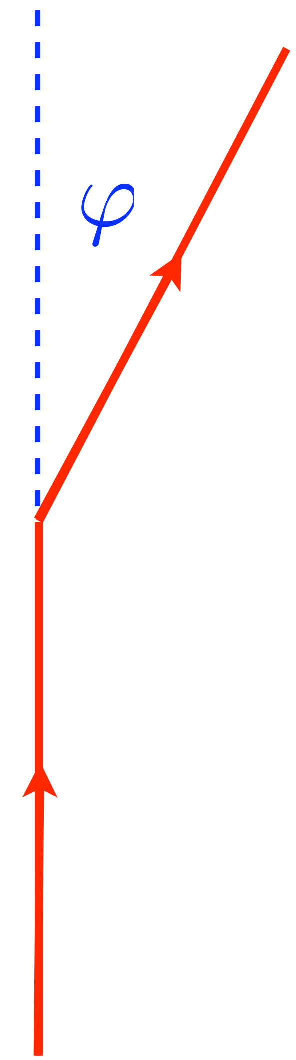


J.J. Thomson

Bremsstrahlung function

$$\Delta E = 2\pi B(\lambda, N) \int dt\ (\dot{v})^2$$

Cusped Wilson loop

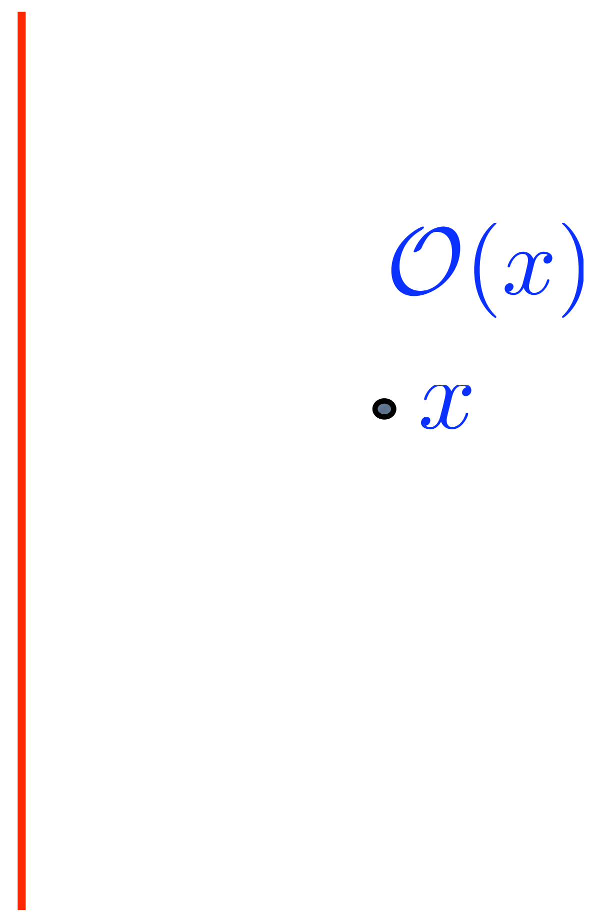


Cusp anomalous dimension

Polyakov 80

$$< W > \sim e^{- \, \Gamma_{cusp}(\varphi, \lambda) \; Log \frac{L}{\epsilon}}$$

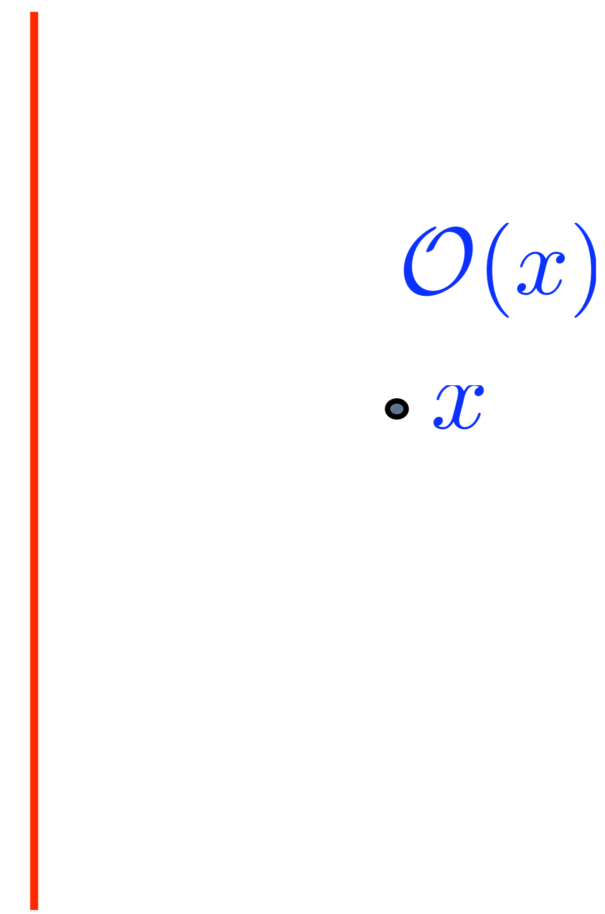
Line operators and local operators



$$< \mathcal{O}(x) >_W \equiv \frac{< \mathcal{O}(x) W >}{< W >}$$

Kapustin 05

Line operators and local operators

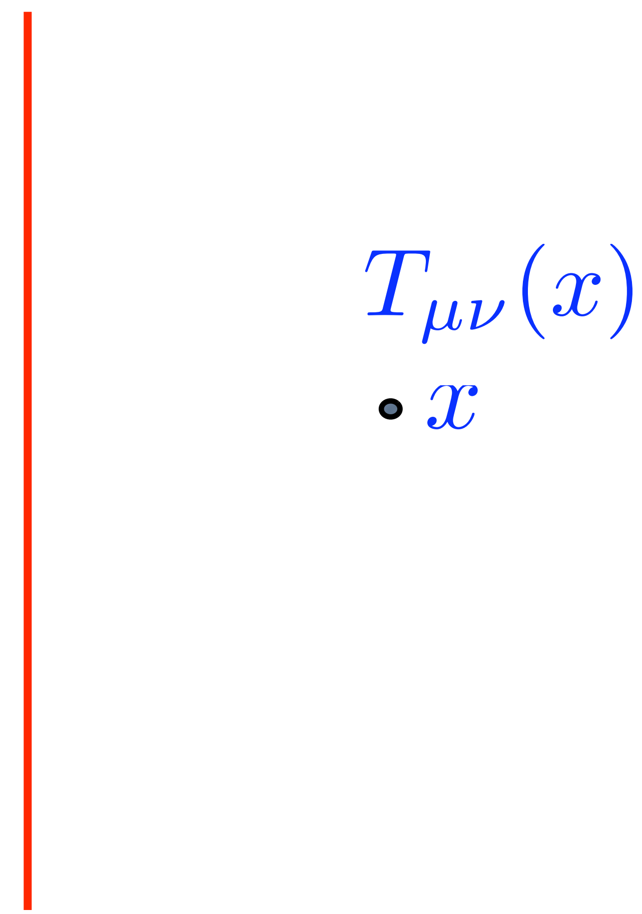


Conformal symmetry fixes $< \mathcal{O} >_W$ **up to a coefficient.**

$< \mathcal{O}(x_1)\mathcal{O}(x_2) >_W$ **is no longer fixed.**

Buchbinder, Tseytlin 12

Line operators and local operators



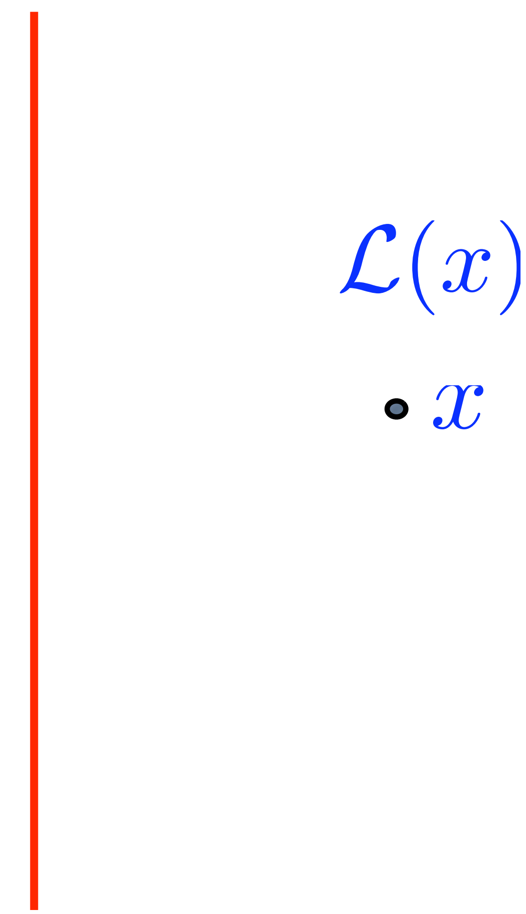
For a generic CFT

$$< T_{00}(x) >_W = h(\lambda, N)\,\frac{1}{|\vec{x}|^4}$$

$$< T_{ij}(x) >_W = h(\lambda, N)\,\frac{-\delta_{ij} + 2\frac{x_i x_j}{|\vec{x}|^2}}{|\vec{x}|^4}$$

Kapustin 05

Line operators and local operators



For a CFT with $\mathcal{L}$

$$< \mathcal{L}(x) >_W = f(\lambda, N)\ \frac{1}{|\vec{x}|^4}$$

World-line Operators: Displacement Operators

Polyakov, Rychkov 00
Semenoff, Young 04
Drukker, Kawamoto 06

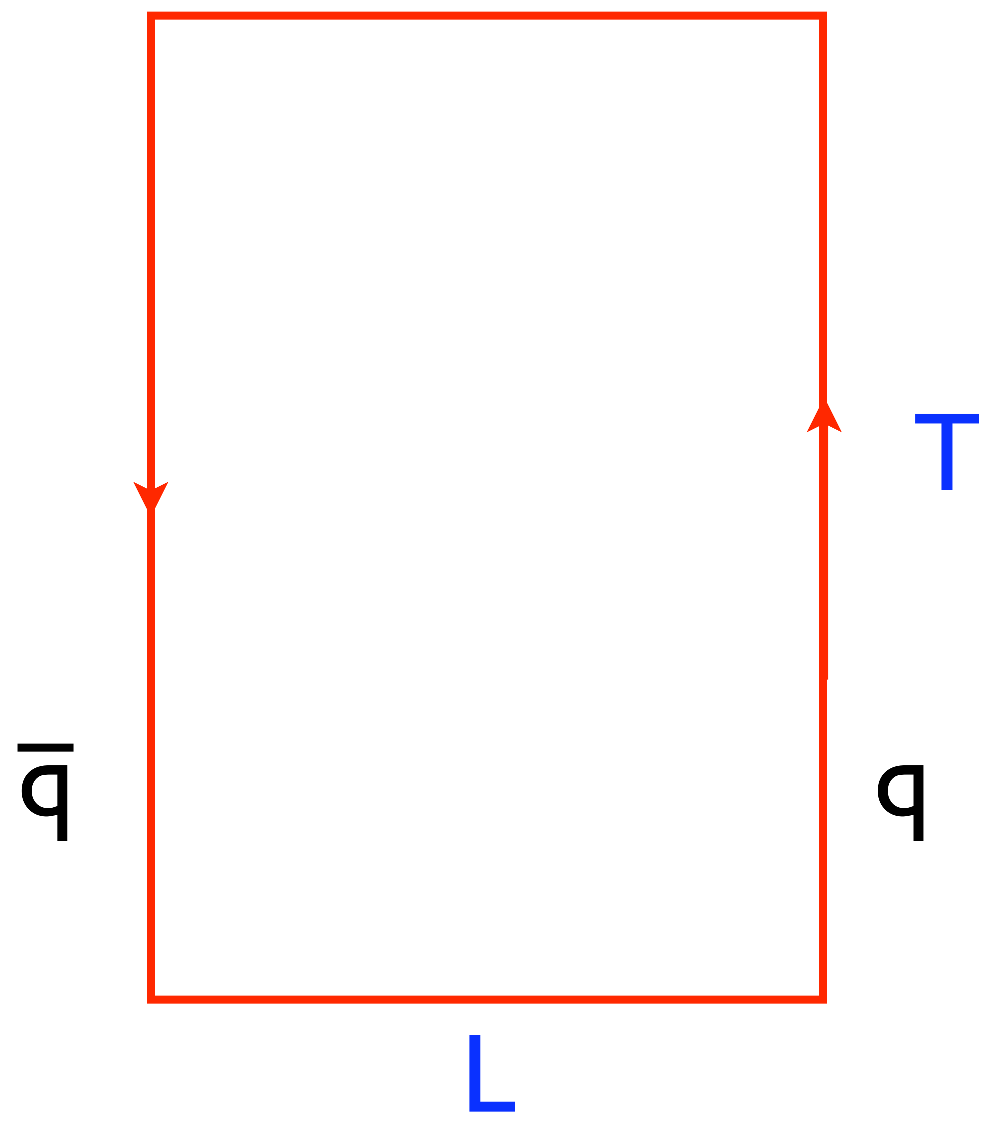
$\mathbb{D}_j(t_2)$ $\delta x^j(t_2)$

$\mathbb{D}_i(t_1)$ $\delta x^i(t_1)$

$$\Delta(\mathbb{D}_i) = 2$$

$$<< \mathbb{D}_i(t_1)\mathbb{D}_j(t_2) >> = \tilde{\gamma}(\lambda, N) \frac{\delta_{ij}}{(t_1 - t_2)^4}$$

The Holy Grail: q-$\bar{q}$ potential



For a CFT: $$V_{q\bar{q}}(L) = -\frac{c(\lambda, N)}{L}$$

For any line operator in any 4d CFT, we have defined:

$$< W > \sim e^{-\,\Gamma_{cusp}(\varphi,\lambda)\,Log\frac{L}{\epsilon}}$$

$$\Delta E = 2\pi B(\lambda, N)\int dt\ (\dot{v})^2$$

$$<< \mathbb{D}_i(t_1)\mathbb{D}_j(t_2) >>= \ \tilde{\gamma}(\lambda, N)\ \frac{\delta_{ij}}{(t_1 - t_2)^4}$$

$$< T_{00}(x) >_W = h(\lambda, N)\,\frac{1}{|\vec{x}|^4}$$

$$< \mathcal{L}(x) >_W = f(\lambda, N)\ \frac{1}{|\vec{x}|^4}$$

These coefficients are actually not independent...

Expand the cusp anomalous dimension at small angles,

$$\Gamma(\varphi) = \Gamma(\lambda, N)\,\varphi^2 + \mathcal{O}(\varphi^4) + \ldots$$

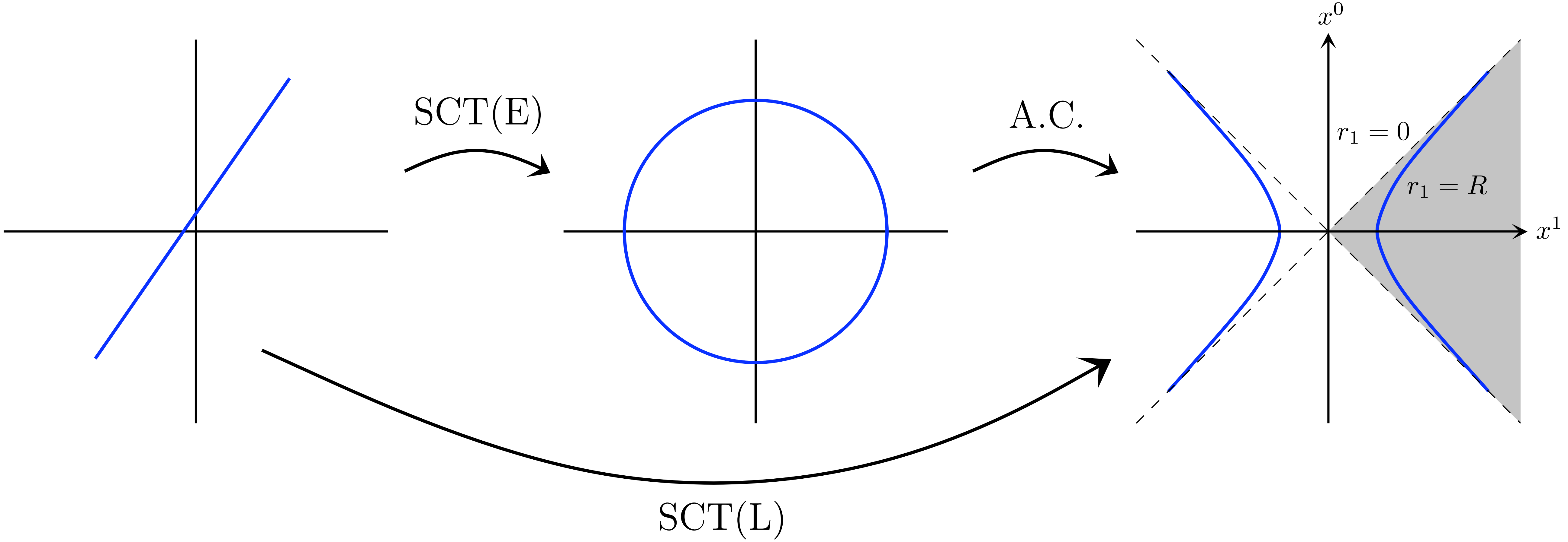
Then, for any line operator and any 4d CFT,

$$\boxed{\Gamma = \frac{\tilde{\gamma}}{12} = B}$$

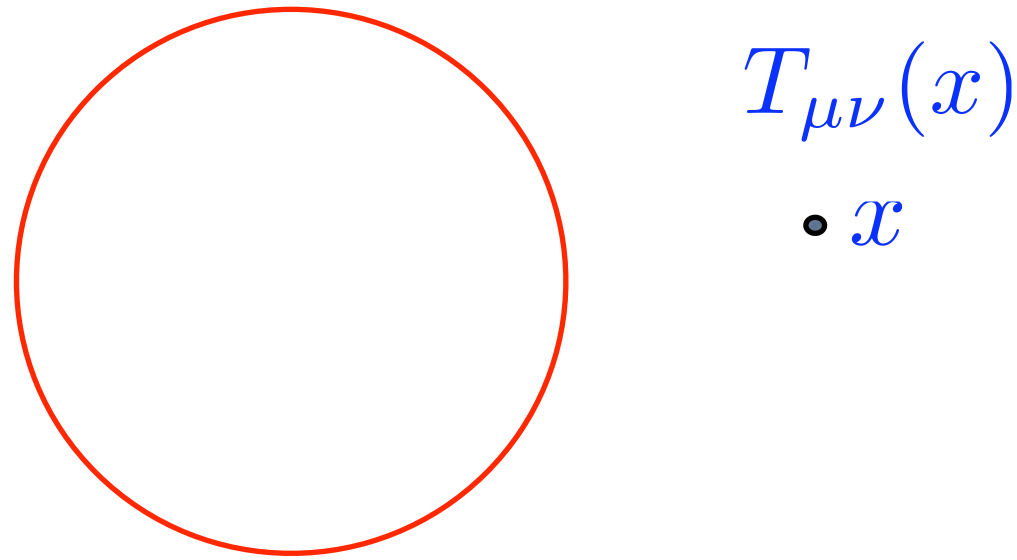
Correa, Henn, Maldacena, Sever 12

But wait!, there is more...

Hyperbolic Wilson line: accelerated probe



Hyperbolic Wilson line: accelerated probe

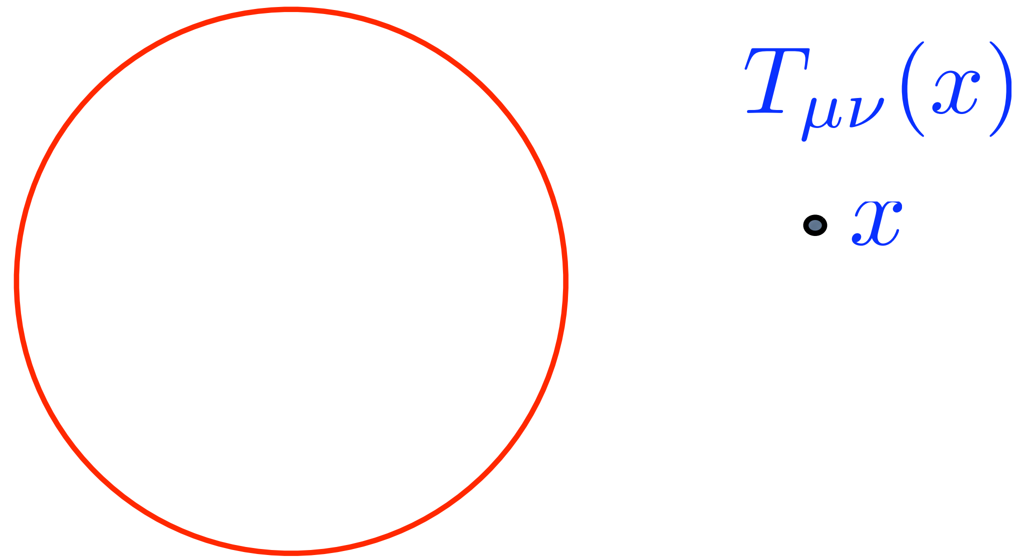


Since T_{0i} is the Poynting vector, this should give an alternative way to compute energy loss by radiation.

Actually, one needs to use an improved $\bar{T}_{0i}$.

Lewkowycz, Maldacena 13
Agón, Güijosa, Pedraza 14

Hyperbolic Wilson line: accelerated probe

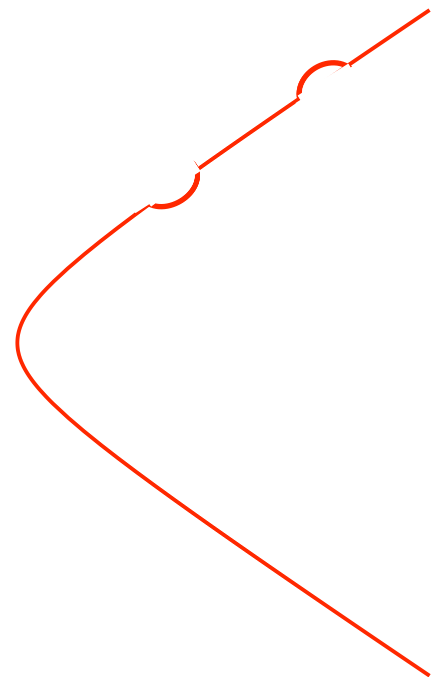


An alternative way to compute energy loss by radiation

$$P = 2\pi\, B(\lambda, N)\, a^{\mu} a_{\mu}$$

BF, Garolera, Lewkowycz 12

Hyperbolic Wilson line: accelerated probe



BF, Garolera, Torrents 13

$$<<\mathbb{D}_i(\tau)\mathbb{D}_j(0)>>= 12\ B(\lambda,N)\ \frac{\delta_{ij}}{16R^4\sinh^4(\frac{\tau}{2R})}$$

$$\kappa=\lim_{w\to 0}\int d\tau e^{iw\tau}<<\mathbb{D}_i(\tau)\mathbb{D}_j(0)>>= 16\pi^3 B(\lambda,N)\,T^3$$

Unruh temperature

Very pretty.... but can we actually compute $B(\lambda, N)$ for any probe in any CFT?

Yes! 1/2-BPS probe coupled to $\mathcal{N}=4$ SYM.

Computing the Bremsstrahlung function

- Conformal symmetry gives **constraints** on correlation functions, but it doesn't fix all coefficients.

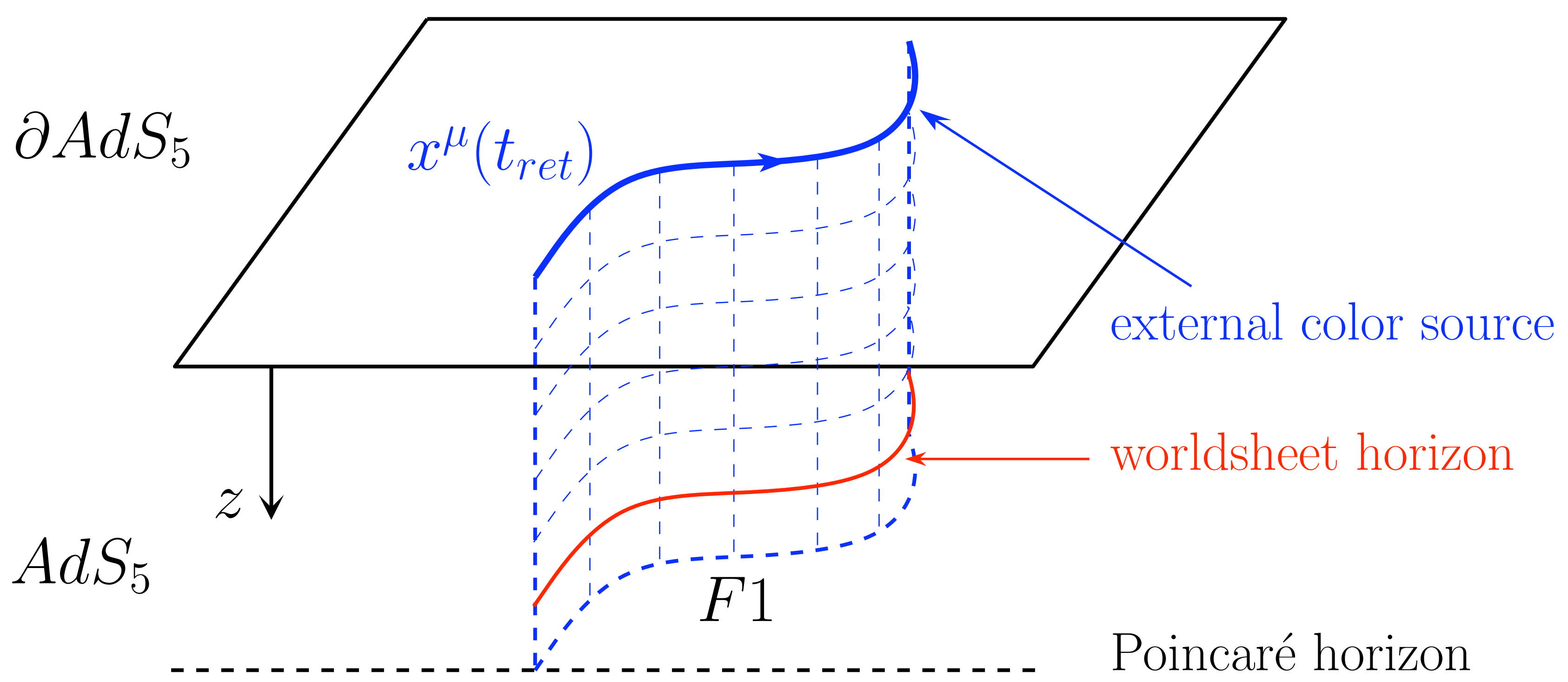
- We need **additional tools** to compute coefficients:

  - ☆ Pert. Theory (finite N, small λ)
  - ☆ AdS/CFT (large N, large λ)
  - ☆ Integrability (large N, finite λ)
  - ☆ Localization (finite N, finite λ)

External Probes in AdS/CFT

External Probes in AdS/CFT

Consider a particle in the fundamental representation. Its dual is a fundamental string, reaching the boundary of AdS at the particle word-line.



External Probes in AdS/CFT

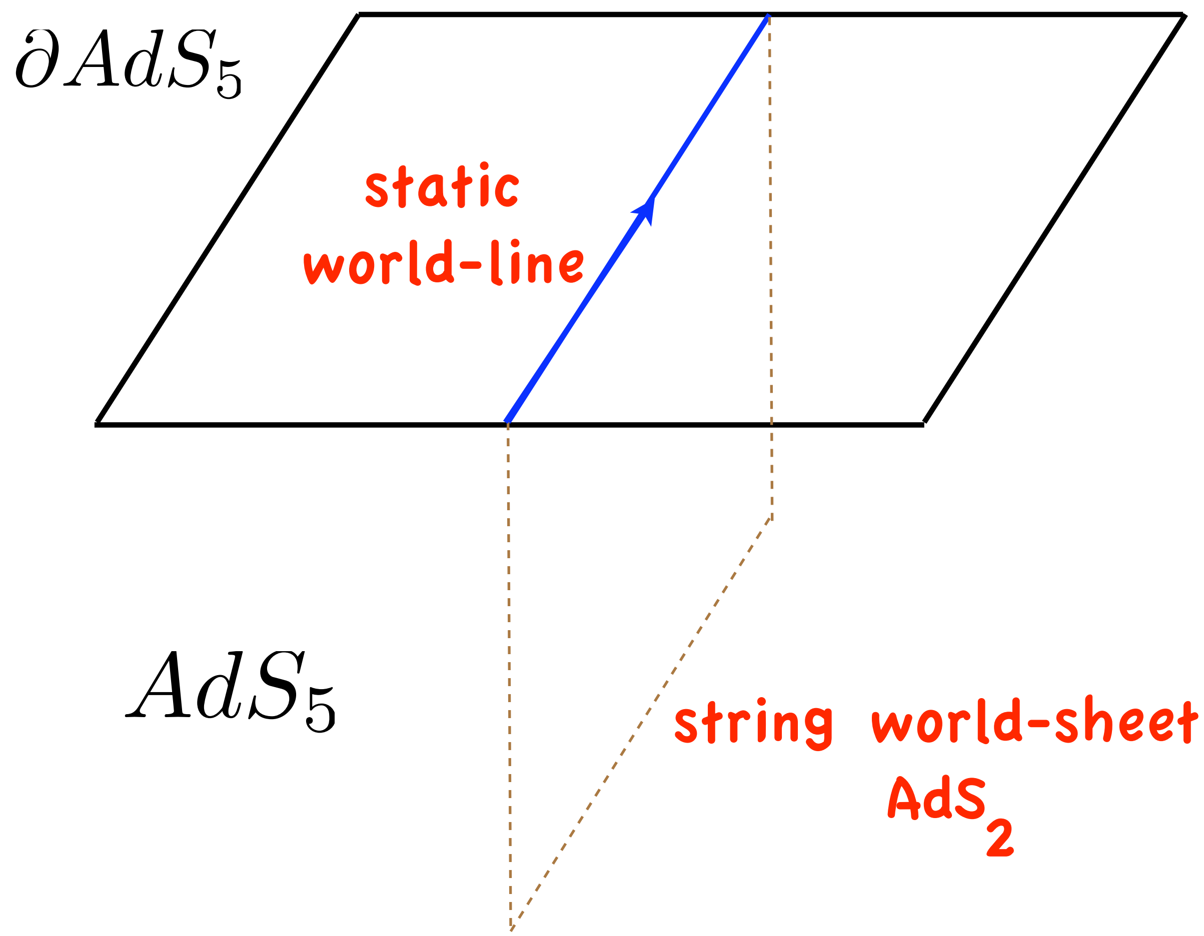
In the absence of other scales, the effective charge is

$$e^2_{\Box} \sim \sqrt{\lambda}$$

It signals screening of the charge at strong coupling.

$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{-|g|} = -\frac{\sqrt{\lambda}}{2\pi L^2}\int d^2\sigma\sqrt{-|g|}$$

First Example: static particle



$$\mathcal{L} = \frac{1}{2g_{YM}^2} \mathrm{Tr} \ \left(F^2 + [X_I, X_J][X^I, X^J] + \text{fermions} \ \right)$$

?

$$< \mathcal{L}(\vec{x}) >= f(\lambda, N) \frac{1}{|\vec{x}|^4}$$

First Example: static particle

The Lagrangian density is dual to the dilaton.

The string backreaction perturbs the AdS background.

$$\phi(x) = \int d^5x' G(x,x') J(x')$$

The dilaton was constant in the unperturbed solution $\Rightarrow$ its linearized perturbation decouples.

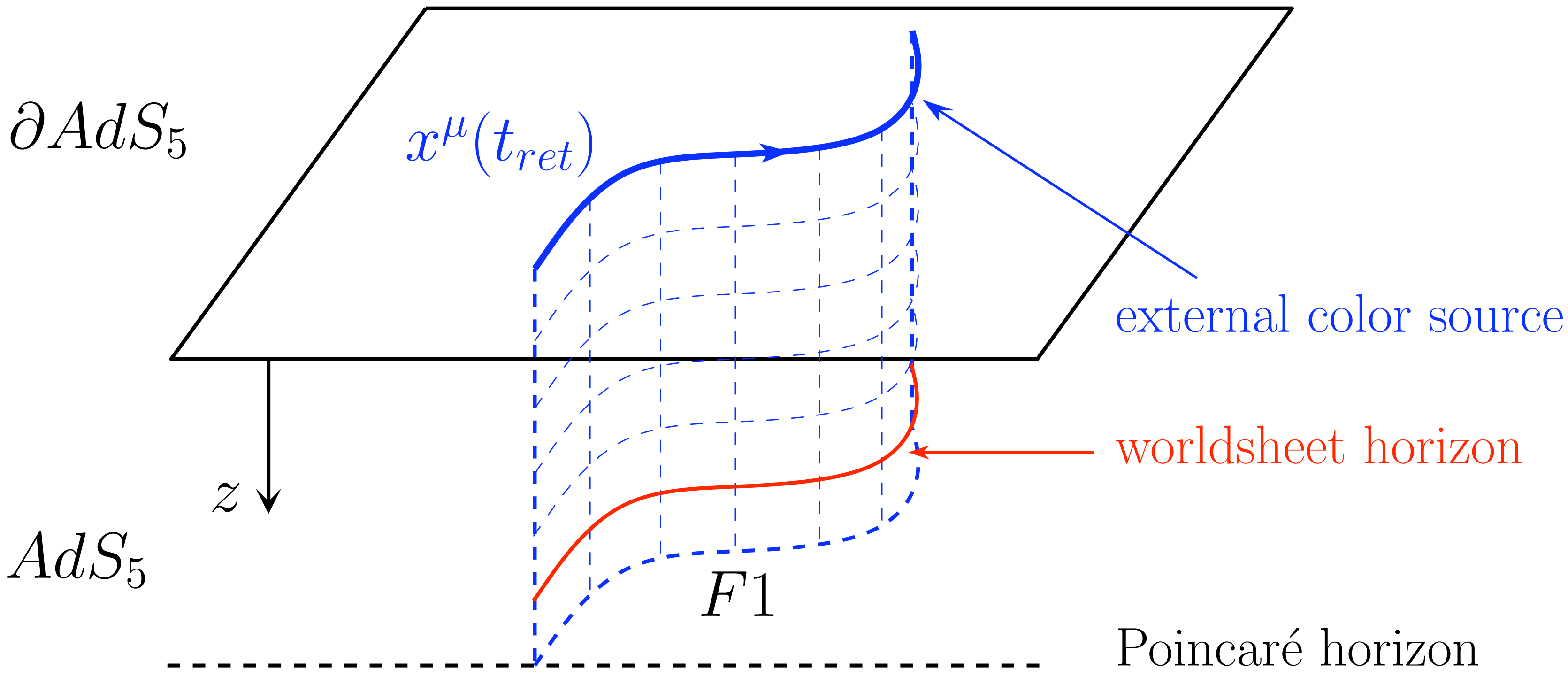
$$< \mathcal{L}(\vec{x}) > = \frac{1}{16\pi^2} \frac{\sqrt{\lambda}}{|\vec{x}|^4}$$

Danielsson, Keski-Vakkuri, Kruczenski
Callan, Güijosa

Second Example: accelerated particle

Mikhailov found the fundamental string dual to a particle following an arbitrary timelike trajectory.

Mikhailov 03

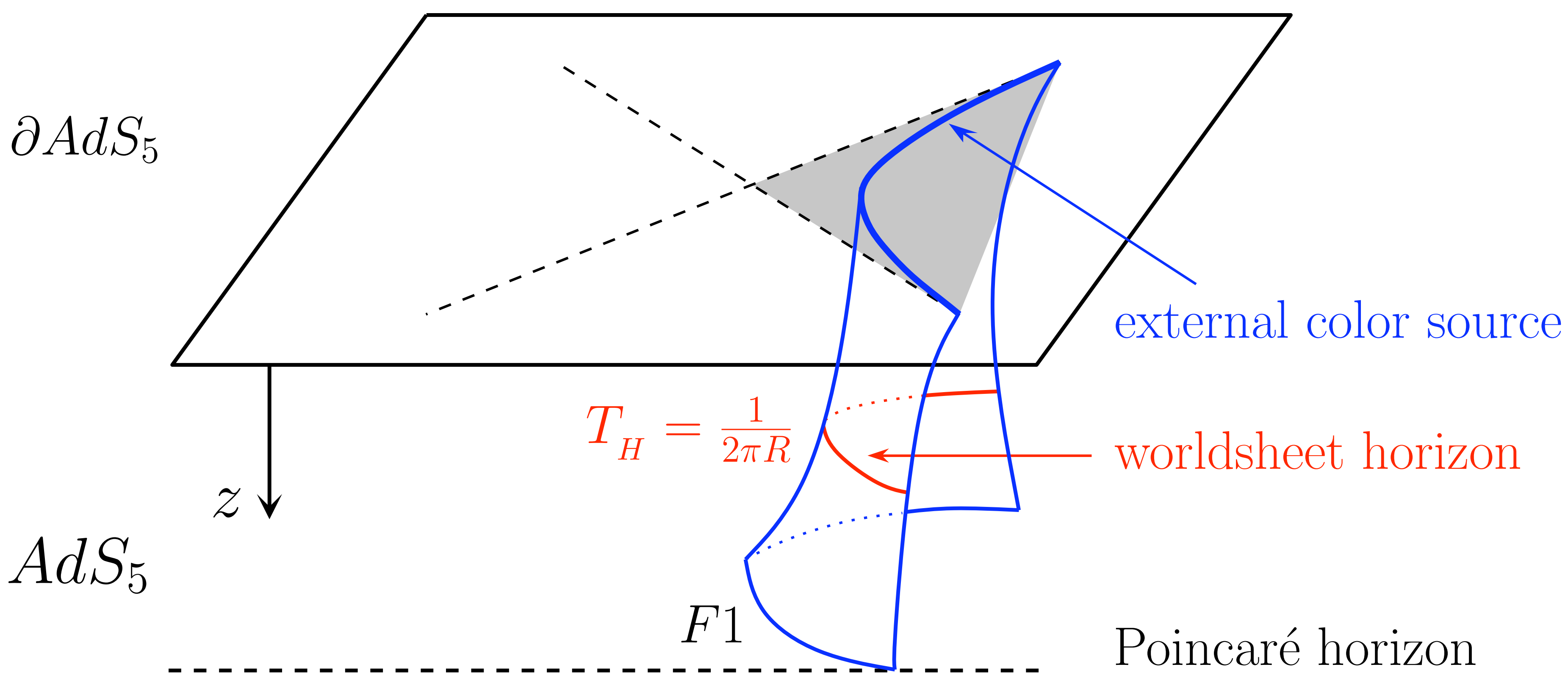


$$P = \frac{\sqrt{\lambda}}{2\pi} a^\mu a_\mu$$

Mikhailov 03

Second Example: accelerated particle

The world-sheet horizon splits the gluonic cloud into a Coulombic and a radiative part.



$$E = \int d\sigma\, \mathcal{E}$$

External Probes in AdS/CFT

Static Particle

$$< \mathcal{L}(\vec{x}) >= \frac{1}{16\pi^2} \frac{\sqrt{\lambda}}{|\vec{x}|^4}$$

Danielsson *et al.*
Callan, Güijosa 98

Accelerated particle

$$P = \frac{\sqrt{\lambda}}{2\pi} a^\mu a_\mu$$

Mikhailov 03

$$\kappa = 4\pi \sqrt{\lambda} \;\; T^3$$

Xiao 08

Circular Wilson loop

$$\ln \;\; < W_{\bigcirc} >= \sqrt{\lambda}$$

Berenstein *et al.* 98

$q\bar{q}$ Potential

$$V_{q\bar{q}} = -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)} \frac{\sqrt{\lambda}}{L}$$

Rey,Yee
Maldacena 98

External Probes in AdS/CFT

All these computations yield

$$\boxed{B(\lambda, N) = \frac{\sqrt{\lambda}}{4\pi^2}}$$

The $\sqrt{\lambda}$ in these results appears from evaluating classical string solutions to the NG action. There are two types of corrections:

$1/\sqrt{\lambda}$ **world-sheet fluctuations.**

Forste, Ghoshal, Theisen 99
Drukker, Gross,Tseytlin 00
....
Buchbinder, Tseytlin 13

$1/N$ **higher genus world-sheets.**

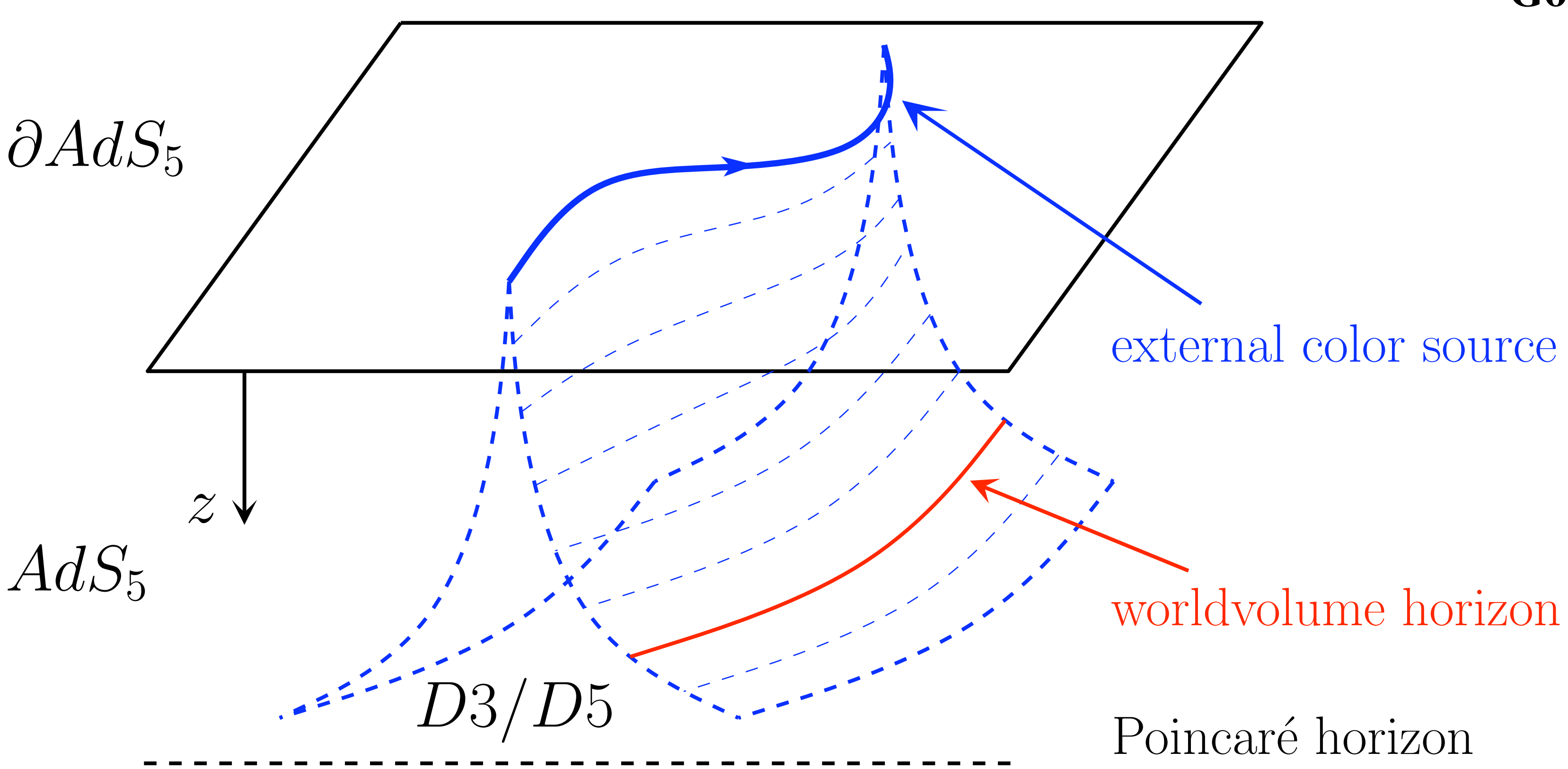
1/N Corrections with AdS/CFT

Probes in higher rank representations

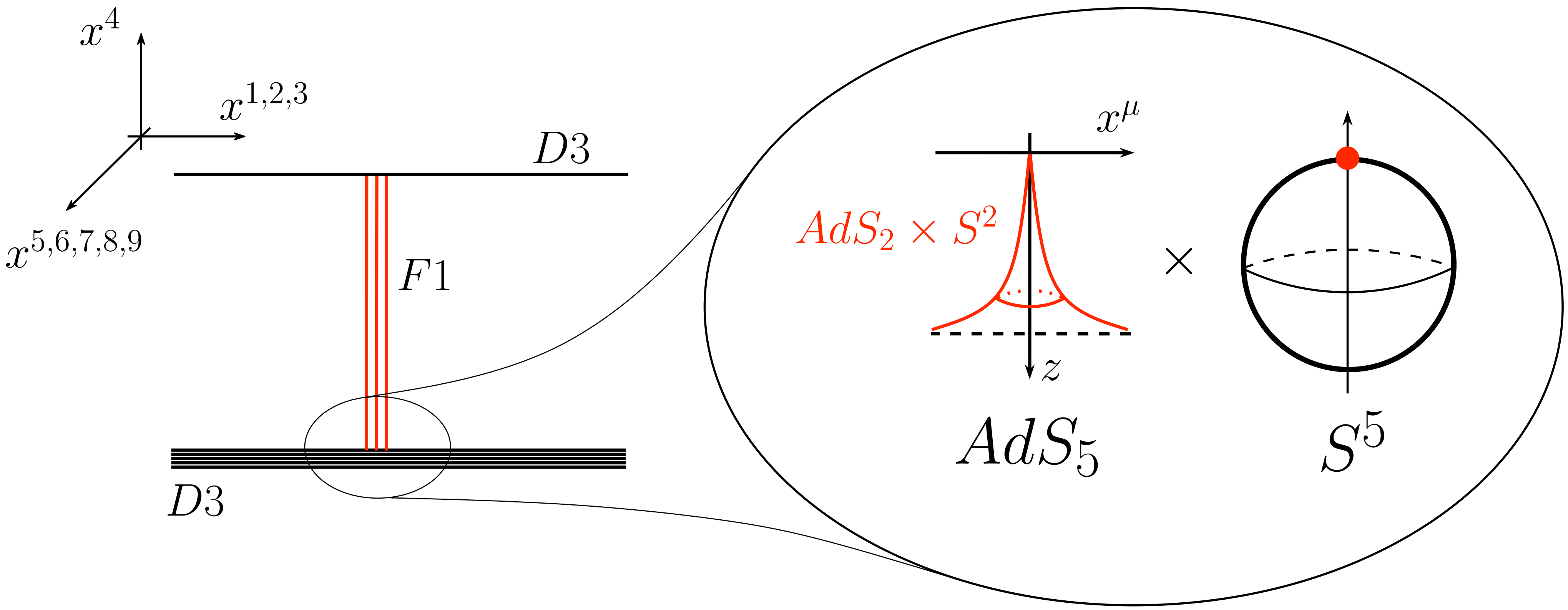
D3, k-units of flux ⇔ **k-symmetric rep.**

D5, k-units of flux ⇔ **k-antisymmetric rep.**

Hartnoll, Prem Kumar 06
Yamaguchi
Gomis, Passerini



First Example: static particle



Drukker, BF 05

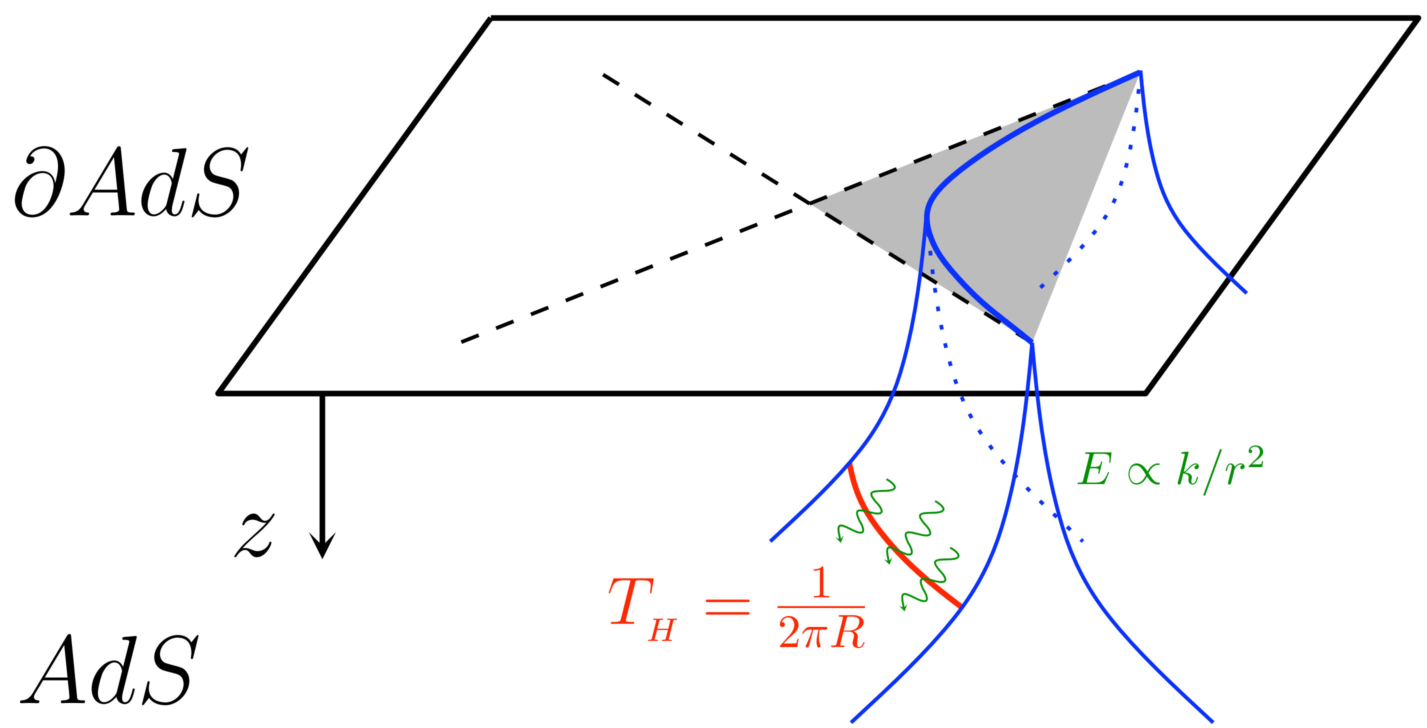
Again, the D3-brane backreacts on the AdS solution. From the linearized perturbations for the dilaton,

$$< \mathcal{L}(\vec{x}) >_{S_k} = \frac{k\sqrt{\lambda}\sqrt{1+\frac{k^2\lambda}{16N^2}}}{16\pi^2} \frac{1}{|\vec{x}|^4}$$

BF, Garolera, Lewkowycz 12

Second Example: accelerated particle

We can use a hyperbolic D3-brane to evaluate the energy loss by radiation in hyperbolic motion.



$$P = \frac{k\sqrt{\lambda}}{2\pi}\sqrt{1+\frac{k^2\lambda}{16N^2}}\quad a^\mu a_\mu$$

BF, Garolera 11

Probes in k-symmetric representation

Static Particle

$$< \mathcal{L}(\vec{x}) >_{S_k} = \frac{k\sqrt{\lambda}}{16\pi^2}\sqrt{1+\frac{k^2\lambda}{16N^2}}\,\frac{1}{|\vec{x}|^4}$$

BF, Garolera, Lewkowycz 12

Accelerated Particle

$$P = \frac{k\sqrt{\lambda}}{2\pi}\sqrt{1+\frac{k^2\lambda}{16N^2}}\;a^\mu a_\mu$$

BF, Garolera 11

$$\kappa = 4\pi k\sqrt{\lambda}\sqrt{1+\frac{k^2\lambda}{16N^2}}\;T^3$$

BF, Garolera, Torrents 13

Circular Wilson loop

$$\ln\ < W(\bigcirc) > = \frac{k\sqrt{\lambda}}{2}\sqrt{1+\frac{k^2\lambda}{16N^2}} + 2N\sinh^{-1}\frac{k\sqrt{\lambda}}{4N}$$

Drukker, BF 05

Probes in k-symmetric representation

All these computations yield

$$\boxed{B(\lambda, N) = \frac{k\sqrt{\lambda}}{4\pi^2}\sqrt{1+\frac{k^2\lambda}{16N^2}}}$$

A priori, not justified to trust this result for k=1.

$$\underbrace{\frac{N^2}{\lambda^2} >> k}_{\text{probe approx.}} \qquad \overbrace{k >> \frac{N}{\lambda^{3/4}}}^{\text{SUGRA approx.}}$$

Probes in k-antisymmetric representation

Universal result:

String world-sheet $\Sigma \hookrightarrow M$ (M Ricci flat: AdS, S-AdS,...)

D5 world-volume $\Sigma \times S^4 \hookrightarrow M \times S^5$ **Hartnoll 06**

This amounts to $$\sqrt{\lambda} \to \frac{2N}{3\pi} \sin^3 \theta_k \sqrt{\lambda}$$

where $$\sin\theta_k \cos\theta_k - \theta_k = \pi\left(\frac{k}{N} - 1\right)$$

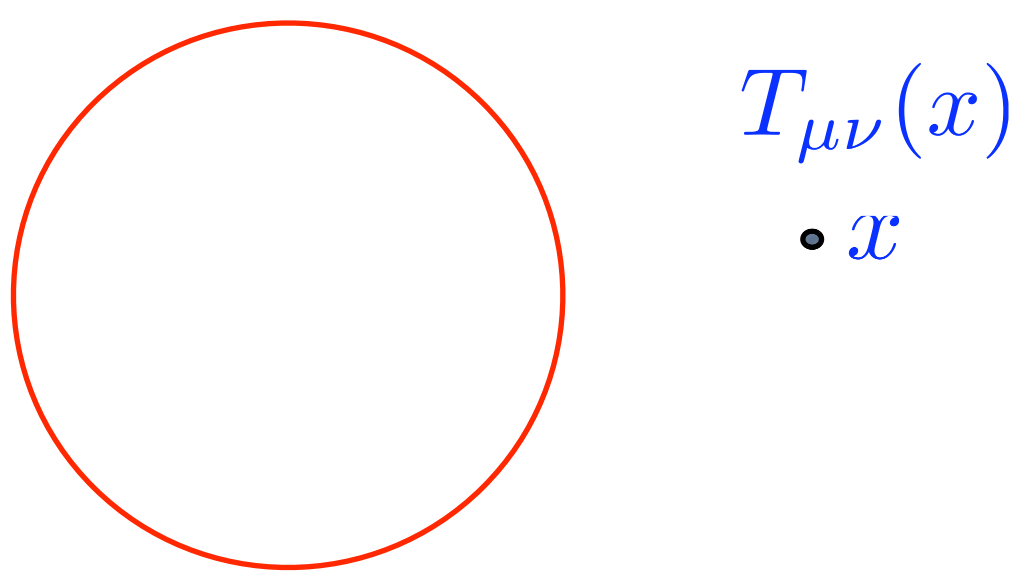
Valid even at finite temperature ! (e.g. drag force) **Chernicoff, Güijosa 06**

Exact Results for External Probes

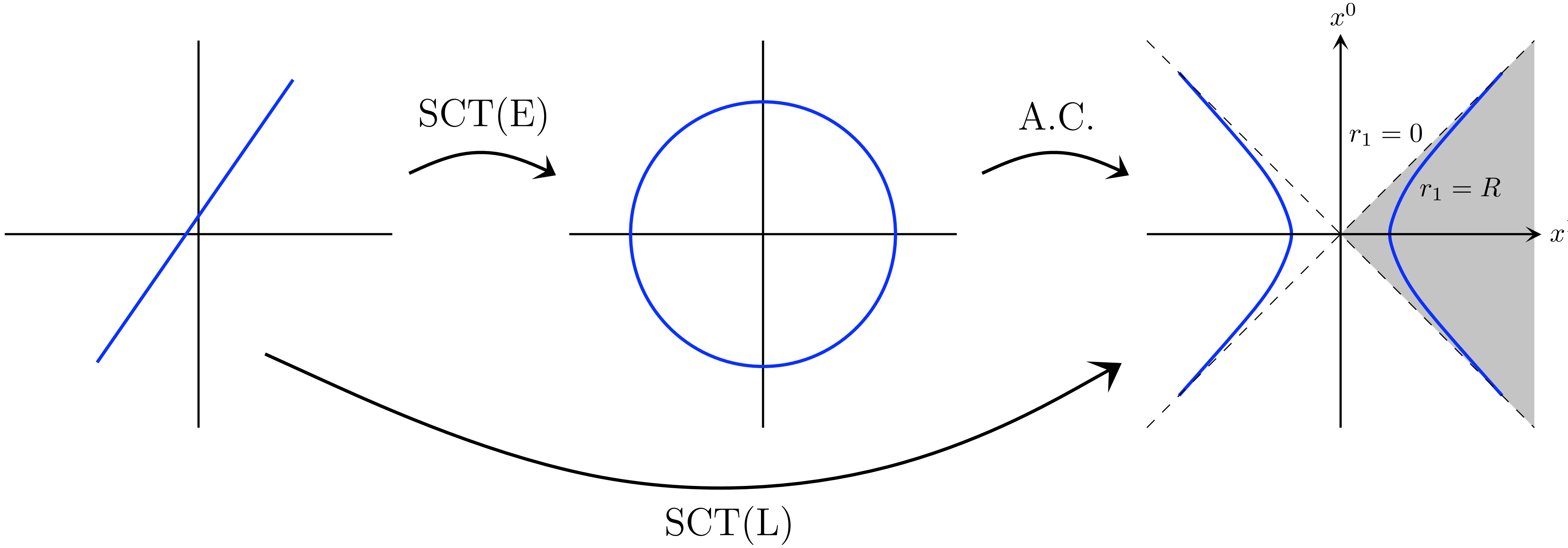
An exact Bremsstrahlung function

We will derive the Bremsstrahlung function for an electric 1/2 BPS probe in the fundamental rep. of N=4 U(N) SYM.

Our strategy: compute $< T_{\mu\nu} >_W = \dfrac{< T_{\mu\nu}(x) W_{\bigcirc} >}{< W_{\bigcirc} >}$



Start with <W>. Recall the Special Conformal Transformation,



$$< W_{|} >= 1 \qquad < W_{\bigcirc} >\neq 1$$

Conformal Anomaly !

The anomaly is localized at a point in space-time $\Rightarrow$

It is perturbatively captured by a matrix model.

$$< W_{\bigcirc} >= \frac{1}{N} L^{1}_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\frac{\lambda}{8N}}$$

Erickson, Semenoff, Zarembo 00

Drukker, Gross 00

Using localization techniques, Pestun proved the result to be correct, and exact.

Pestun 07

What about $< T_{\mu\nu}(x) W_{\bigcirc} >$?

In $\mathcal{N}=4$ SYM, the Lagrangian density and the stress energy tensor belong to a short multiplet, the supercurrent multiplet.

$$\mathcal{O}_2 = \mathrm{Tr}\,\left(\Phi^{\{I}\Phi^{J\}}\right) \qquad T_{\mu\nu} \sim Q^2\bar{Q}^2\mathcal{O}_2 \qquad \mathcal{L} \sim Q^4\mathcal{O}_2$$

$< W_{\bigcirc}\mathcal{O}_2(x) >$ **is computed with a normal matrix model**

Okuyama, Semenoff 06

$$< W(\bigcirc)_{\Box}\mathcal{O}_2 >= \frac{\sqrt{2}\lambda}{4N^3}\left[L^2_{N-1}\left(-\frac{\lambda}{4N}\right) + L^2_{N-2}\left(-\frac{\lambda}{4N}\right)\right]e^{\frac{\lambda}{8N}}$$

and finally we arrive at the **Bremsstrahlung function** for an **electric** 1/2 BPS probe in the **fundamental** rep. of N=4 U(N) SYM,

$$\boxed{B_{U(N)}(\lambda, N) = \frac{\lambda}{16\pi^2 N} \frac{L_{N-1}^2\left(-\frac{\lambda}{4N}\right) + L_{N-2}^2\left(-\frac{\lambda}{4N}\right)}{L_{N-1}^1\left(-\frac{\lambda}{4N}\right)}}$$

It is a **rational** function (why?)

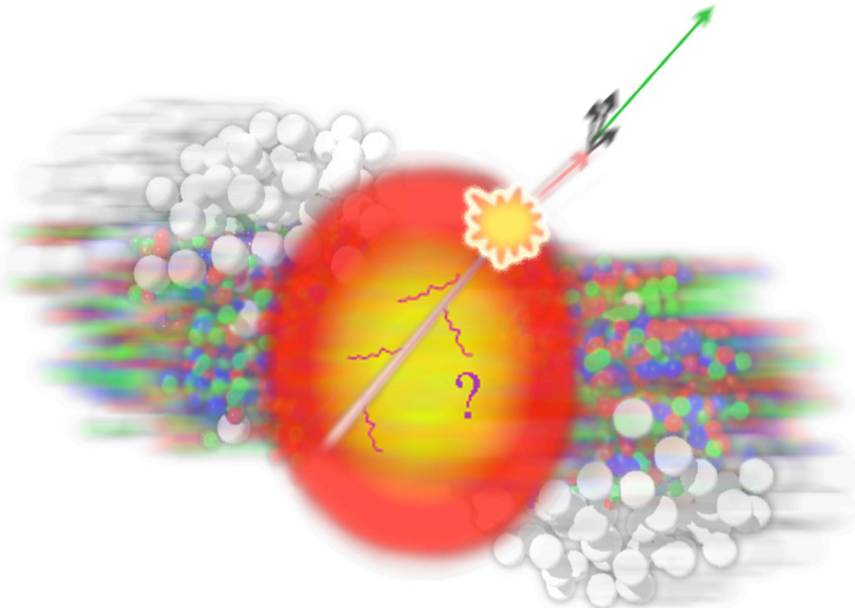
Equivalently,

$$B = \frac{1}{2\pi^2}\lambda\partial_\lambda \log \;< W_\bigcirc >$$

Correa, Henn, Maldacena, Sever 12

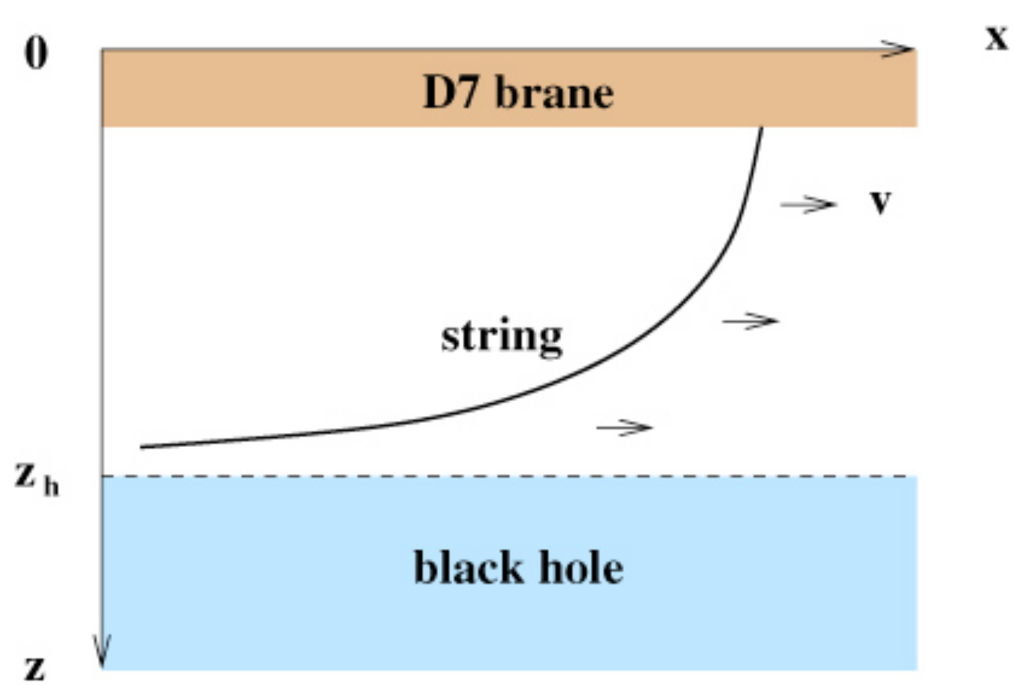
An application:
A benchmark for transport coefficients

Momentum broadening in Q.G.P.



$$\kappa = g(\lambda, N)T^3$$

Modelling QGP by N=4 SYM : trailing string



Herzog *et al.* 06
Casalderrey-Solana, Teaney 06
Gubser 06

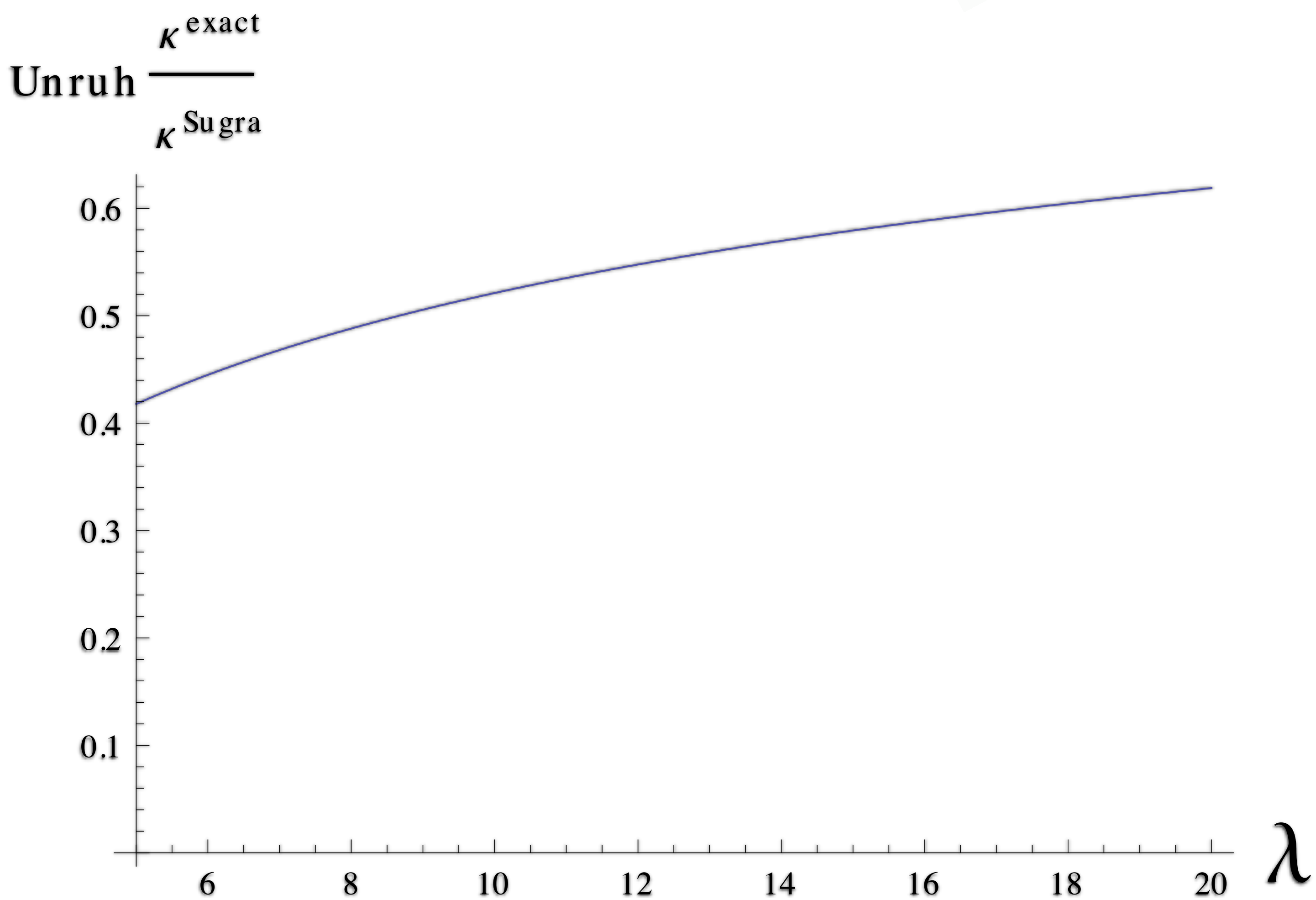
SUGRA $\Rightarrow$ $\kappa = \pi\sqrt{\lambda}T^3$

SUGRA vs. exact results

SU(3), finite λ $$\kappa = 4\pi \frac{\lambda}{18} \frac{\lambda^2 + 144\lambda + 3456}{\lambda^2 + 72\lambda + 864} T^3$$

Large N, large λ $$\kappa = 4\pi\sqrt{\lambda}\, T^3$$

Unruh temperature



QGP-modelling range

Roughly, in this range

$$\boxed{\kappa^{\mathbf{SUGRA}}_{\mathbf{Unruh}} \approx 2\,\kappa^{\mathbf{Exact}}_{\mathbf{Unruh}}}$$

IF the same were true for $\kappa_{\mathbf{thermal}}$

$$\boxed{\mathbf{D}^{\mathbf{Exact}}_{\mathbf{thermal}} \approx 2\,\mathbf{D}^{\mathbf{SUGRA}}_{\mathbf{thermal}}}$$

push towards QGP value ...

Higher rank reps.

Localization works for arbitrary gauge groups and representations.

For instance, consider antisymmetric rep. of U(N).

BF, Torrents 13

Define $g = \frac{\lambda}{4N}$, $A_{ij}(g) = L_i^{j-i}(-g)e^{g/2}$

We find

$$< F_A(t) >= \sum_t t^{N-k} < W_{A_k}(g) >= |t + A(g)|$$

$$< W_{A_k}(g) >= e^{\frac{kg}{2}} \sum_{j=0}^{k(N-k)} d_j \frac{g^j}{j!} \qquad d_j \in \mathbb{N}$$

Back to String Theory...

BF, Garolera, Torrents
work in adiabatic progress

From Localization to String Theory

We can compute exactly certain quantities in CFT. Can we use them to learn about string theory beyond the SUGRA regime?

"Ask not what the world-sheet can do for you; ask what you can do for the world-sheet."

(Understandably) Anonymous

From Localization to String Theory

N=4 SYM
G=SO(N),Sp(N)

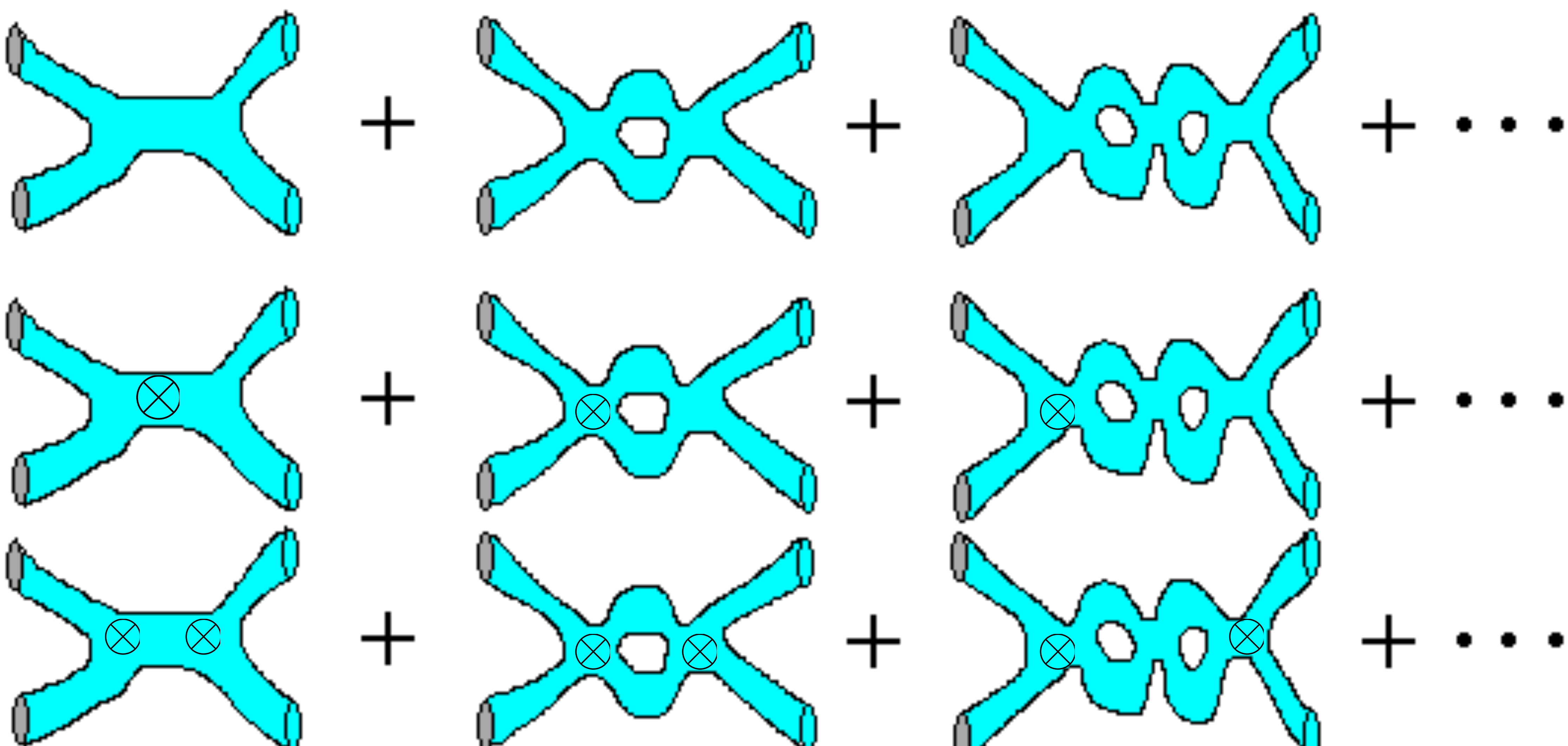
$\Longleftrightarrow$

IIB string
$AdS_5 \times RP^5$

Witten 98

Novel feature: 1/N expansion involves non-orientable surfaces.

Cicuta 82

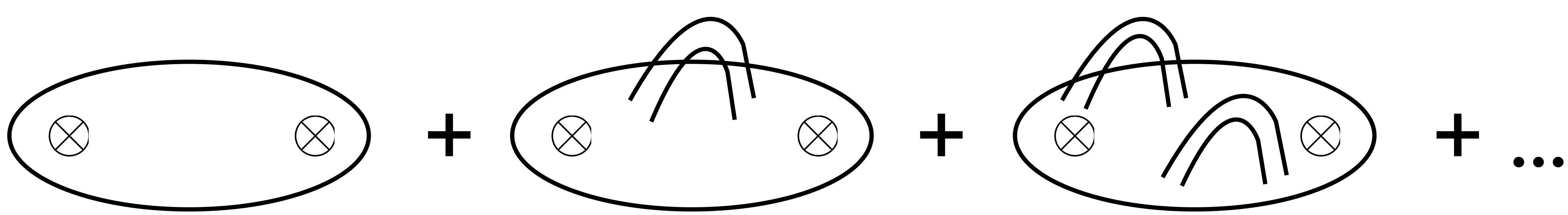


From Localization to String Theory

Exact relation among vevs:

$$< W(g) >_{\substack{SO(2N) \\ Sp(N)}} = < W(g) >_{U(2N)} \mp \frac{1}{2} \int_0^g dg' \; < W(g') >_{U(2N)}$$

World-sheets with two crosscaps don't contribute:



= 0 + 0 + 0 + ...

Prediction for string theory !

Conclusions and Outlook

The Bremsstrahlung function, the small angle limit of the cusp anomalous dimension, determines many properties of heavy probes coupled to CFTs.

Thanks to localization, the Bremsstrahlung functions of probes in various reps. of N=4 SU(N) SYM **can be determined exactly via matrix model computations.**

In the regime of validity of SUGRA, these results reduce to functions of $\sqrt{\lambda}/N$ **, and D-brane probe computations capture them precisely.**

Conclusions and Outlook

- **Compute the full cusp anomalous dimension.**
- **Compute** $B(\lambda, N)$ **for other probes/CFTs.**
- **Finite mass ? (OK with localization).**

Adding Flavor

Karch, Katz 02

Schwinger effect and critical electric field

$$E_c = \frac{2\pi m^2}{\sqrt{\lambda}}$$

Semenoff, Zarembo 11

- **Beyond the vacuum state... Finite** μ **, T ?**